Choice of Regression Models in Time Series Data

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Abstract

Time series data possess distinct properties compared to cross section data. They have high temporal dependence, trend component and may have seasonal as well as cyclical patterns. A much-discussed issue in time series data is non-stationarity that highly influences the efficiency and consistency of regression estimates. In addition, time series regressions are most likely to suffer from spurious relationship. Thus, correct choice should be made regarding the regression models to obtain consistent estimates of the parameters and avoid spurious regression. This paper discusses the properties of time series data and the choice of appropriate regression models specifically in the context of finite samples. By discussing the relative strengths and limitations of the regression models that are used in time series data, this paper aims to contribute to the selection framework of regression models in time series analysis.

Keywords: stationarity, time series models, cointegration, VAR, volatility modelling

JEL Classification: C2, C22

1. INTRODUCTION

Time series data is data collected at regular intervals over time. Such data are different from cross section data as they are not independently distributed and have persistence and temporal disturbance with them. Some time series exhibit seasonality and thus require a completely different approach of analysis. In addition, some financial time series exhibit high volatility and demand a specific set of tools for estimating the causal relationship and produce forecasts.

The application of time series analysis methods varies based on a wide range of time series properties. Fulcher, Little and Jones (2015) mention the basic statistics of the distribution, correlations, stationary, information theoretic and entropy measures, linear and nonlinear model fits and others as important aspects of time series analysis. Traditionally, time series analysis highly relied on the assumption of stationarity and erogodic processes. The methodology of stationary time series analysis has been extended to trending series and time series modelling with heterogeneously distributed errors (Durlauf and Phillips, 1988). Furthermore, statistical techniques were invented to address the problems of unit root tests while vector autoregressions, granger causality and cointegration techniques were invented to address the sources of trends and cycles, causality and the nature of equilibrium.

Autoregressive integrated moving average (ARIMA) models was introduced by Box and Jenkins (1970) to estimate the time series observations for equally spaced periods. They derived autoregressive (AR), moving average (MA), and AR autoregressive moving average (ARMA) models from ARIMA models. ARIMA is applied to non-stationary series and requires to be differentiated to make it stationary. MA models are used where the past errors determine its future values and ARMA models are appropriate when series is functionally related with unobserved errors and its own past behaviors (Maçaira et al., 2018). Later, Generalized AR Conditional Heteroscedasticity (GARCH) model was introduced by Bollerslev (1986) with more flexible lag structure.

Granger (1969) introduced the causal lag and causal strength in time series data to analyze the direction of causality between two related variables. Various model selection criteria were developed to estimate the relative quality of models. Akaike information criterion (AIC) by Akaike (1973), Schwarz (1978), and Ljung and Box (1978) contributed to the problem of selecting a model from different dimensions. Dickey and Fuller (1979) introduced the unit root test for the limiting distributions of the AR estimator while Phillips and Perron (1988) introduced the Phillip-Perron unit root test that makes a nonparametric correction to the t-test statistic present in the Dickey-Fuller test (Macaira et al., 2018).

One distinguishing feature of time series is that of temporal dependence: the distribution of an observation at a certain time point conditional on previous value of the series depends on the outcome of those previous observations (Charlton and Caimo, 2012). After Granger (2003) highlighted the issue of nonsense regression in time series analysis that was earlier introduced by Yule (1926), assessing the properties of time series data and selecting the model accordingly has received more attention. Following the developments in unit root testing and the increasing interest on identifying nonsense regressions, a number of cointegration models have been used extensively in the literature including Engle and Granger approach, Johansen approach and autoregressive distributive lag model (ARDL).

This paper attempts at reviewing the techniques used for assessing the time series properties of the variables and selecting appropriate techniques for identifying such properties depending on the sample size and nature of the data. In addition, it discusses the choice of regression models for forecasting as well as for estimating the dynamic causal relationship for various nature of time series data.

The rest of the paper is structured as follows: section II discusses the properties of time series data and use of appropriate techniques to identify their properties, section III discusses the selection of appropriate models on the basis of the time series properties and the last section concludes the paper.

2. PROPERTIES OF TIME SERIES DATA

Time series data may have a number of features that are almost absent in cross section data- it may have a trend component, a cyclical pattern, seasonal fluctuations and temporal dependence. Besides these issues, the issue of stationarity has received much attention from the researchers during the last two decades. It is important because in the presence of non-stationary series, persistence of shocks in the series will be infinite, the standard assumptions of asymptotic analysis will not be valid and the regression models suffer from spurious relationship (Gujarati, 2011).

A time series is said to be stationary if its mean, variance and auto-covariance remain the same no matter at what point they are measured. Such a time series tend to return to its mean value and any fluctuations around the mean are broadly uniform. On the other hand, if the mean, variance and auto-covariance depend on time, it is called a non-stationary time series. Figure 1 shows a plot of a stationary series simulated by an AR(1) process given by $y_t = 0.5y_{t-1} + \epsilon_t$ and Figure 2 shows a plot of a non-stationary process simulated from the AR (1) process given by $y_t = y_{t-1} + \epsilon_t$. The process in Figure 1 has a constant mean of zero and a broadly uniform variation around it whereas the process in Figure 2 exhibits a non-constant mean as well as changing amplitudes around it.

Figure 1: A Simulated Stationary Process

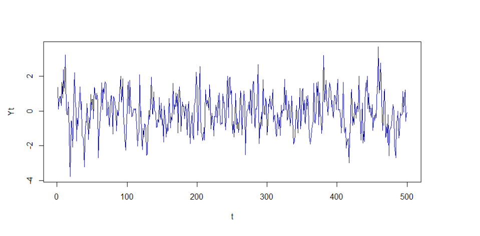
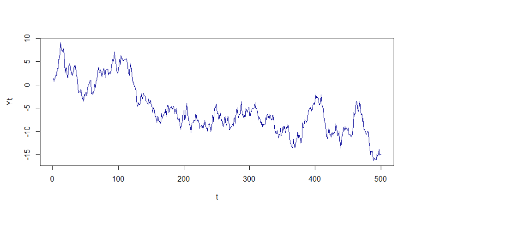


