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Journal of Institute of Science and Technology

Volume 22, Issue 2, January 2018

ISSN: 2469-9062 (print), 2467-9240 (e)

JIST, **22** (2): 76-85 (2018)

Published by:

Institute of Science and Technology

Tribhuvan University

Kirtipur, Kathmandu, Nepal

JIST 2018, 22 (2): 76-85
ISSN: 2469-9062 (p), 2467-9240 (e)

Research Article

ZERO-TRUNCATED DISCRETE TWO-PARAMETER POISSON-LINDLEY DISTRIBUTION WITH APPLICATIONS

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***Received:** 31 March, 2017; **Revised:** 25 September, 2017; **Accepted**: 27 September, 2017*

ABSTRACT

A zero-truncated discrete two-parameter Poisson-Lindley distribution (ZTDTPPLD), which includes zero-truncated Poisson-Lindley distribution (ZTPLD) as a particular case, has been introduced. The proposed distribution has been obtained by compounding size-biased Poisson distribution (SBPD) with a continuous distribution. Its raw moments and central moments have been given. The coefficients of variation, skewness, kurtosis, and index of dispersion have been obtained and their nature and behavior have been studied graphically. Maximum likelihood estimation (MLE) has been discussed for estimating its parameters. The goodness of fit of ZTDTPPLD has been discussed with some data sets and the fit shows satisfactory over zero – truncated Poisson distribution (ZTPD) and ZTPLD.

Keywords: Zero-truncated distribution, Discrete two-parameter Poisson-Lindley distribution, Moments, Maximum Likelihood estimation, Goodness of fit.

INTRODUCTION

In probability theory, zero-truncated distributions are certain discrete distributions having support the set of positive integers. Zero-truncated distributions are suitable models for modeling data when the data to be modeled originate from a mechanism which generates data excluding zero counts.

Suppose $P_0\left(x;\theta\right)$ is the original distribution. Then the zero-truncated version of $P_0\left(x;\theta\right)$ can be defined as

$$P_1\left(x;\theta\right)=\frac{P_0\left(x;\theta\right)}{1-P_0\left(0;\theta\right)}\quad ;x=1,2,3,... \tag{1.1}$$

Shanker *et al.* (2012) has obtained a discrete two-parameter Poisson-Lindley distribution (DTPPLD) defined by its probability mass function (pmf)

$$P_0\left(x;\theta,\alpha\right)=\frac{\theta^2}{\theta+\alpha}\frac{\alpha x+\left(\alpha+\theta+1\right)}{\left(\theta+1\right)^{x+2}};x=0,1,2,...,\theta>0,\theta>\alpha \tag{1.2}$$

It can be easily verified that at $\alpha=1$, DTPPLD (1.2) reduces to the one parameter Poisson-Lindley distribution (PLD) introduced by Sankaran (1970) having pmf

$$P_2\left(x;\theta\right)=\frac{\theta^2\left(x+\theta+2\right)}{\left(\theta+1\right)^{x+3}}\;;x=0,1,2,...,\theta>0 \tag{1.3}$$

Shanker *et al.* (2012) have studies the mathematical and statistical properties, estimation of parameters of DTPPLD and its applications to model count data. It should be noted that PLD is also a Poisson mixture of Lindley distribution, introduced by Lindley (1958). Shanker and Hagos (2015) have discussed the applications of PLD for modeling data from biological sciences.

The DTPPLD is a Poisson mixture of a two-parameter Lindley distribution (TPLD) of Shanker *et al.* (2013) having probability density function (pdf)

$$f_1\left(x;\theta,\alpha\right)=\frac{\theta^2}{\theta+\alpha}\left(1+\alpha x\right)e^{-\theta x};x>0,\theta>0,\theta>\alpha \tag{1.4}$$

In this paper, a ZTDTPPLD, of which zero-truncated Poisson-Lindley distribution (ZTPLD) is a particular case, has been obtained by compounding size-biased Poisson distribution

(SBPD) with a continuous distribution. Its raw moments and central moments have been obtained and thus the expressions for coefficient of variation, skewness, kurtosis, and index of dispersion have been obtained and their nature and behavior have been discussed graphically. Maximum likelihood estimation has been discussed for estimating the parameters of ZTDTPPLD. The goodness of fit of ZTDTPPLD has also been discussed with some data sets and its fit has been compared with zero - truncated Poisson distribution (ZTPD) and zero-truncated Poisson- Lindley distribution (ZTPLD).

