Developing stem taper model for Shorea robusta in far-western Terai of Nepal

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Received: 2, November 2023 Revised: 21, January 2024 Accepted: 4, February 2024 Published: 26, February 2024

Tree taper functions expressed in terms of height and diameter at breast height (DBH) provide essential information for precise estimation of current growing stock. Taper models play a crucial role in calculating timber volumes in forest inventories. However, such models are still unavailable at the required level in Nepal. This study aimed to develop taper equations for *Shorea robusta*, enabling predictions of diameter anywhere along the stem and estimation of tree volumes at desired sections. A destructive sampling method was employed, involving 81 sample trees of *S. robusta* across ten locations within Kailali and Kanchanpur districts in the far-western Terai region of Nepal. The trees were felled for measurement of upper stem diameters. Two independent models - B-spline and 5th degree polynomial - were used to predict upper stem diameters. Both models were applied to the entire dataset, irrespective of DBH, to create common fitted taper models. Subsequently, these models were tested for three DBH classes to compare and identify the best fitting model. The B-spline polynomial taper models exhibited a strong dependency on tree DBH size. Hence, improved B-spline models were derived by classifying the dataset based on DBH. Conversely, the 5th degree polynomial model showed no DBH size dependency, offering a better fit for the unclassified dataset encompassing all DBH ranges.

Keywords: *B-Spline,* DBH, destructive sampling, polynomial, stem taper model, taper

Accurate and up-to-date information on current growing stock and its future growth potential is crucial for informed and sustainable forest management strategies. This information allows forest managers to make data-driven decisions regarding harvesting, conservation, and resource allocation. Timely and precise data on current growing stock can be efficiently obtained through taper functions, which express the relationship between tree diameter and height (Muhairwe, 1999). As such, stem taper functions are considered fundamental inputs for forest planning and management at all levels (Kublin et al., 2013; Heidarsson & Pukkala, 2011).

While form factors provide a general idea of a tree's shape, they fail to capture the crucial detail of how diameter changes along the stem as height increases. This dynamic relationship is precisely what taper equations excel at representing. Taper equations offer a predictive tool for estimating diameters at any desired point along the trunk, unlocking essential information for various forestry applications. Modeling stem taper thus becomes a cornerstone for deriving upper stem diameters and calculating stem volumes at different heights - both vital for accurate resource assessments and sustainable forest management practices.

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The stem taper represents the rate of change (decrease) in stem diameter with increasing tree height (Kohler et al., 2016). This change, captured by taper equations, holds significance for both the current state and future potential of forest resources. Various methods have been developed for constructing taper equations, with past studies often focusing on softwood species like pine and spruce (Max & Burkhart, 1976; Fang et al., 2000). However, valuable hardwood species like *Shorea robusta* have received considerably less attention, despite their significant economic and ecological value in regions such as Nepal's Terai and Siwalik where *Shorea robusta* is abundant (DFRS, 2014). This multipurpose tree can reach impressive heights of 45–50 m and constitutes a primary timber source for the Nepalese market (Jackson, 1994).