Figure 2: A Simulated Non-stationary Process

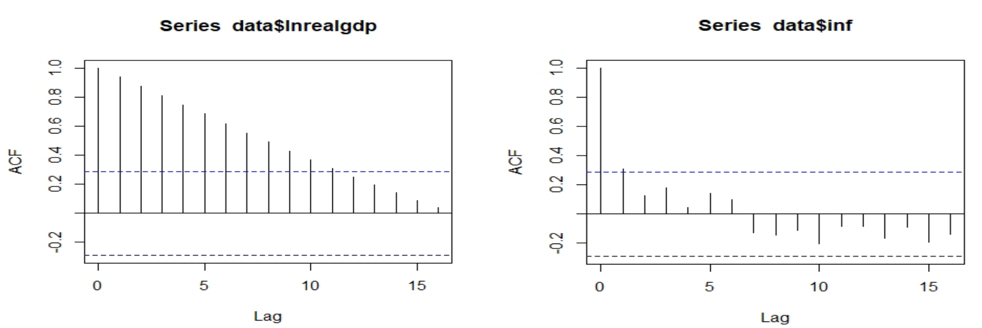


Source: Authors' estimation

A stationary time series is also called a time series integrated of order zero or I (0) process. If a time series is non-stationary at level but stationary at first difference, it is said to be integrated of order one or I (1) process. In general, if a non-stationary time series has to be differenced d times to get a stationary series, it is said to be integrated of order d or an I (d) process. In economics, most time series are generally I(1); that is, they generally become stationary only after taking their first difference (Granger, 1986).

One of the simplest way of investigating the time series properties is a visual inspection of correlogram. It depicts the autocorrelation and partial autocorrelation coefficients of the series up to a certain order. If the autocorrelation coefficients start with a very high value and decays slowly with the increase in the lags, the time series is likely to be non-stationary. Figure 3 presents the correlogram of the log of real GDP and inflation of Nepal. The autocorrelation function (ACF) of real GDP is decaying very slowly and remains well above the significance range (dotted blue lines) on the left panel, indicating a non-stationary process. On the other hand, the ACF of inflation follows an exponential decay and alternates in sign indicating that inflation may be a stationary process. However, a formal test is needed to have a precise inference about the stationarity.

Figure 3: Autocorrelation function of log Real GDP and Inflation of Nepal

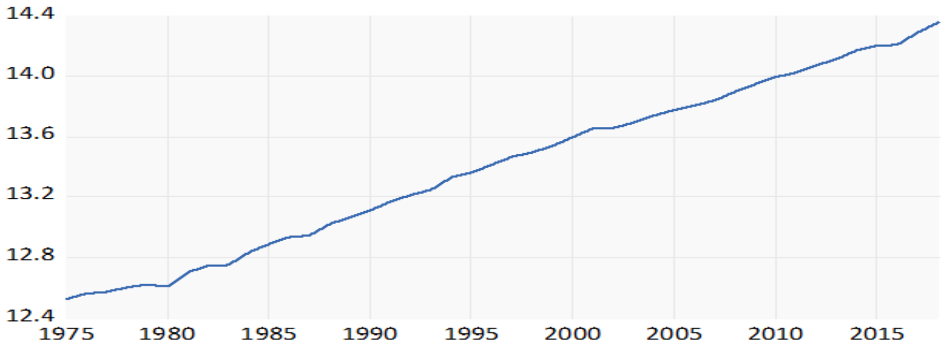


Source: Authors' estimation from NRB (2021) data

The formal test for stationary is the test for unit root. Many such tests have been proposed for this purpose including ADF test, PP test, KPSS test, DF-GLS test and NG Perron test (see Baum (2000) and Herranz (2017) and for a detailed treatment on these tests). However, the choice of the unit root test and specifying the parameters during the test is extremely important to draw correct inferences.

A simple AR(1) model for testing the unit root test is $y_t = \phi y_{t-1} + \epsilon_t$; ϵ_t is a white noise error term. The tests for unit root is essentially a test for the null hypothesis $\phi = 1$ against the alternative hypothesis of $\phi < 1$. In the ADF version of the test, if the calculated t statistics is greater than the critical value, the series is a stationary series.

Figure 4: Log of Real GDP



Source: Nepal Rastra Bank (2021)

Figure 4 depicts the log of Real GDP of Nepal. As GDP has a clear upward trend, it is sensible to include both intercept and trend component while performing the unit root test. Moreover, there are no significant breaks in the series, hence the standard unit root test is appropriate. Table 1 presents the results of ADF test of the log of Real GDP.

Table 1: ADF test of Log of Real GDP

Null Hypothesis: LNREALGDP has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=9)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.3744	0.0677
Test critical values:	1% level	-4.1756	
	5% level	-3.5130	
	10% level	-3.1868	

*MacKinnon (1996) one-sided p-values.

Source: Authors' Estimation

The test results in Table 1 show that the calculated value of the t-statistics is smaller than the critical value at 5 percent level of significance. This implies that the null hypothesis of non-stationary GDP series cannot be rejected.

Table 2 : ADF test of Log of Real GDP at First Difference

Null Hypothesis: D(LNREALGDP) has a unit root

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-7.7435	0.0000
Test critical values:	1% level	-3.5885	
	5% level	-2.9297	
	10% level	-2.6030	

*MacKinnon (1996) one-sided p-values.
Source: Authors' Estimation

To determine the exact order of integration of log of real GDP, the series can be tested for the presence of the unit root at the first difference. Table 2 reports the results from the ADF tests for the log of real GDP at first difference. The probability associated with the ADF statistics shows that log of real GDP is stationary at first difference. It is also supported by the value of t statistic that is higher than the 5 percent critical value. Thus, log of real GDP is an I (1) variable.