ZERO-TRUNCATED DISCRETE TWO-PARAMETER POISSON-LINDLEY DISTRIBUTION

Using (1.1) and (1.2), the pmf of zero-truncated discrete two-parameter Poisson-Lindley distribution (ZTDTPPLD) can be obtained as

$$P_3\left(x;\theta,\alpha\right)=\frac{\theta^2}{\theta^2+2\theta\alpha+\theta+\alpha}\,\frac{\alpha\,x+\left(\theta+\alpha+1\right)}{\left(\theta+1\right)^x}\quad ;x=1,2,3,...,\ \theta>0,\theta^2+2\theta\alpha+\theta+\alpha>0 \qquad (2.1)$$

It can be easily verified that at $\alpha=1$, (2.1) reduces to the pmf of ZTPLD introduced by Ghitany *et al.* (2008) having pmf

$$P_4\left(x;\theta\right)=\frac{\theta^2}{\theta^2+3\theta+1}\,\frac{x+\theta+2}{\left(\theta+1\right)^x}\quad ;x=1,2,3,....,\theta>0 \qquad (2.2)$$

Shanker *et al.* (2015) have done extensive study on the comparison between ZTPD and ZTPLD with respect to their applications to data sets excluding zero-counts and showed that in demography and biological sciences ZTPLD gives better fit than ZTPD while in social sciences ZTPD gives better fit than ZTPLD.

The pmf of zero-truncated Poisson distribution (ZTPD) is given by

$$P_5\left(x;\theta\right)=\frac{e^{-\theta}\,\theta^x}{\left(1-e^{-\theta}\right)x!}\quad ;x=1,2,3,....,\theta>0 \qquad (2.3)$$

To study the nature and behavior of ZTDTPPLD for varying values of parameters θ and α, a number of graphs of the pmf of ZTDTPPLD have been drawn and presented in the figure 1.

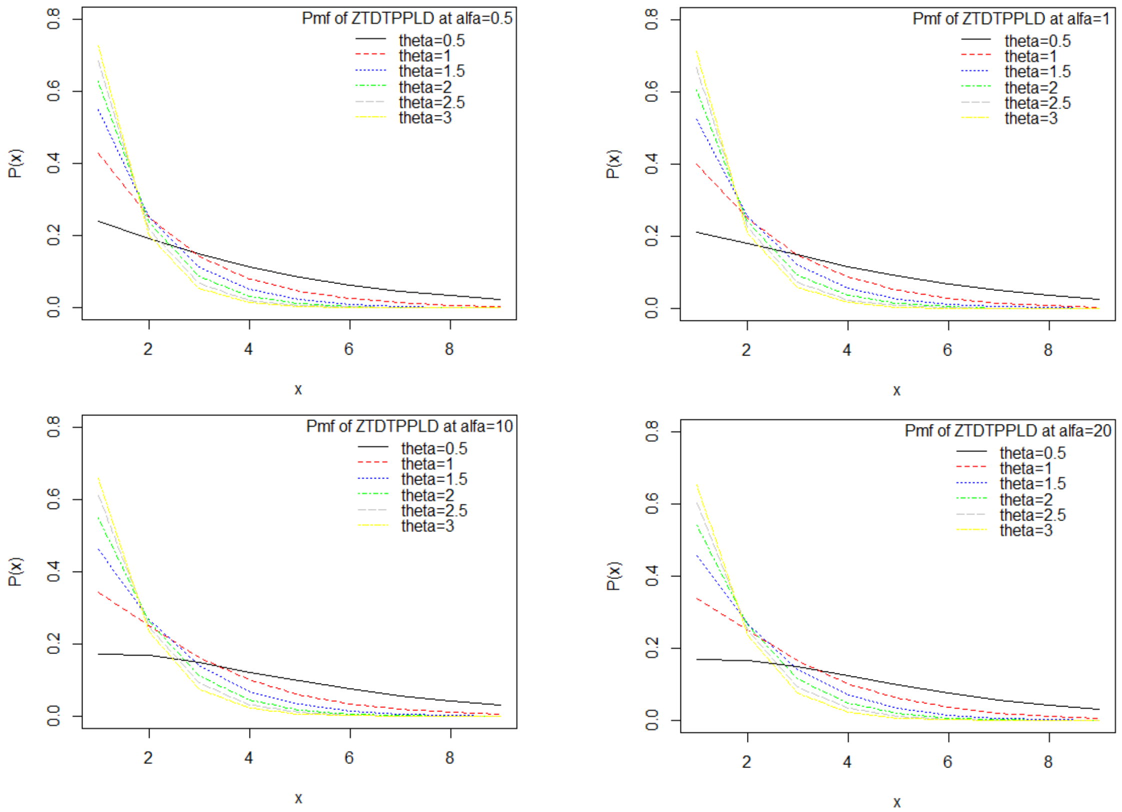

***Fig.1**. Graph of the probability mass function of ZTDTPPLD for varying values of parameters α and θ.*

The ZTDTPPLD (2.1) can also be obtained from size-biased Poisson distribution (SBPD) having pmf

$$g\left(x\,|\,\lambda\right)=\frac{e^{-\lambda}\lambda^{x-1}}{\left(x-1\right)!}\quad ;x=1,2,3,\dots\,,\lambda>0 \tag{2.4}$$

when the parameter λ of SBPD follows a continuous distribution having pdf

$$h\left(\lambda;\theta,\alpha\right)=\frac{\theta^2}{\theta^2+2\theta\alpha+\theta+\alpha}\left[\alpha\left(\theta+1\right)\lambda+\left(\theta+\alpha+1\right)\right]e^{-\theta\lambda};\lambda>0,\theta>0,\theta^2+2\theta\alpha+\theta+\alpha>0 \tag{2.5}$$