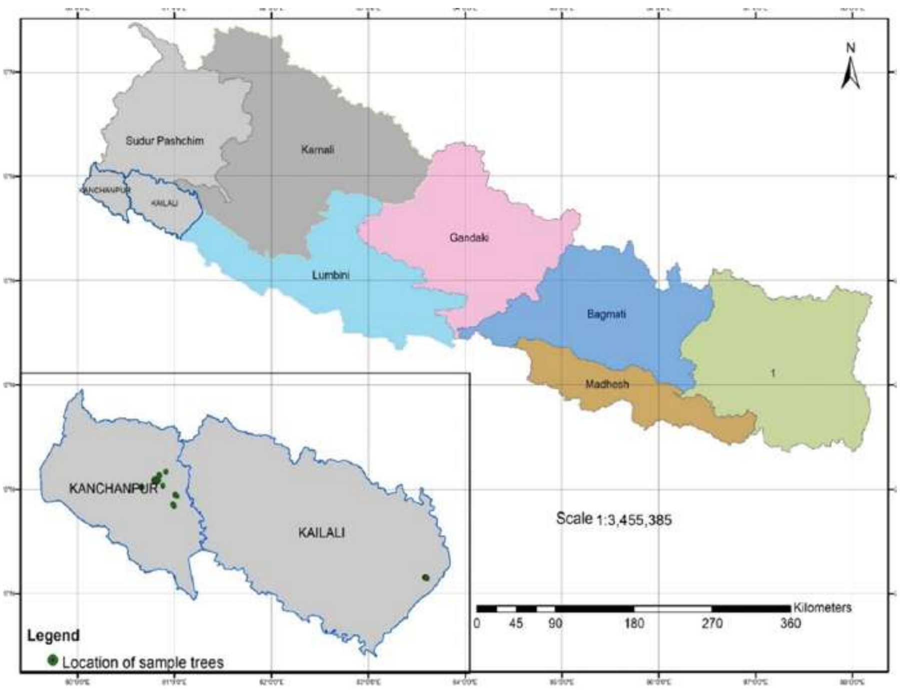

Figure 1: Location of the sample sites in far-western Terai region of Nepal

Despite its crucial role, information on volume and taper functions for *Shorea robusta* in Nepal remains scarce. Consequently, national forest resource assessments have had to rely on rough volume estimates derived from polynomial taper equations developed using data from exotic tree plantations in Zambia - a far cry from the specific context of Nepal's native *Shorea robusta* populations (Heinonen et al., 1996). This highlights the critical need for locally relevant taper equations tailored for *Shorea robusta* in Nepal. The present study aims to address this gap by developing robust and localized taper equations specifically for *Shorea robusta* in Nepal to empower forest managers with the ability to reliably predict tree volumes and diameters.

Materials and methods

Study area

The data were collected from ten different sites located in the far western Terai region of Nepal, specifically within Kailali and Kanchanpur districts of Sudurpaschhim Province (Figure 1). The study was conducted in the year 2022. The study sites are located between 28.8314°N and 28.8372°N and between 80.8987°E and 80.3213°E. This region, extending from the Karnali River in the east to the country's western border, holds 97,622 hectares of forests outside protected areas (DFRS, 2014). The elevation of the study area ranges from 109 m to 200 m above mean sea level (MSL). The climate varies from sub-tropical to tropical, with an annual precipitation of 1130 mm to 2680 mm (DFRS, 2014). Summers are hot, with peak temperatures exceeding 40°C, while winters are dry with lows below 15°C. The native forest type in the region is characterized as *Terai mixed hardwood*, dominated by *Shorea robusta*, which is the most common species in terms of basal area.

Data collection

The data collection was done to prepare local volume tables for *S. robusta* in 2018. The data consisted of 81 sample trees with different DBH (over-bark) classes (Figure 2). A destructive sampling approach was adopted to measure the data. The recorded tree characteristics included the tree height, DBH, crown height, location, sectional diameters, and height from base to diameter measuring points at several sections of the stem.

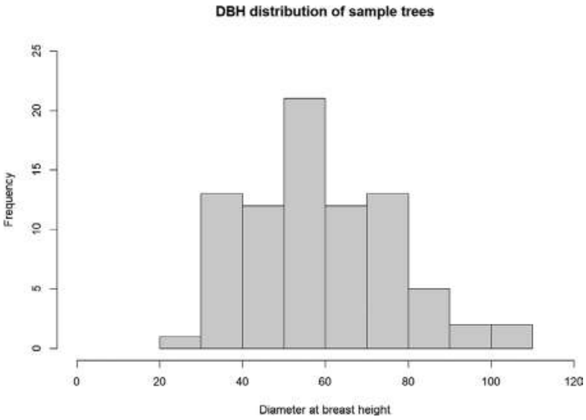

Figure 2: Histogram showing the distribution of DBH (cm) of the sampled trees

After felling of sample trees, the first three over-bark diameter measurements were done at 0.5m intervals at the lowermost sections. The upper stem diameters (over-bark) were measured at 0.3m, 0.8m, 1.3m, 1.8m, 2.8m, 4.0m, and at every 2m (Sharma & Pukkala , 1990; Eerikäinen, 2001) up to the tip of the trees from their base (Figure 3) using a diameter tape at 0.1cm accuracy (Subedi, 2017).