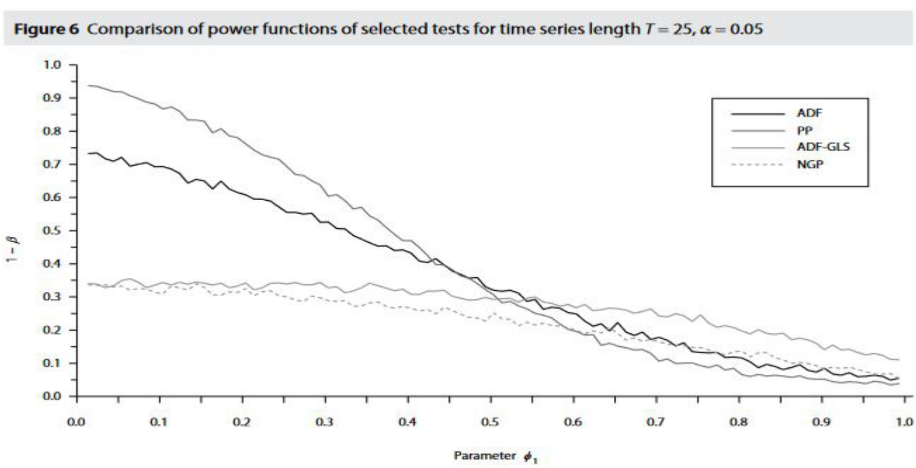
The unit root tests are, however, not always the perfect benchmark for identifying the order of integration of economic time series. They are beset by four important issues. First, the unit root tests are sensitive to the choice of lags. The number of lags used in the test equation should be chosen appropriately. If the number of lags is too small, the test may suffer from the presence of serial correlation and if the lags is too large, the power of the test will suffer. A useful rule of thumb has been suggested by Schwert (1978) to select the maximum number of lags by the rule $p_{max}=12.\left(\frac{T}{100}\right)^{1/4}$, where T is the length of the time series. In addition, Ng and Perron (2001) suggest using the modified AIC criteria for choosing the lags in unit root tests.

Secondly, the power of the unit root test is very low in finite sample as such overreliance on formal unit root tests can be misleading (Cochrane, 1991). Figure 5 shows that the power of most of the unit root tests appears to be low in case of a sample size of 25. For the parameter ϕ of the AR (1) model $y_t= \phi\, y_{t-1}+\varepsilon_t$ in the unit root test equation, the power of the ADF-GLS and NGP tests is very low for $\phi < 0.5$. In the interval of $0.5 < \phi$

< 0.7, the power functions intersect and for higher values of the parameter ADF-GLS and NGP tests perform better than ADF and PP tests. In the case of stationarity tests for finite sample and lower values of the parameter, the KPSS test can be a suitable complement.

Zivot (2006) and Pesaran (2015) emphasize that both ADF and PP test have low power where ϕ is close to one. Both tests fail to reject the invalid null hypothesis, i.e. time series is classified as non-stationary when it is actually stationary. Caner and Killian (2001) argue that even the KPSS test has this drawback. Despite these limitations, the most suitable tests for very short time series are the ADF and PP tests (Fedorová, 2016). DeJong et.al. (1992) argue that the augmented Dickey-Fuller procedure is reasonably well-behaved in case the errors follow autoregressive order.

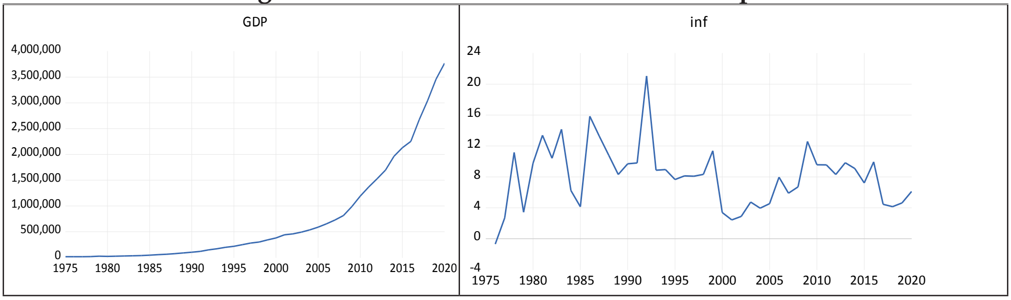
Figure 5 : Power Functions of Selected Unit Root Tests for Sample Size of 25 and 5 Percent Significance



Source: Fedorová (2016)

Thirdly, choice of intercept and trend parameter in the unit root test specification is important. A common rule of thumb in this case is that the test regression should include both intercept and trend when the underlying variable is trended in nature such as asset prices and macroeconomic variables including GDP. On the other hand, the test regression should include the constant only when the underlying series is non-trending such as interest rate, exchange rate, inflation and economic growth rate. Figure 6 shows the graph of real GDP of Nepal whose unit root test should include trend as it is trended whereas the unit root test for inflation should include constant only as it is a non-trended series.

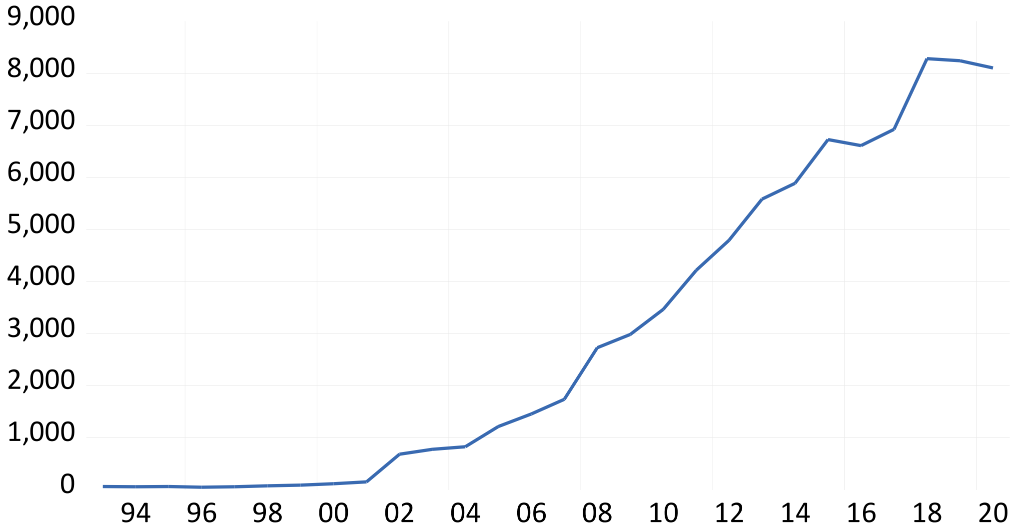
Figure 6 : Plot of GDP and Inflation of Nepal



Source: Nepal Rastra Bank (2021)

One another issue related with the unit root tests is the issue of structural breaks in the series. In the presence of such breaks, the null hypothesis of unit root in the data can rarely be rejected (Perron, 1989). The Dickey- Fuller and Phillips-Perron type unit root tests have low power if there is structural break in the series and fail to reject the unit root null hypothesis. The larger the break and the smaller the size, the lower the power of the test. For such series, Zivot and Andrews (1992), Banerjee et al. (1992) and Perron (1997) have introduced unit root tests in the presence of structural change allowing the break point endogenously for the data. Further Perron (1989) argues that most macroeconomic series are characterized by large and frequent shocks, rather than by unit root. He introduced single exogenous break and uses a modified Dickey- Fuller (DF) unit root test that includes dummy variables or exogenous structural break. The importance of using structural unit root test is to prevent result of non-rejection or biased towards the non- rejection and it can detect the presence of structural break in the series. But the important issues that need to be considered is the power of the test and the multiple breaks in particular variable (Glynn et al, 2007).