Thus, the pmf of ZTDTPPLD can be obtained as

$$\begin{aligned}
P\left(x;\theta,\alpha\right)&=\int_0^\infty g\left(x\,|\,\lambda\right)\cdot h\left(\lambda;\theta,\alpha\right)d\lambda\\
&=\int_0^\infty \frac{e^{-\lambda}\lambda^{x-1}}{\left(x-1\right)!}\cdot\frac{\theta^2}{\theta^2+2\theta\alpha+\theta+\alpha}\left[\alpha\left(\theta+1\right)\lambda+\left(\theta+\alpha+1\right)\right]e^{-\theta\lambda}\,d\lambda \qquad (2.6)\\
&=\frac{\theta^2}{\left[\theta^2+2\theta\alpha+\theta+\alpha\right]\left(x-1\right)!}\int_0^\infty e^{-(\theta+1)\lambda}\left[\alpha\left(\theta+1\right)\lambda^x+\left(\theta+\alpha+1\right)\lambda^{x-1}\right]d\lambda\\
&=\frac{\theta^2}{\left[\theta^2+2\theta\alpha+\theta+\alpha\right]\left(x-1\right)!}\left[\frac{\alpha\left(\theta+1\right)\Gamma\left(x+1\right)}{\left(\theta+1\right)^{x+1}}+\frac{\left(\theta+\alpha+1\right)\Gamma\left(x\right)}{\left(\theta+1\right)^x}\right]\\
&=\frac{\theta^2}{\theta^2+2\theta\alpha+\theta+\alpha}\frac{\alpha x+\left(\theta+\alpha+1\right)}{\left(\theta+1\right)^x};x=1,2,3,...,\ \theta>0,\ \theta^2+2\theta\alpha+\theta+\alpha>0
\end{aligned}$$

which is the pmf of ZTDTPPLD with parameter θ and α as given in (2.1).

MOMENTS OF ZTDTPPLD

The r th factorial moment about origin of ZTDTPPLD (2.1) can be obtained as

$\mu_{(r)}{}'=E\left[E\left(X^{(r)}\,|\,\lambda\right)\right]$; where $X^{(r)}=X\left(X-1\right)\left(X-2\right)....\left(X-r+1\right)$.

Using (2.6), we have

$$\begin{aligned}
\mu_{(r)}{}'&=\frac{\theta^2}{\theta^2+2\theta\alpha+\theta+\alpha}\int_0^\infty\left[\sum_{x=1}^\infty x^{(r)}\frac{e^{-\lambda}\lambda^{x-1}}{\left(x-1\right)!}\right]\cdot\left[\alpha\left(\theta+1\right)\lambda+\left(\theta+\alpha+1\right)\right]e^{-\theta\lambda}d\lambda\\
&=\frac{\theta^2}{\theta^2+2\theta\alpha+\theta+\alpha}\int_0^\infty\left[\lambda^{r-1}\sum_{x=r}^\infty x\frac{e^{-\lambda}\lambda^{x-r}}{\left(x-r\right)!}\right]\cdot\left[\alpha\left(\theta+1\right)\lambda+\left(\theta+\alpha+1\right)\right]e^{-\theta\lambda}d\lambda
\end{aligned}$$

Taking $y=x-r$, we get

$$\begin{aligned}
\mu_{(r)}{}'&=\frac{\theta^2}{\theta^2+2\theta\alpha+\theta+\alpha}\int_0^\infty\left[\lambda^{r-1}\sum_{y=0}^\infty\left(y+r\right)\frac{e^{-\lambda}\lambda^{y}}{y!}\right]\cdot\left[\alpha\left(\theta+1\right)\lambda+\left(\theta+\alpha+1\right)\right]e^{-\theta\lambda}d\lambda\\
&=\frac{\theta^2}{\theta^2+2\theta\alpha+\theta+\alpha}\int_0^\infty\lambda^{r-1}\left(\lambda+r\right)\cdot\left[\alpha\left(\theta+1\right)\lambda+\left(\theta+\alpha+1\right)\right]e^{-\theta\lambda}d\lambda
\end{aligned}$$

Using gamma integral and a little algebraic simplification, we get the expression for the r th factorial moment about origin of ZTDTPPLD (2.1) as

$$\mu_{(r)}' = \frac{r!(\theta+1)^2\left\{\theta+(r+1)\alpha\right\}}{\theta^r\left(\theta^2+2\theta\alpha+\theta+\alpha\right)} \; ; r=1,2,3,... \tag{3.1}$$