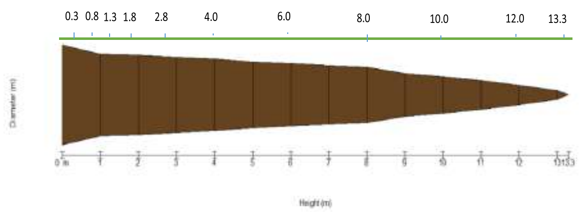

Figure 3: Diameter measurement points along the tree stem

All the trees with serious defects and abnormalities were excluded from the sampling frame.

Observation of tree stem diameters (cm) at different heights (m) from its base for all the sampled trees is presented in Figure 4.

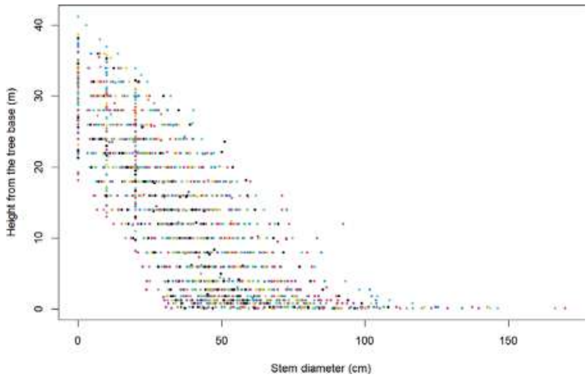

Fig. 4: Observation of stem diameters (cm) at different heights from the base (m); Different colours stand for different sample trees, i.e. same coloured downward inclined dotted lines represent unique sample trees.

Data analysis and tools

Stem taper functions were used to model the relative decrease in upper stem diameters (D_x) with an increase in relative heights (H_x). The diameter at breast height (DBH), due to its easy-to-access property, has been taken as the relative value to derive proportional decrease in D_x which were modelled as a function of the Hx. The following two independent modelling approaches were followed to develop the stem taper equations:

i. B-spline cubic polynomial model

Spline interpolation is a common mathematical approach of generating a set of new points within the boundaries of known points. These new points are function values of an interpolation function (referred to as spline), consisting of multiple cubic piece-wise polynomials (Fornberg & Zuev, 2007). Cubic spline has a continuous second derivative while quadratic spline only has a continuous first derivative. Thus, cubic spline being smoother, was chosen (Equation 1).

$$y = a + bx + cx^2 + dx^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where,

y is dependent variable, i.e. upper stem diameter (cm); x is independent variable, i.e. ratio of upper stem diameter and DBH; and a, b, c, and d are equation parameters.

ii. Polynomial 5th degree

Polynomial 5th degree modelling approach was based on the functions implemented in rForest Package that allowed to fit the following taper model (Equation 2) along with plot visualization in 2D and 3D (Silva, 2021):

$di/dbh=(hi/ht)+(hi/ht)^2+(hi/ht)^3+(hi/ht)^4+(hi/ht)^5$ ……………… (2)

Where,

d_i/dbh is the ratio of upper stem diameter and tree DBH;

hi is the height of tree from its base at the point of sectional diameter measurement

ht is the total height of tree

Two independent taper models namely ***B-spline*** and ***polynomial 5th-degree*** were initially tested for the trees with different DBH classes. Later, both the models were tested for three different DBH classes: i) <55 cm, ii) 55–70 cm, & iii) >75 cm, and the model outputs were compared to find the best-fit model. Several R packages such as splines (R Core Team, 2022), rForest (Silva *et al.*, 2021), tidyverse (Wickham *et al.*, 2019), and ggplot2 (Wickham, 2016) were used for the purpose of data analysis and visualization.

