BIBECHANA

A Multidisciplinary Journal of Science, Technology and Mathematics

ISSN 2091-0762 (online)

Journal homepage: http://nepjol.info/index.php/BIBECHANA

The sensitivity of L_4 and L_5 in the framework of restricted problem of three bodies when the primaries are different

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Article history: Received 2 October, 2012; Accepted 16 November, 2012

Abstract

In the paper, the location of liberation points have been observed and triangular liberation points L_4 and L_5 are determined.

Keywords: Libration points; Jacobi integral; triaxial rigid body

1. Introduction

Two primaries the point masses, the bigger one is spherical and the second primary; (smaller primary) is a triaxial rigid body are moving around their centre of mass in circular orbits under the influence of their mutual gravitational attraction and a third body, influenced by the primaries but not influencing their motion moves in the plane defined by the two revolving primaries.

If one body is triaxial and other is spherical then the triangular libration points in restricted problem of three bodies are infinitesimal or linearly stable.

Leontovic [1] established a non-linear stability of the triangular libration point. Deprit and Palmore [2] established the family of short orbits originating at the equilateral triangular libration point. Deprit [2,3] studied geometrically the long periodic and short periodic obits around L_4 with the help of D'Alembert's series. Henrard [4] discovered that the long periodic orbits at L_4 doesn't evolve in a continuous way. And discontinuity appears not only at mass ratios but also at singular bifurcation points. Tuckness [6] has used all the stability criteria numerically and investigated the sensitivities of third body around L_4 when it is given positional deviations away from L_4 with a suitable condition. The above mentioned mathematicians studied the different aspects of stability of libration points with different approaches in circular restricted problem of three bodies.

Here we have taken the bigger primary is a spherical and smaller as a triaxial rigid body under the same condition of Mc kanzie_ and _Szebehely and Tukness.

2. Equation of Motion

The equations of motion of the third body are:

$$\left.\begin{aligned} \ddot{x} - 2\eta\dot{y} &= \frac{\partial\Omega}{\partial x} \\ \ddot{y} + 2\eta\dot{x} &= \frac{\partial\Omega}{\partial y} \end{aligned}\right\} \tag{1}$$

where $r_1^2 = (x-\mu)^2 + y^2$ (2)

$$r_2^2 = (x-\mu+1)^2 + y^2 \tag{3}$$

and $\Omega = \frac{\eta^2}{2}\left[(1+\mu)r_1^2 + \mu r_2^2\right] + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{\mu(2\sigma_1 - \sigma_2)}{2r_2^3} - \frac{3\mu}{2r_2^5}(\sigma_1 - \sigma_2)y^2$ (4)

$$\mu = \frac{m_2}{m_1 + m_2} \leq \frac{1}{2}, m_1 \geq m_2$$

Where r_1,r_2 are distances between first and second primary from third body respectively, and m_1,m_2 are masses of the first and second primaries respectively.

The mean angular motion, $\eta = \sqrt{1 + \frac{3}{2}(2\sigma_1 - \sigma_2)}$ (5)

where $\sigma_1 = A_1 - A_3$ *and* $\sigma_2 = A_2 - A_3$ *and* $A_1 = \frac{a^2}{5R^2}, A_2 = \frac{b^2}{5R^2}, A_3 = \frac{c^2}{5R^2}$,

a, b, c being semi-axes of triaxial rigid body and R is distance between the primaries.

3. Location of Libration Points

From equation (1)

$$\frac{d}{dt}\left(\dot{x}^2 + \dot{y}^2\right) = 2\left(\frac{\partial\Omega}{\partial x}.\dot{x} + \frac{\partial\Omega}{\partial y}.\dot{y}\right) = 2\frac{d}{dt}\Omega(x,y)$$

Integrating w. r. t. t we get $\dot{x}^2 + \dot{y}^2 = 2\Omega - c$ (6)

which is Jacobi's integral.

Libration points are solutions of the equations

$$\frac{\partial\Omega}{\partial x}. = 0, \frac{\partial\Omega}{\partial y} = 0,$$

we get, $\frac{\partial r_1}{\partial y} = \frac{y}{r_1}, \frac{\partial r_2}{\partial y} = \frac{y}{r_2}$

or,

$$\frac{\partial\Omega}{\partial x}. = \eta^2 x - \frac{(1-\mu)(x-\mu)}{r_1^3} - \frac{\mu(x-\mu)+1}{r_2^3} - \frac{3\mu(2\sigma_1 - \sigma_2)}{2r_2^5}(x-\mu+1) + \frac{15\mu(\sigma_1 - \sigma_2)}{2r_2^7}(x-\mu+1)y^2$$

$$\frac{\partial \Omega}{\partial y}. = y\left[\eta^2 - \frac{(1-\mu)}{r_1^3} - \frac{\mu}{r_2^3} - \frac{3\mu(4\sigma_1 - 3\sigma_2)}{2r_2^5} + \frac{15\mu(\sigma_1 - \sigma_2)}{2r_2^7} y^2\right]$$