Substituting $r=1,2,3,\text{and}\,4$ in equation (3.1), the first four factorial moments about origin can be obtained and using the relationship between moments about origin and factorial moments about origin, the first four moments about origin of ZTDTPPLD (2.1) are obtained as

$$\mu_1' = \frac{(\theta+1)^2(\theta+2\alpha)}{\theta\left(\theta^2+2\theta\alpha+\theta+\alpha\right)}$$

$$\mu_2' = \frac{(\theta+1)^2\left\{\theta^2+2(\alpha+1)\theta+6\alpha\right\}}{\theta^2\left(\theta^2+2\theta\alpha+\theta+\alpha\right)}$$

$$\mu_3' = \frac{(\theta+1)^2\left\{\theta^3+2(\alpha+3)\theta^2+6(3\alpha+1)\theta+24\alpha\right\}}{\theta^3\left(\theta^2+2\theta\alpha+\theta+\alpha\right)}$$

$$\mu_4' = \frac{(\theta+1)^2\left\{\theta^4+2(\alpha+7)\theta^3+6(7\alpha+6)\theta^2+24(6\alpha+1)\theta+120\alpha\right\}}{\theta^4\left(\theta^2+2\theta\alpha+\theta+\alpha\right)}$$

Again using the relationship between moments about origin and moments about mean, the moments about mean of ZTDTPPLD (2.1) are obtained as

$$\mu_2 = \sigma^2 = \frac{(\theta+1)^2\left\{\theta^3+(5\alpha+1)\theta^2+\left(6\alpha^2+4\alpha\right)\theta+2\alpha^2\right\}}{\theta^2\left(\theta^2+2\theta\alpha+\theta+\alpha\right)^2}$$

$$\mu_3 = \frac{(\theta+1)^2\begin{Bmatrix}\theta^6+(7\alpha+4)\theta^5+\left(16\alpha^2+28\alpha+5\right)\theta^4+\left(12\alpha^3+59\alpha^2+33\alpha+2\right)\theta^3\\ +\left(38\alpha^2+54\alpha+12\right)\theta^2\alpha+(22\alpha+12)\theta\alpha^2+4\alpha^3\end{Bmatrix}}{\theta^3\left(\theta^2+2\theta\alpha+\theta+\alpha\right)^3}$$

$$\mu_4 = \frac{(\theta+1)^2\begin{Bmatrix}\theta^9+(9\alpha+12)\theta^8+\left(30\alpha^2+114\alpha+39\right)\theta^7+\left(44\alpha^3+389\alpha^2+363\alpha+55\right)\theta^6\\ \left(24\alpha^4+572\alpha^3+1147\alpha^2+492\alpha+36\right)\theta^5+\left(308\alpha^4+1497\alpha^3+1376\alpha^2+306\alpha+9\right)\theta^4\\ \left(686\alpha^3+1508\alpha^2+720\alpha+72\right)\theta^3\alpha+\left(554\alpha^2+636\alpha+132\right)\theta^2\alpha^2+(192\alpha+96)\theta\alpha^3+24\alpha^4\end{Bmatrix}}{\theta^4\left(\theta^2+2\theta\alpha+\theta+\alpha\right)^4}$$

The coefficient of variation (C.V), coefficient of Skewness $\left(\sqrt{\beta_1}\right)$, coefficient of Kurtosis $\left(\beta_2\right)$ and index of dispersion (γ) of ZTDTPPLD (2.1) are obtained as

$$C.V. = \frac{\sigma}{\mu_1'} = \frac{\sqrt{\left\{\theta^3+(5\alpha+1)\theta^2+\left(6\alpha^2+4\alpha\right)\theta+2\alpha^2\right\}}}{(\theta+1)(\theta+2\alpha)}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{(\mu_2)^{3/2}} = \frac{\begin{Bmatrix} \theta^6 + (7\alpha+4)\theta^5 + (16\alpha^2+28\alpha+5)\theta^4 + (12\alpha^3+59\alpha^2+33\alpha+2)\theta^3 \\ +(38\alpha^2+54\alpha+12)\theta^2\,\alpha + (22\alpha+12)\theta\,\alpha^2 + 4\alpha^3 \end{Bmatrix}}{(\theta+1)\left\{\theta^3+(5\alpha+1)\theta^2+(6\alpha^2+4\alpha)\theta+2\alpha^2\right\}^{3/2}}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^{\,2}} = \frac{\begin{Bmatrix} \theta^9 + (9\alpha+12)\theta^8 + (30\alpha^2+114\alpha+39)\theta^7 + (44\alpha^3+389\alpha^2+363\alpha+55)\theta^6 + \\ (24\alpha^4+572\alpha^3+1147\alpha^2+492\alpha+36)\theta^5 + (308\alpha^4+1497\alpha^3+1376\alpha^2+306\alpha+9)\theta^4 + \\ (686\alpha^3+1508\alpha^2+720\alpha+72)\theta^3\alpha + (554\alpha^2+636\alpha+132)\theta^2\alpha^2 + (192\alpha+96)\theta\alpha^3 + 24\alpha^4 \end{Bmatrix}}{(\theta+1)^2\left\{\theta^3+(5\alpha+1)\theta^2+(6\alpha^2+4\alpha)\theta+2\alpha^2\right\}^2}$$

$$\gamma = \frac{\sigma^2}{\mu_1'} = \frac{\theta^3+(5\alpha+1)\theta^2+(6\alpha^2+4\alpha)\theta+2\alpha^2}{\theta(\theta+2\alpha)(\theta^2+2\theta\alpha+\theta+\alpha)}$$