Results

i) B-spline cubic polynomial

The B-spline cubic polynomial taper models developed independently for three different DBH classes: i) <55 cm, ii) 55–70 cm, & iii) >70 cm were found to have lesser standard errors of 6.25, 5.48, and 8.87 respectively with higher adjusted R^2 of 0.84, 0.90, and 0.96 respectively (Table 1). On the other hand, a single taper model developed for all DBH sizes had a lower adjusted R^2 of 0.78 and a greater standard error of 12.85 as compared to the former ones (Figure 5).

Table 1: Estimated parameters of B-spline cubic polynomial taper models

S.N.	DBH class (cm)	a	b	c	d	SE (residuals)	Adj. R^2
1.	< 55	50.3	−25.4	−19.1	−51.0	6.25	0.84
2.	55–70	75.0	−40.3	−31.6	−76.8	5.48	0.90
3.	> 70	82.3	−37.6	−36.7	−84.2	8.87	0.96
All DBH classes		64.5	−32.7	−26.8	−65.7	12.85	0.78

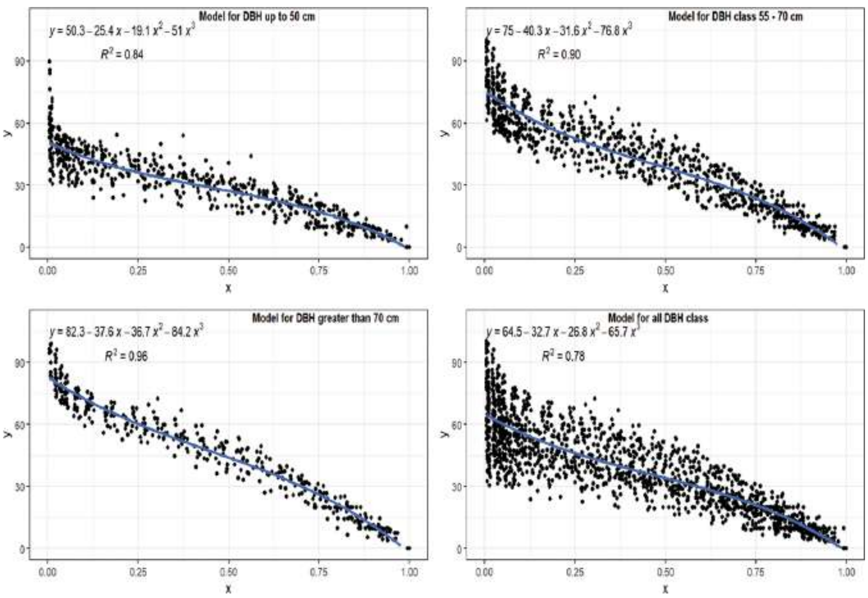

Figure 5: B-spline cubic polynomial taper models

ii) 5th degree polynomial

The model built for the three DBH classes i) <55 cm, ii) 55–70 cm, & iii) >70 cm predicted a large deviation from the observed value, although the outcome varied for each of the three DBH classes. However, the standard errors for each model parameter of the model developed for all DBH sizes were smaller than that were estimated for the former ones (Figure 6).