Figure 7 : Shows the plot of remittances received by Nepal from 1993 to 2020. The series seems to have a structural break around 2000.



Source: World Bank (2021)

Accordingly, the standard ADF test shows that the series is not stationary even after first differencing (Table 3) whereas the structural break unit root test shows that the series is stationary at first differences.

Table 3: Standard ADF and Structural Break Unit Root Test Results

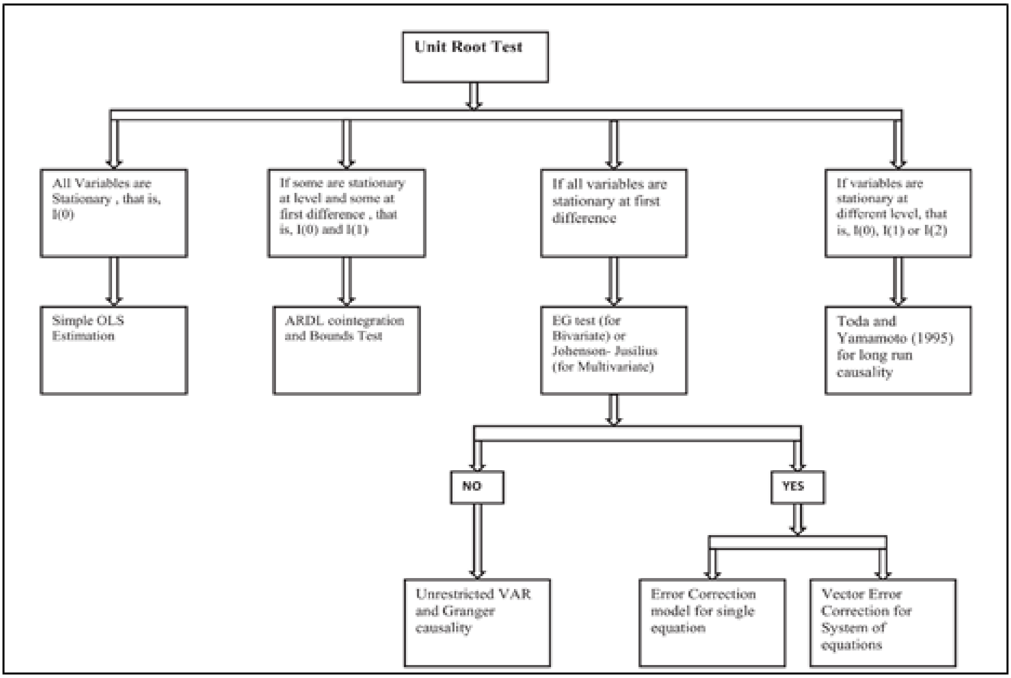
Standard ADF Test Null Hypothesis: D(REMIT) has a unit roott-Statistic Prob.* ADF test statistic -1.5754 0.4793 critical values: 1% level -3.7378 5% level -2.9918 10% level -2.6355 *MacKinnon (1996) one-sided p-values.	Structural Unit Root Test Null Hypothesis: D(REMIT) has a unit root t-Statistic Prob.* ADF test statistic -5.8706 < 0.01 critical values 1% level -4.9491 5% level -4.4436 10% level -4.1936 *Vogelsang (1993) asymptotic one-sided p-values.

Source: Authors' Estimation

3. CHOICE OF REGRESSION MODELS

The choice of regression models is largely determined by the time series properties of the underlying data. If the data are stationary in nature, use of OLS will result into unbiased and consistent estimates and thus it can be safely used to draw inferences. However, if the data are non-stationary, use of OLS may result into spurious relationship among the variables. In such a case, cointegration techniques can be used to check whether there is any long run relationship among the variables. If cointegration exists, short-run dynamics of the relationship as well as long run coefficients can be determined. Figure 8 summarizes the possiblemodels that can be used depending on the stationarity property of the data.

Figure 8: Choice of Regression Models in Time Series Data following Unit Root Test



Source: Jalil and Rao (2019)

Cointegration

The regression analysis on time series has been much benefited from the concept of cointegration by Granger (1981) and Engle and Granger (1987). They show that using OLS in case of I (1) variables could be misleading because non-stationary series violates the basic assumptions of OLS as such one cannot get the best linear unbiased estimators and there may exist spurious or nonsense correlation between the non-stationary variables.

Table 4 presents the results from a regression run on a non-stationary series on another non-stationary series generated by the mechanism $y_t = y_{t-1} + \epsilon_t$ and $x_t = x_{t-1} + \epsilon_t$. The regression results are statistically significant. The x variable is statistically significant even though it should not have any relationship with y since both x and y have been independently generated from a random walk model.

Table 4: Regression Results from a Random Walk Series

	Coefficient	Std. Error	t-Ratio	Probability
Constant	-5.21	0.24	-21.74	0.00
X	0.63	0.24	2.60	0.01
F: 6.786	Prob. (F) : 0.0094 n=1000			

Source: Authors' Estimation

In case, where the variables are non-stationary at levels but are difference stationary, cointegration methodology allows researchers to test for the presence of long run equilibrium relationship among the variables. If the separate economic time series are stationary after differencing or they are I(1), but a linear combination of them is stationary, then the series are said to be cointegrated. Formally, given x_t and y_t are I (1) or are difference stationary processes, they are said to be cointegrated if there exists a parameter α such that $u_t = y_t - \alpha x_t$ is a stationary process or I (0). The estimated α is no longer spurious rather is super-consistent (Gujarati, 2004).

Tests for cointegration seek to discern whether or not a long-run relationship exists among such a set of variables. The existence of a common trend among the variables means that in the long run, the behavior of the common trend will drive the behavior of the variables. In such a case, the shocks in time series will die out as the variables adjust back to their common trend.