The nature of coefficient of variation, coefficient of skewness, coefficient of kurtosis, and index of dispersion of ZTDTPPLD (2.1) are shown graphically in figure 2.

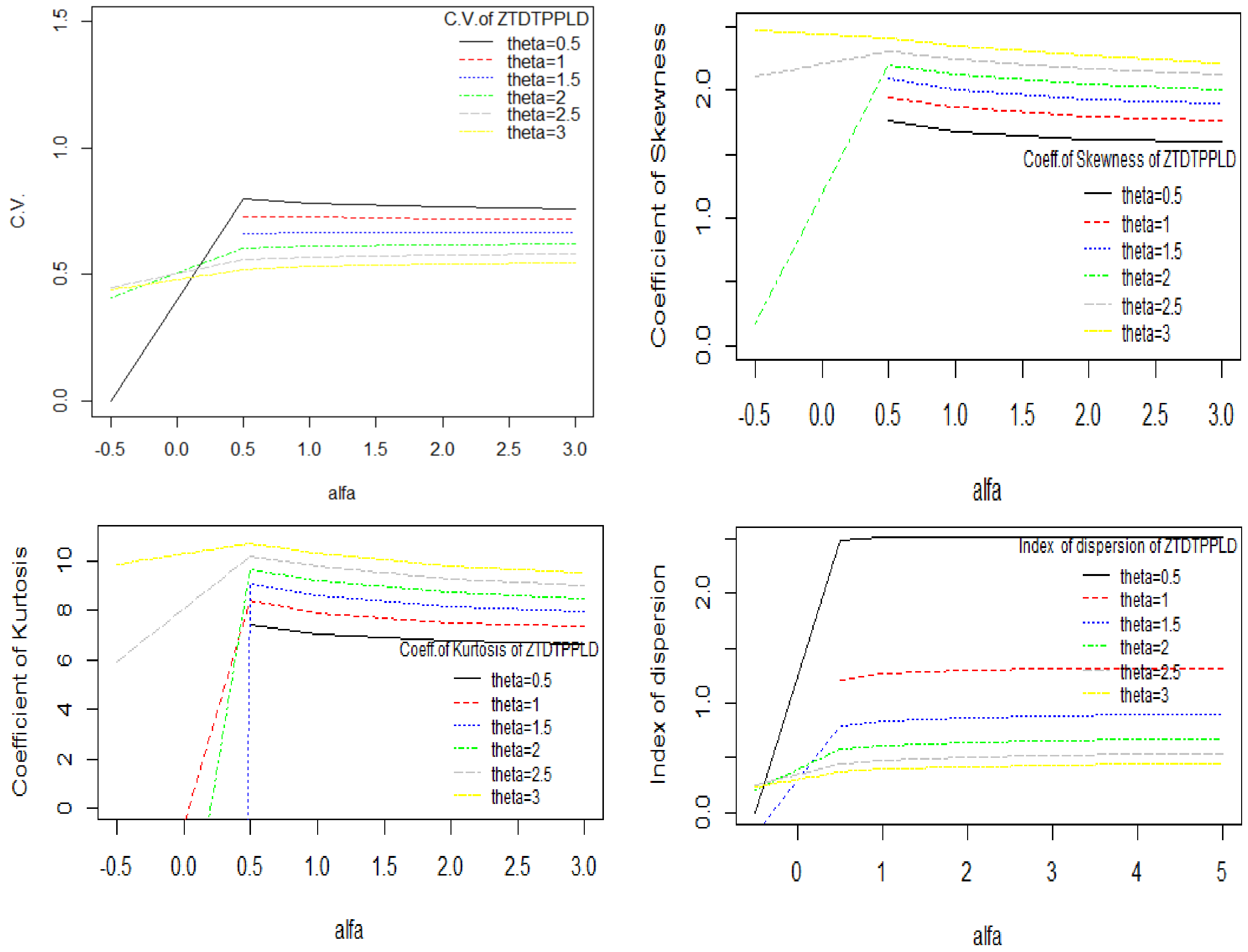

Fig. 2. Coefficient of variation (CV), Coefficient of skewness, coefficient of kurtosis and index of dispersion plot for different values of α and θ.