4. Triangular Libration Points

The triangular libration points are solutions of the equations:

$$\eta^2 x - \frac{(1-\mu)(x-\mu)}{r_1^3} - \frac{\mu(x-\mu+1)}{r_2^3} - \frac{3\mu(2\sigma_1 - \sigma_2)}{2r_2^5}(x-\mu+1) + \frac{15\mu(\sigma_1 - \sigma_2)}{2r_2^7}(x-\mu+1)y^2 = 0 \tag{7}$$

$$y\left[\eta^2 - \frac{(1-\mu)}{r_1^3} - \frac{\mu}{r_2^3} - \frac{3\mu(4\sigma_1 - 3\sigma_2)}{2r_2^5} + \frac{15\mu(\sigma_1 - \sigma_2)}{2r_2^7} y^2\right] = 0 \tag{8}$$

Multiplying equation (7) by $(x - \mu + 1)$ and subtracting the result from equation (8); we set.

$$-\eta^2(1-\mu) + \frac{(1-\mu)}{r_1^3} + \frac{3\mu(\sigma_1 - \sigma_2)}{r_2^5}(x-\mu+1) = 0 \tag{9}$$

Multiplying equation 8 by $(x-\mu)$and subtracting the result from equation (7)

we get, $$\left[\eta^2 - \frac{1}{r_2^3} + \frac{3(\sigma_1 - \sigma_2)(x-\mu)}{r_2^5} - \frac{3(2\sigma_1 - \sigma_2)}{2r_2^5} + \frac{15(\sigma_1 - \sigma_2)}{2r_2^7} y^2\right] = 0 \tag{10}$$

If we put $\sigma_1 = \sigma_2 = 0$ then equations (9) and (10) become

$\eta^2 - \frac{1}{r_2^3}$ =0 and $-\eta^2 + \frac{1}{r_1^3} = 0$

Adding them,

we get, $r_1 = r_2$ (11)

Using equation 5,

we get, $r_1 = r_2 = \eta = 1$

If $\sigma_1 \neq 0, \sigma_2 \neq 0$ then we get

$$x = \mu - \frac{1}{2} + \beta - \alpha \tag{12}$$

$$y = \pm\frac{\sqrt{3}}{2}\left[1 + \frac{2}{3}(\alpha + \beta)\right] \tag{13}$$

where $r_1 = 1 + \alpha,\ r_2 = 1 + \beta,\ 0 < \alpha, \beta << 1$

Putting the values of η, r_1, r_2, x, y in equations (9) and (11) We get,

$$1 + \frac{3}{2}(2\sigma_1 - \sigma_2) - (1+\beta)^{-3} + 3(\sigma_1 - \sigma_2)\left(\beta - \alpha - \frac{1}{2}\right)(1+\beta)^{-5} - \frac{3}{2}(2\sigma_1 - \sigma_2)(1+\beta)^{-5} + \frac{15}{2}(\sigma_1 - \sigma_2)(1+\beta)^{-7}\left(\frac{3}{4} + \alpha + \beta\right) = 0$$

and

$$-\left(1 + 3\sigma_1 - \frac{3}{2}\sigma_2\right)(1-\mu) + (1-\mu)(1+\alpha)^{-3} + 3\mu(\sigma_1 - \sigma_2)\left(\frac{1}{2} + \beta - \alpha\right)(1+\beta)^{-5} = 0$$

We get $\beta = -\frac{11}{8}\sigma_1 + \frac{11}{8}\sigma_2$ and $\alpha = \left[\frac{-2+3\mu}{2(1-\mu)}\right]\sigma_1 + \left[\frac{1-2\mu}{2(1-\mu)}\right]\sigma_2$ respectively

From equation (12)

$$x = \mu - \frac{1}{2} - \left[\frac{3+\mu}{8(1-\mu)}\right]\sigma_1 + \left[\frac{7-3\mu}{8(1-\mu)}\right]\sigma_2$$

and from equation (13)

$$y = \pm\frac{\sqrt{3}}{2}\left[1 - \frac{19-23\mu}{12(1-\mu)}\sigma_1 + \frac{15-19\mu}{12(1-\mu)}\sigma_2\right]$$

Thus,

$L_4 \equiv \left[\mu - \frac{1}{2} + \beta - \alpha, \frac{\sqrt{3}}{2}\left\{1 + \frac{2}{3}(\alpha+\beta)\right\}\right]$ and

$L_5 \equiv \left[\mu - \frac{1}{2} + \beta - \alpha, -\frac{\sqrt{3}}{2}\left\{1 + \frac{2}{3}(\alpha+\beta)\right\}\right]$ can be written as

$$L_4 \equiv \left[\mu - \frac{1}{2} - \frac{3+\mu}{8(1-\mu)}\sigma_1 + \frac{7-3\mu}{8(1-\mu)}\sigma_2, \frac{\sqrt{3}}{2}\left\{1 - \frac{(19-23\mu)}{12(1-\mu)}\sigma_1 + \frac{(15-19\mu)}{12(1-\mu)}\sigma_2\right\}\right]$$

$$L_5 \equiv \left[\mu - \frac{1}{2} - \frac{3+\mu}{8(1-\mu)}\sigma_1 + \frac{7-3\mu}{8(1-\mu)}\sigma_2, -\frac{\sqrt{3}}{2}\left\{1 - \frac{(19-23\mu)}{12(1-\mu)}\sigma_1 + \frac{(15-19\mu)}{12(1-\mu)}\sigma_2\right\}\right]$$

5. Conclusion

The triaxial rigid body can be affected by analytical effect on the values of the critical mass. The libration points L_4 & L_5 are depending on μ, σ_1, σ_2.

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