Error Correction Modeling

Even if y_t and x_t variables are cointegrated, that is, there is a long run equilibrium relationship between them, there may be disequilibrium in the short run. Thus, the error term $u_t = y_t - \beta_1 - \beta_2 x_t$ in the regression equation $y_t = \beta_1 + \beta_2 x_t + u_t$ is called the equilibrium error. This error term can be used to tie the short-run behavior of Y to its long-run value. The error correction models (ECM) first used by Sargan and later popularized by Engle and Granger corrects for disequilibrium (Bhatta, 2013). The Granger Representation Theorem says that if two variables y_t and x_t are cointegrated, then the relationship between the two can be expressed as Error Correction Model by:

$$\Delta y_t = \alpha_0 + \alpha_1 \Delta x_t + \alpha_2 u_{t-1} + \varepsilon_t$$

Where,

Δ= first difference operator,

$\boldsymbol{\varepsilon}_t$= a white noise error term,

u_{t-1} = one period lagged value of the error term from the cointegrating regression

The ECM states that Δy_t depends on Δx_t and on the equilibrium error term. If the error term is non-zero, the model is out of equilibrium. Here the value of α_2 decides how quickly the equilibrium is restored.

Cointegration techniques are used to address the problem of spurious regression in time series data and can be used when data are non-stationary. Many economic theories imply that a linear combination of variables is stationary although individually they are not. If there is such a stable linear combination among the variables, they are said to be cointegrated. Thus, in time series analysis of macroeconomic series, it becomes imperative to check for stationarity and cointegration to avoid non-sense regressions.

Several techniques have been proposed to test the existence of cointegration in time series data including Engle Granger Test, Johansen Test and ARDL Test. However, each test has its own strengths and weaknesses. A careful selection of the test can provide us more precise and valid estimates of the parameters.

Engle Granger Test

The Engle Granger test basically tests whether a common trend exists between the variables or not. In the first step, y_t is regressed on x_t and residual series is generated and on the second step, unit root test is applied to check whether the residual series is stationary or not. If the residual series is stationary, the variables y_t and x_t are said to be cointegrated and error correction model is estimated to observe the behavior of the variables in the short run.

Table 5 and Table 6 present the estimates of the Engle Granger cointegration test between the log of real money demand and log of real GDP of Nepal. The first step results show that log of real GDP is highly significant in explaining the money demand. However, the model suffers from autocorrelation and R^2 is higher than the DW statistics implying that we cannot rely on the results. To confirm whether there is any long run relationship between them or not, the residual series generated from the regression results in Table 5 has to be tested for the presence of unit root.

Table 5: Engle and Granger two-step procedure - Step 1

Dependent Variable: LNRM2
Sample: 1975 2020

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LNREALGDP	1.9580	0.0247	79.032	0.0000
C	-12.141	0.3172	-38.268	0.0000

R-squared	0.9930	Mean dependent var	12.9071
Adjusted R-squared	0.9928	S.D. dependent var	1.1554
F-statistic	6246.144	Durbin-Watson stat	0.4382
Prob(F-statistic)	0.0000		

Source: Authors' Estimation

The results in Table 6 show that the residual series is stationary at the conventional 5 percent level of significance. This confirms the existence of long-run relationship between money demand and real GDP of Nepal.

Table 6: Unit root test of Residual

Null Hypothesis: Residual has a unit root

Exogenous: Constant

Lag Length: 0 (Automatic - based on SIC, maxlag=9)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.4828	0.0130
Test critical values:	1% level	-3.5847	
	5% level	-2.9281	
	10% level	-2.6022	

*MacKinnon (1996) one-sided p-values.

Source: Authors' Estimation

The cointegration between money demand and GDP is further supported by the results from error correction model presented in Table 7. The coefficient of the lagged residual term has the expected negative sign and is statistically significant supporting that money demand and GDP in Nepal have long run relationship. In addition, the coefficient of the residual shows that 22 percent of the disequilibrium is corrected in each period and the variables returns back to equilibrium in nearly 5 years.

Table 7: Error Correction Model

Dependent Variable: D(LNRM2)
Method: Least Squares
Sample (adjusted): 1976 2020

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LNREALGDP)	0.3001	0.3067	0.9783	0.3335
RESIDUAL(-1)	-0.2166	0.0720	-3.0074	0.0044
C	0.0792	0.0146	5.3938	0.0000

R-squared	0.1781	Mean dependent var	0.0925
Adjusted R-squared	0.1390	S.D. dependent var	0.0478
F-statistic	4.5519	Durbin-Watson stat	1.6234
Prob(F-statistic)	0.0162		

Source: Authors' Estimation

The Engle Granger test for cointegration has several weaknesses that make it inappropriate in small samples. The most important among them is that it uses the standard unit root tests that have power and size distortions in finite samples. Thus, in case of small samples, results from this test should be carefully interpreted. In addition, it also requires that all variables be I (1) and thus cannot be applied in case of mixed order of variables.

ARDL Approach to Cointegration

The ARDL bounds testing approach to cointegration was developed by Pesaran and Shin (1999) and Pesaran *et al.* (2001). Due to the low power and other problems associated with other cointegration tests, the ARDL approach to cointegration has become popular in the recent years.

The ARDL cointegration approach has numerous advantages in comparison to other cointegration methods such as Engle and Granger (1987), Johansen (1988), and Johansen and Juselius (1990) procedures: (i) The ARDL procedure can be applied whether the regressors are I(1) and/or I(0), while Johansen cointegration techniques require that all the variables in the system be of equal order of integration, (ii) While the Johansen cointegration technique requires large sample size for validity, the ARDL procedure is statistically more valid in small samples, (iii) The ARDL procedure allows that the variables may have different optimal lags, while it is impossible with conventional cointegration procedures, (iv) The ARDL technique generally provides unbiased estimates of the long-run model and valid t-statistics even when some of the regressors are endogenous, (v) The ARDL procedure employs only a single reduced form equation compared to a system of equations in Johansen approach (Bhatta, 2013).

Table 8 presents the results from the Bound Test used to test the presence of long run relationship between money demand and GDP of Nepal by using the ARDL model. Since the data is annual, maximum lag is set to 2. The appropriate lag order selected by using the SBC criterion is ARDL (1,0). The computed F-statistics is greater than the appropriate upper bound of the critical value and thus the null hypothesis of no cointegration is rejected. However, the results from the error correction model are more reliable as a test of cointegration in this procedure (Bahmani-Oskooee & Bohl, 2000). The results from the error correction model in Table 9 show that the lagged residual series have a negative sign and is statistically significant at 5 percent level of significance, thus supporting the existence of cointegration.