MAXIMUM LIKELIHOOD ESTIMATION OF PARAMETERS

Let $(x_1, x_2, ..., x_n)$ be a random sample of size n from the ZTDTPPLD (2.1) and let f_x be the observed frequency in the sample corresponding to $X = x\ (x = 1, 2, 3, ..., k)$ such that $\sum_{x=1}^{k} f_x = n$, where k is the largest observed value having non-zero frequency. The likelihood function L of the ZTDTPPLD (2.1) is given by

$$L = \left(\frac{\theta^2}{\theta^2 + 2\theta\alpha + \theta + \alpha}\right)^n \frac{1}{(\theta+1)^{\sum_{x=1}^{k} x f_x}} \prod_{x=1}^{k} \left[\alpha x + (\theta + \alpha + 1)\right]^{f_x}$$

The log likelihood function is thus obtained as

$$\log L = n \log\left(\frac{\theta^2}{\theta^2 + 2\theta\alpha + \theta + \alpha}\right) - \sum_{x=1}^{k} x\, f_x \log(\theta+1) + \sum_{x=1}^{k} f_x \log\left[\alpha x + (\theta + \alpha + 1)\right]$$

The maximum likelihood estimates $(\hat{\theta}, \hat{\alpha})$ of (θ, α) of ZTDTPPLD (2.1) is the solutions of the following log likelihood equations

$$\frac{\partial \log L}{\partial \theta} = \frac{2n}{\theta} - \frac{n(2\theta + 2\alpha + 1)}{\theta^2 + 2\theta\alpha + \theta + \alpha} - \frac{n\bar{x}}{\theta+1} + \sum_{x=1}^{k} \frac{f_x}{\left[\alpha x + (\theta + \alpha + 1)\right]} = 0$$

$$\frac{\partial \log L}{\partial \alpha} = \frac{-n(2\theta+1)}{\theta^2 + 2\theta\alpha + \theta + \alpha} + \sum_{x=1}^{k} \frac{(x+1)\, f_x}{\left[\alpha x + (\theta + \alpha + 1)\right]} = 0$$

where $\bar{x}$ is the sample mean.

These two log likelihood equations do not seem to be solved directly. However, the Fisher's scoring method can be applied to solve these equations. We have

$$\frac{\partial^2 \log L}{\partial \theta^2} = -\frac{2n}{\theta^2} + \frac{n\left\{2\theta(\theta + 2\alpha + 1) + (4\alpha^2 + 2\alpha + 1)\right\}}{\left(\theta^2 + 2\theta\alpha + \theta + \alpha\right)^2} + \frac{n\bar{x}}{(\theta+1)^2} - \sum_{x=1}^{k} \frac{f_x}{\left[\alpha x + (\theta + \alpha + 1)\right]^2}$$

$$\frac{\partial^2 \log L}{\partial \alpha^2} = \frac{n(2\theta+1)^2}{\left(\theta^2 + 2\theta\alpha + \theta + \alpha\right)^2} - \sum_{x=1}^{k} \frac{(x+1)^2\, f_x}{\left[\alpha x + (\theta + \alpha + 1)\right]^2}$$

$$\frac{\partial^2 \log L}{\partial \theta \partial \alpha} = \frac{n\left(2\theta^2 + 2\theta + 2\alpha + 1\right)}{\left(\theta^2 + 2\theta\alpha + \theta + \alpha\right)^2} - \sum_{x=1}^{k} \frac{(x+1)\, f_x}{\left[\alpha x + (\theta + \alpha + 1)\right]^2} = \frac{\partial^2 \log L}{\partial \alpha \partial \theta}$$

For the maximum likelihood estimates $(\hat{\theta}, \hat{\alpha})$ of (θ, α) of ZTDTPPLD (2.1), following equations can be solved

$$\begin{bmatrix} \frac{\partial^2 \log L}{\partial \theta^2} & \frac{\partial^2 \log L}{\partial \theta \partial \alpha} \\ \frac{\partial^2 \log L}{\partial \theta \partial \alpha} & \frac{\partial^2 \log L}{\partial \alpha^2} \end{bmatrix}_{\substack{\hat{\theta}=\theta_0 \\ \hat{\alpha}=\alpha_0}} \begin{bmatrix} \hat{\theta} - \theta_0 \\ \hat{\alpha} - \alpha_0 \end{bmatrix} = \begin{bmatrix} \frac{\partial \log L}{\partial \theta} \\ \frac{\partial \log L}{\partial \alpha} \end{bmatrix}_{\substack{\hat{\theta}=\theta_0 \\ \hat{\alpha}=\alpha_0}}$$

where θ_0 and α_0 are the initial values of θ and α, respectively. These equations are solved iteratively till sufficiently close values of $\hat{\theta}$ and $\hat{\alpha}$ are obtained. In this paper R software has been used to estimate parameters of the ZTDTPPLD.