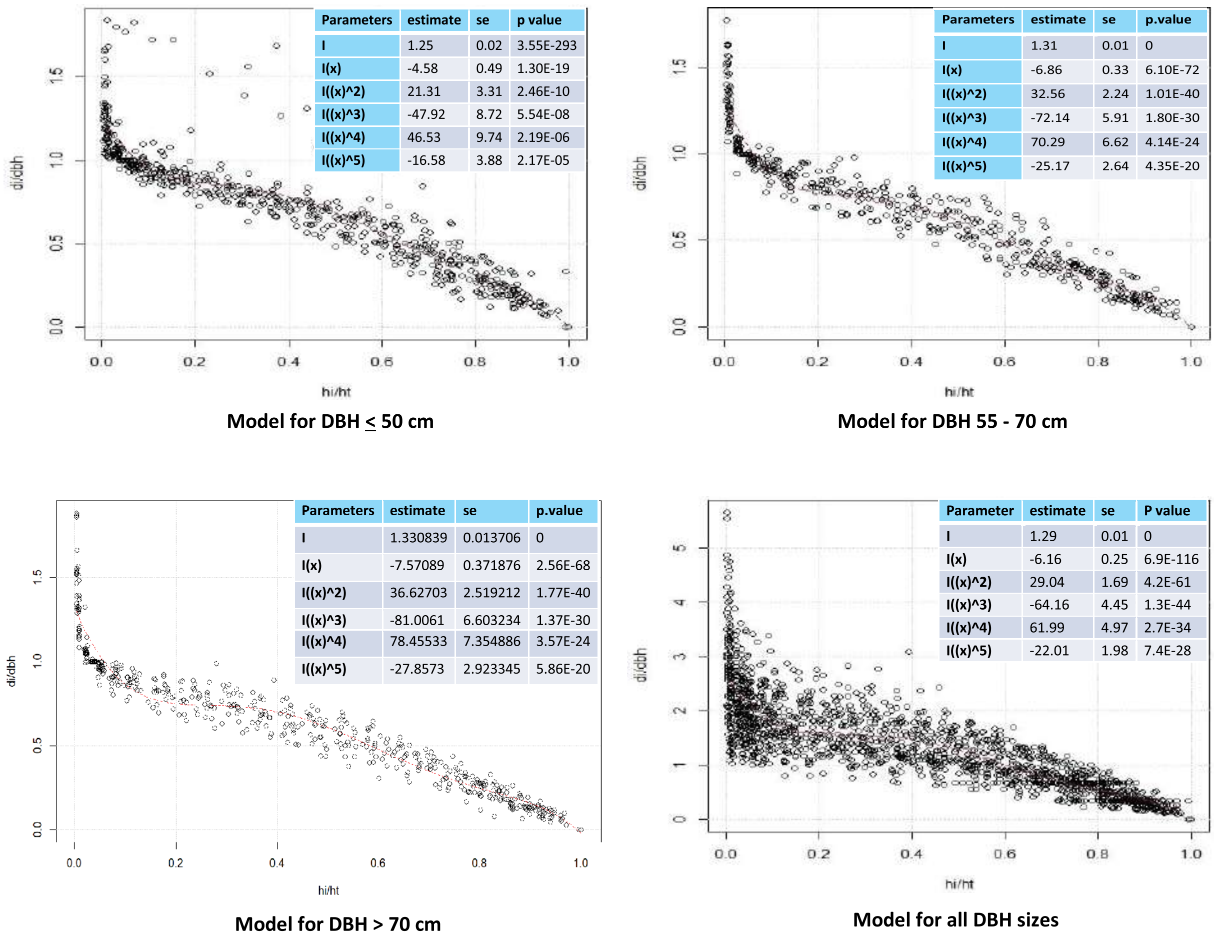

Figure 6: 5th degree polynomial taper models

Discussion

Taper equations of *Eucalyptus* species in New south wales revealed that changes in diameter and height growth along the stem over time contribute to taper variations between individual trees (Muhairwe, 1999). Several factors influence this variability, including intrinsic characteristics like genetics and extrinsic factors like climate change, site quality, tree and stand age, crown size, canopy position, defoliation, species, and stand density. A single "universal" taper model is difficult because of the intricate interactions between these components (McClure & Czaplewski, 1986). This emphasizes the requirement for species-specific and context-sensitive models tailored to accurately represent the unique growth patterns of different tree species.

This study investigated the influence of diameter at breast height (DBH) classes on the development of taper models for *Shorea robusta*. Our findings show that considering DBH classes is crucial for achieving optimal model accuracy, particularly when using the B-spline cubic polynomial function. Models developed for individual DBH classes significantly outperformed the single model built for all sizes combined, confirming the importance of accounting for inherent variations within different size groups. This finding aligns with prior research emphasizing the benefits of fitting statistics and size-specific approaches for improved model selection (Kozak & Kozak, 2003; Bellocchi et al., 2010). Moreover, our approach implicitly incorporates independent tree accuracy checks as recommended by Ducey & Williams (2011), as each DBH class represents a distinct size group.