Table 8: ARDL Bound Test Results

F-Bounds Test		Null Hypothesis: No levels relationship		
Test Statistic	Value	Signif.	I(0)	I(1)
			n=1000	
F-statistic	68.1127	10%	3.02	3.51
K	1	5%	3.62	4.16
		2.5%	4.18	4.79
		1%	4.94	5.58
Actual Sample Size	45		Finite Sample: n=45	
		10%	3.19	3.73
		5%	3.877	4.46
		1%	5.607	6.193

Source: Authors' Estimation

The coefficient of the error correction model is similar to that of the Engle Granger procedure suggesting that nearly one fifth of the disequilibrium is corrected each period.

Table 9: ARDL Error Correction Model Results

Dependent Variable: D(LNRM2)

Selected Model: ARDL(1, 0)

Sample: 1975 2020

Variable	Coefficient	Std. Error	t-Statistic	Prob.
CointEq(-1)*	-0.2135	0.0145	-14.6311	0.0000
R-squared	0.1783	Mean dependent var		0.0925
Adjusted R-squared	0.1783	S.D. dependent var		0.0478
Durbin-Watson stat	1.6420			

Source: Authors' Estimation

Johansen Test

Johansen (1988) applies the maximum likelihood principle to determine the presence of cointegrating vectors in nonstationary time series. The Johansen (1988) procedure relies heavily on the relationship between the rank of a matrix and its characteristics roots. It is simply a multivariate generalization of the Dickey-Fuller test.

In the univariate case, it is possible to view the stationarity of y_t as being dependent on the magnitude of a_1.

$$y_t = a_1 y_{t-1} + \varepsilon_t$$

$$y_t - y_{t-1} = a_1 y_{t-1} - y_{t-1} + \varepsilon_t$$

$$\Delta y_t = (a_1 - 1) y_{t-1} + \varepsilon_t$$

If $(a_1 - 1) = 0$, then y_t process has a unit root.

Now, consider the simple generalization to n variables.

$$x_t = A_1 x_{t-1} + \varepsilon_t$$

$$\Delta x_t = (A_1 - I) x_{t-1} + \varepsilon_t$$

$$\Delta x_t = \pi x_{t-1} + \varepsilon_t$$

Where,

x_t and ε_t are $n \times 1$ vector; A_1 is an $n \times n$ matrix of parameters; I is an identity matrix of $n \times n$; π is defined to be $A_1 - I$

Here, rank of $A_1 - I$ equals to the number of cointegrating vectors. If $A_1 - I$ consists of all zeros so that rank (π) = 0, all the x_{it} sequences are unit root processes. Since, there is no linear combination of x_{it} processes that is stationary, the variables are not cointegrated. Instead, if π is of rank n, the vector process is stationary. If rank (π) = 1, there is a single cointegrating vector and the expression πx_{t-1} is error correction term. The multiple cointegrating vectors exist if $1 < rank(\pi) < n$. The number of distinct cointegrating vectors can be obtained by checking the significance of the characteristic roots of π. The rank of matrix is equal to the number of its characteristic roots that differ from zero.

While carrying out the Johansen test, the variables need to have the same order of integration and appropriate lag length need to be selected through a VAR model in level by using the multivariate generalizations of the AIC or SBC.

Table 10 presents the results from the Johansen test. The trace as well as the eigenvalue test results show that there is one cointegrating relationship between log of real money demand and log of real GDP thus supporting the results from the ARDL approach.

Table 10: Results from the Johansen Test

Sample (adjusted): 1977 2020
Trend assumption: No deterministic trend (restricted constant)
Series: LNRM2 LNREALGDP
Lags interval (in first differences): 1 to 1

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.5258	39.5848	20.2618	0.0000
At most 1	0.1422	6.7489	9.1645	0.1403

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.5258	32.8358	15.8921	0.0001
At most 1	0.1422	6.7489	9.1645	0.1403

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Source: Authors' Estimation

One major weakness of the Johansen test is that the limiting distribution of the test is often a poor approximation to the finite sample distribution (Johansen, 2002). Type I error rates for the Johansen test depend heavily on the number of groups and the ratio of the smallest sample size to the number of dependent variables. The larger sample sizes are required to control Type I error rate (Coombs & Algina, 1996).

VAR Models

VAR models are used for multivariate time series. The structure is that each variable is a linear function of past lags of itself and past lags of the other variables. One of the prerequisites of the VAR Models is that the data need to be stationary in nature. Since VAR is estimated by OLS technique equation by equation, use of non-stationary data could lead to misleading standard errors and unstable VAR system as well as explosive impulse response functions (Groenewold, Guoping, & Anping, 2007).

Estimating a VAR model requires a careful consideration of the following steps: selecting the lag order of the VAR, estimating the VAR and checking the VAR stability. Table 11 shows the results for the lag order selection according to the various criteria from a bivariate VAR model with money demand and GDP as the two variables.

Table 11: Lag Order Selection for the VAR

Endogenous variables: LNRM2 LNREALGDP

Exogenous variables: C

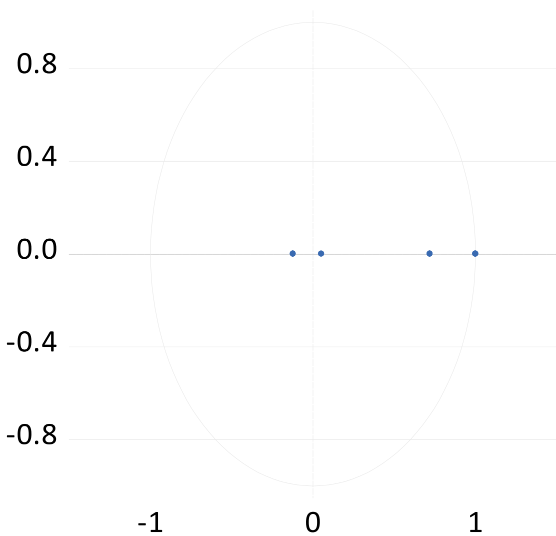
Sample: 1975 2020

Lag	LogL	LR	FPE	AIC	SC	HQ
0	11.1361	NA	0.002219	-0.4350	-0.3523	-0.4047
1	177.9893	309.8700*	9.51e-07*	-8.1899*	-7.9417*	-8.0989*
2	179.5172	2.6920	1.07e-06	-8.0722	-7.6585	-7.9205
3	182.1653	4.4135	1.15e-06	-8.0078	-7.4286	-7.7955
4	185.6817	5.5258	1.18e-06	-7.9848	-7.2401	-7.7118

Source: Authors' Estimation

As shown in Table 11, most of the criteria prefer lag 1 in the VAR model. The estimated impulse response functions from the model are presented in Figure 10.