GOODNESS OF FIT

In this section, we present the goodness of fit of ZTDTPPLD, ZTPD and ZTPLD for four count data sets. The first data set is due to Finney and Varley (1955) who gave counts of number of flower having number of fly eggs. The second data set is due to Singh and Yadav (1971) regarding the number of households having at least one migrant from households according to the number of migrants. The third data set is regarding the number of counts of sites with particles from Immunogold data, reported by Mathews and Appleton (1993). The fourth data set is regarding the number of snowshoe hares counts captured over 7 days, reported by Keith and Meslow (1968).

Table 1: Number of flower heads with number of fly eggs, reported by Finney and Varley (1955).

Number of fly eggs	Number of flowers	Expected Frequency		
		ZTPD	ZTPLD	ZTDTPPLD
1	22	15.3	26.8	25.0
2	18	21.8	19.8	20.3
3	18	20.8	14.0	14.8
4	11	14.9	9.5	10.1
5	9	8.5	6.3	6.6
6	6	4.0	4.2	4.2
7	3	1.7	2.7	2.6
8	0	0.6	1.7	1.6
9	1	0.4	3.0	2.8
Total	**88**	**88.0**	**88.0**	**88.0**
ML Estimate		$\hat{\theta} = 2.8604$	$\hat{\theta} = 0.7186$	$\hat{\theta} = 0.82407$ $\hat{\alpha} = 25.41431$
χ^2		6.648	3.780	2.39
d.f.		4	4	3
P-value		0.1557	0.4366	0.4955

Table 2: Number of households having at least one migrant according to the number of migrants, reported by Singh and Yadav (1971).

Number of migrants	Observed frequency	Expected Frequency		
		ZTPD	ZTPLD	ZTDTPPLD
1	375	354.0	379.0	376.4
2	143	167.7	137.2	140.2
3	49	52.9	48.4	49.0
4	17	12.5	16.7	16.5
5	2	2.4	5.7	5.3
6	2	0.4	1.9	1.7
7	1	0.1	0.6	0.6
8	1	0.0	0.5	0.3
Total	590	590.0	590.0	590.0
ML Estimate		$\hat{\theta} = 0.9475$	$\hat{\theta} = 2.2848$	$\hat{\theta} = 2.56082$ $\hat{\alpha} = 3.33174$
χ^2		8.922	1.138	0.51
d.f.		2	3	2
P-value		0.0115	0.7679	0.7749

Table 3: The number of counts of sites with particles from Immunogold data, reported by Mathews and Appleton (1993)

Number of sites with particles	Observed frequency	Expected Frequency		
		ZTPD	**ZTPLD**	**ZTDTPPLD**
1	122	115.8	124.7	123.0
2	50	57.4	46.7	48.7
3	18	18.9	17.0	17.5
4	4	4.7	6.1	5.9
5	4	5.9	3.5	2.9
Total	198	198.0	198.0	198.0
ML Estimate		$\hat{\theta}=0.9906$	$\hat{\theta}=2.1831$	$\hat{\theta}=2.65049$ $\hat{\alpha}=15.23738$
χ^2		2.140	0.617	0.11
d.f.		2	2	1
P-value		0.3430	0.7345	0.7401

Table 4: The number of snowshoe hares counts captured over 7 days, reported by Keith and Meslow (1968)

Number of times hares caught	Observed frequency	Expected Frequency		
		ZTPD	**ZTPLD**	**ZTDTPPLD**
1	184	174.6	182.6	183.1
2	55	66.0	55.3	54.6
3	14	16.6	16.4	16.3
4	4	3.2	4.8	4.8
5	4	0.6	1.9	2.2
Total	261	261.0	261.0	261.0
ML Estimate		$\hat{\theta}=0.7563$	$\hat{\theta}=2.8639$	$\hat{\theta}=2.35111$ $\hat{\alpha}=-0.000236$
χ^2		2.464	0.615	0.46
d.f.		1	2	1
P-value		0.1165	0.7353	0.4976

It is obvious from the goodness of fit of ZTDTPPLD, ZTPD, and ZTPLD that ZTDTPPLD gives better fit in tables 1, 2 and 3, while in table 4 ZTPLD gives better fit. The nature of the probability mass functions of the fitted distributions, ZTDTPPLD, ZTPLD, and ZTPD for four data sets has been shown graphically in the figure 3.