The study compared two contrasting methodologies for developing taper models for *Shorea robusta*. Firstly, the available data was grouped into three DBH classes (small, medium, large) based on established classification systems in the region. Separate taper models were then developed for each class using both B-spline cubic polynomial and 5th degree polynomial functions. In addition, models were developed using all data points irrespective of DBH size for both functions, providing a broader representation of tree variability. The findings showcase the effectiveness of the classification by DBH approach for the B-spline cubic polynomial model. Models specific to individual DBH classes exhibited significantly better fits compared to the model built for all sizes combined. This aligns with recommendations by Kozak & Kozak (2003) and Bellocchi et al. (2010) who emphasize the importance of fitting statistics and graphical evaluations in model selection. Additionally, by segmenting data based on size classes, our approach implicitly incorporates the recommendations of Ducey & Williams (2011) for independent tree accuracy checks, as different DBH classes represent distinct tree size groups.

The success of the B-spline model under the classified approach can be attributed to its inherent flexibility. Splines offer a balance between adaptability and complexity, effectively capturing the dynamic changes in stem form observed across different size classes, particularly in hardwoods like *Shorea robusta* prone to stump flare (Kublin et al., 2013). Furthermore, cubic spline interpolation minimizes oscillations at knots, leading to smoother approximations and improved model accuracy (Fornberg & Zuev, 2007). This flexibility allowed the spline model to effectively adapt to different DBH classes, resulting in superior performance compared to the single, unclassified model.

Interestingly, the opposite trend was observed for the 5^{th} degree polynomial model. In this case, a single model encompassing all DBH sizes yielded better results compared to models specific to individual classes. This suggests that for this specific function, incorporating the entire range of tree sizes within one model provided a more accurate representation of tree variability, particularly regarding size class. While further investigation is needed to fully understand this finding, it highlights the potential benefits of considering the broader spectrum of variability when choosing or developing taper models. Our findings regarding the 5^{th} degree polynomial model resonate with similar research by Téo et al. (2018) in Brazil, where this function outperformed other models for taper prediction in their study. However, it's crucial to emphasize that the optimal approach may vary depending on the species, function chosen, and specific application.

Conclusion

The findings highlight the importance of carefully considering both species-specific characteristics and the chosen function when developing taper models. While our study demonstrates the effectiveness of classifying data by DBH for B-spline models in *Shorea robusta*, the optimal approach may differ for other species or functions. Future research could investigate the performance of additional functions or incorporate ecological variables beyond DBH for even more refined model development. Additionally, exploring alternative classification methods, such as crown size or site quality, could further enhance model accuracy and applicability. The insights gained from this study can be invaluable for researchers, forest managers, and stakeholders seeking accurate and reliable estimates of tree volume and stem dimensions in *Shorea robusta* plantations. Our findings provide valuable guidance for choosing appropriate functions and classification methods, ultimately contributing to improved sustainable forest management practices.

Author's contribution statement

A. Khadka: Research ideas, develop the research tools, methods, and select most appropriate packages, data analysis, results generation, revision of the research findings and manuscript preparation. R. Dhakal: Research ideas, select most appropriate packages, data analysis, results generation and first draft preparation. T. Subedi: Field data collection, ideas on data analysis, review and editing. A. K. Acharya: Select appropriate packages, data analysis, results generation, review and editing. B. P. Dhakal: Review and editing. P. Lamichhane: Review and editing.K. K. Pokharel: Review and editing. S. Khanal: develop the research tools, methods, and select most appropriate packages, revision of the findings, review and final editing

Data availability

The data collected for this study is available from the Figshare repository (10.6084/m9.figshare.25225511).

Conflict of Interest

The authors declare no conflict of interest.

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