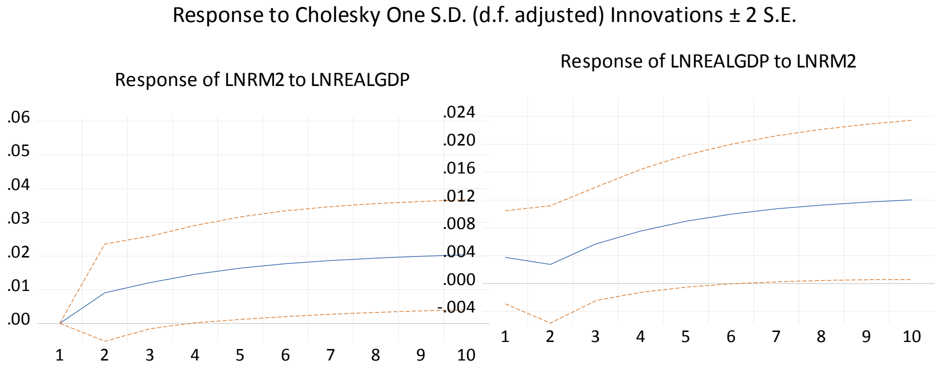
Figure 9 : Inverse Roots of AR Characteristic Polynomial



Source: Author's estimation

The impulse responses in Figure 10 show explosive evolution over time which is caused by the instability in the VAR system. The instability of the VAR is evident in Figure 9 where some of the inverse roots of the AR polynomial are not within the unit circle.

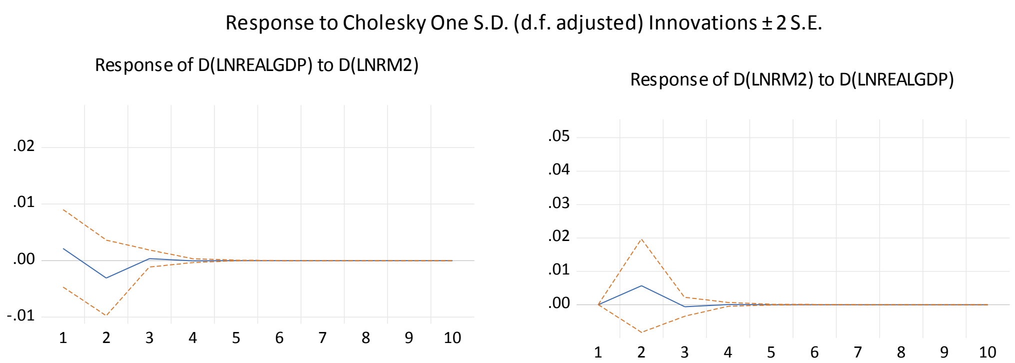
Figure 10: Impulse Response Function from the VAR Model with I(1) Variables



Source: Authors' Estimation

The problem of VAR instability can be corrected by making the variables stationary. Figure 11 show that using the variables in differenced form removes the problem of instability and produces convergent impulse response functions.

Figure 11: Impulse Response Function from the VAR Model with I(0) Variables



Source: Authors' Estimation

ARCH and GARCH Models

The OLS model relies on the assumption that the expected value of all error terms squared is the same at any given point. Non-fulfillment of this assumption though yields unbiased regression coefficients but the standard errors and confidence intervals estimated will be narrow giving a false sense of precision. The ARCH model by Engle (1982) and GARCH model by Bollerslev (1986) treat heteroskedasticity as a variance to be modeled.

An ARCH(m) process is one for which the variance at time t is conditional on observations at the previous m times and the relationship is:

$$Var(y_{t-1}, \dots, y_{t-m}) = \sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \cdots + \alpha_m y_{t-m}^2$$

With certain constraints imposed on the coefficients, the y_t series squared will theoretically be AR(m) and must satisfy following conditions:

$\alpha_0 > 0$ and $\alpha_i > 0$: to guarantee positive variance.

$0 \le \sum_{i=1}^{i=m} \alpha_i < 1$

$\alpha_1 > \alpha_2 \dots > \alpha_m$: Recent past has more influence than older lags.

The GARCH (p,q) model estimates conditional variance as a function of weighted average of the past squared residuals till q lagged term and lagged conditional variance till p terms. Consider the regression model:

$$Y_t = \alpha + \beta_1 X_t + \mu_t$$

Where, $\mu_t \sim N(0, \sigma^2)$ and σ^2 is not constant but changes over time and dependent on the past history.

$$\mu_t = \epsilon_t \sqrt{h_t}\ ; h_t = \sigma^2$$

where, ϵ_t is the white noise and $\sim N(0,1)$ and h_t is the systematic variance which changes over time.

$$h_t = \gamma_0 + \sum_{i=0}^{p} \delta_i h_{t-i} + \sum_{j=0}^{q} \phi_j u_{t-j}^2$$ n as

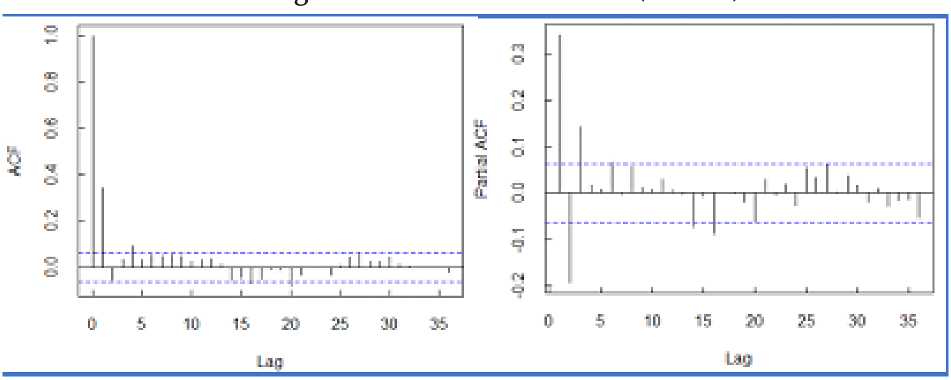
Now, h_t depends both on past values of the shocks or error which is captured by the lagged squared residual terms and on past values of itself, which is captured by lagged h_t terms. This is called variance equation. The GARCH model is stationary if $0 < \left(\sum_{i=0}^{p} \delta_i + \sum_{j=0}^{q} \phi_j\right) < 1$.

The major difference between ARCH and GARCH model are: ARCH model resembles more of a moving average (MA) specification than an autoregression (AR), but GARCH model includes the lagged conditional variance terms as autoregressive terms. ARCH model is an over-parameterized model while GARCH model is a parsimonious model.

ARCH and GARCH model are models for the variance of a time series. ARCH models are used to describe a changing, possibly volatile variance. It could possibly be used to describe a gradually increasing variance over time; most often it is used in situations in which there may be short periods of increased variation.

In practice, a time series plot of the series is the best identification tool for ARCH and GARCH model. ACF and PACF of both y_t and y_t^2 are fruitful to decide to use ARCH or not. If y_t appears to be white noise and y_t^2 appears to be AR (1), then an ARCH (1) model for the variance is suggested. If the PACF of the y_t^2 suggests AR(m), then ARCH(m) may work. GARCH models may be suggested by an ARMA type look to the ACF and PACF of y_t^2. Before proceeding for GARCH models, it is necessary to test for the presence of possible ARCH (q) effects. In the absence of ACRH effects, it is not necessary to estimate GARCH models.

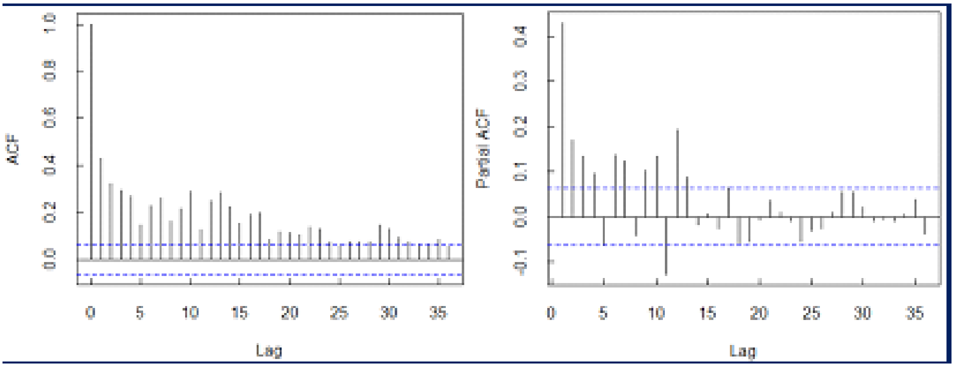
Figure 12: ACF and PACF of D(NEPSE)



Source: Authors' Estimation

Figure 12 shows the autocorrelation and partial autocorrelation of the first difference of the NEPSE index and Figure 13 shows the autocorrelation and partial autocorrelation of the square of the first difference of the index.

Figure 13: ACF and PACF of square of D(NEPSE)



Source: Authors' Estimation

The ADF test in Table 12 confirms that D(LN_NEPSE) is stationary. The ACF and PACF of D(NEPSE) and square of D(NEPSE) confirms that ARCH process is appropriate for NEPSE but ARCH (1) is not appropriate as square of D(NEPSE) is not AR (1) process. The ARCH process can also be confirmed through test of Heteroscedasticity ARCH effect after estimating ARIMA model.

Table 12: ADF Test of NEPSE Index

Null Hypothesis: NEPSE index has a unit root

Variable	Order of integration	t-stat (prob.)
NEPSE Index	I (0)	1.97 (0.99)
NEPSE Index	I (1)	-15.64 (0.00)

Source: Authors' Estimation

The GARCH model is $\mathrm{NEPSE}_t = \alpha + \beta \mathrm{NEPSE}_{t-1} + \gamma e_{t-1} + e_t$

Variance equation: $h_t = \delta + \theta h_{t-1}^2 + \phi e_{t-1}^2$

For stability, $\theta > 0;\ \phi > 0$ and $0 < \theta + \phi < 1$

The estimation results in Table 13 show that the GARCH model is free from autocorrelation and heteroscedasticity. The model is stable as θ *and* ϕ are less than unity and their sum is less than unity. Since, the sum of θ *and* ϕ is close to 1, the volatility in the data is high. The current NEPSE index is affected by its past value and past error term.

Table 13: GARCH Model Estimation

Variable	Coefficient	Std. Error	z-Stat	Prob.
C	7.31	0.21	35.39	0.00
AR(1)	1.01	0.00	572.81	0.00
MA(1)	0.44	0.03	13.20	0.00
Variance Equation				
C	0.01	0.00	4.69	0.00
RESID(-1)^2	0.23	0.03	7.37	0.00
GARCH(-1)	0.74	0.03	25.62	0.00
R-squared	0.99	Adjusted R-squared		0.99
S.E. of regression	0.01	Sum squared resid		0.11
Log likelihood	3214.03	Durbin-Watson stat		2.01
Heteroskedasticity Test: ARCH				
F-statistic	0.063	Prob. F(1,966)		0.81

Source: Author's Estimates

$$NEPSE_t = 7.31 + 1.01\ NEPSE_{t-1} + 0.44e_{t-1} + e_t$$

Variance equation: $h_t = 0.01 + 0.74h_{t-1}^2 + 0.23e_{t-1}^2$

ARCH and GARCH models too have their own limitations. They do not capture leverage and asymmetric effects of favorable and unfavorable news. The ARCH model does not provide any new insight for understanding the source of variation of a financial series (Tsay, 2010). They only provide a mechanical way to describe the behavior of conditional variance. In addition, they provide no indication about what causes such behavior to occur.

5. CONCLUSION

Time series data possess distinct properties such as high temporal dependence, trend component and seasonal as well as cyclical patterns. In addition, the issue of non-stationarity influences the efficiency and consistency of the regression estimates. With the increasing interest on identifying the long run relationship among the trended time series variables, a number of methodologies have evolved in time series literature. Accordingly, a correct choice should be made regarding the regression models to obtain consistent estimates of the parameters. If the data is stationary, the OLS estimators can be taken as reliable, otherwise cointegration techniques, VAR and volatility models can be used depending on the nature of the data. However, the power of the formal unit root tests is low in case of finite samples as such and care is to be taken while examining the time series properties of the data and in selecting the regression model accordingly.

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