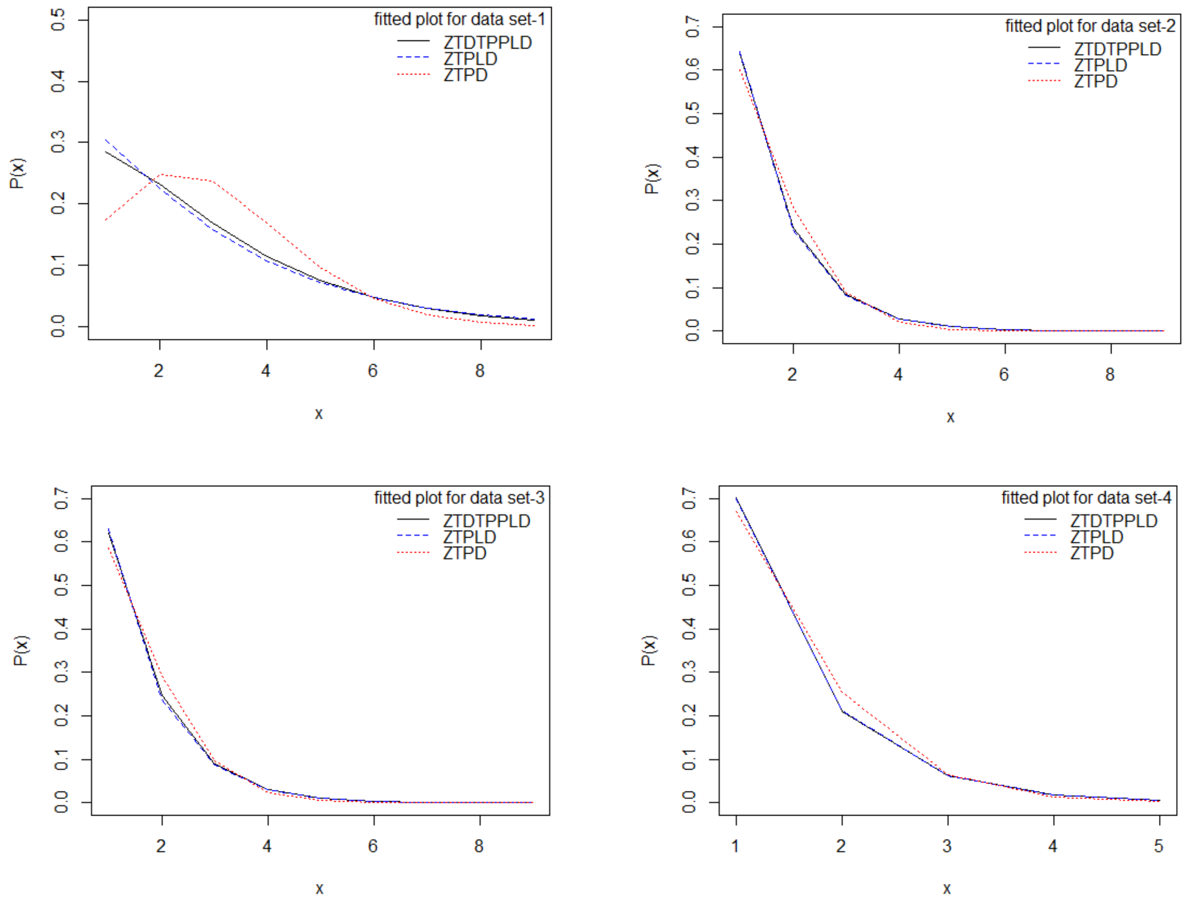

Fig. 3. Probability plots of ZTDTPPLD, ZTPLD and ZTPD for fitted data sets in table 1, 2, 3, and 4.

CONCLUSIONS

In this paper, a zero-truncated discrete two-parameter Poisson-Lindley distribution (ZTDTPPLD), of which zero-truncated Poisson-Lindley distribution (ZTPLD) is a particular case, has been derived by compounding size-biased Poisson distribution (SBPD) with a continuous distribution. Its moments, and moments based measures including coefficient of variation, skewness, kurtosis, and index of dispersion have been obtained and their nature and behavior have been studied graphically. Maximum likelihood estimation has been discussed for estimating the parameters of ZTDTPPLD and the goodness of fit has been discussed with four data sets and in majority of data sets ZTDTPPLD shows quite satisfactory fit over ZTPD and ZTPLD.

ACKNOWLEDGEMENTS

Authors are grateful to the Editor-In-Chief of the Journal and the anonymous reviewer for constructive suggestions which improved the quality and the presentation of the paper.

REFERENCES

Finney, D. J. and Varley, G. C. (1955). An example of the truncated Poisson distribution. *Biometrics*, **11:** 387–394.

Ghitany, M. E.; Atieh, B. and Nadarajah, S. (2008 a). Lindley distribution and Its Applications, *Mathematics Computation and Simulation*, **78**: 493-506.

Ghitany, M. E.; Al-Mutairi, D. K. and Nadarajah, S. (2008b). Zero-truncated Poisson-Lindley distribution and its Applications. *Mathematics and Computers in Simulation*, **79** (3): 279–287.

Keith, L. B. and Meslow, E. C. (1968). Trap response by snowshoe hares. *Journal of Wildlife Management*, **32**: 795–801.

Lindley, D. V. (1958). Fiducial distributions and Bayes' Theorem. *Journal of the Royal Statistical Society*, Series B, **20**: 102–107.

Mathews, J. N. S. and Appleton, D. R. (1993). An application of the Truncated Poisson Distribution to Immunogold Assay. *Biometrics*, **49**: 617 – 621

Sankaran, M. (1970). The discrete Poisson-Lindley distribution. *Biometrics*, **26**: 145-149.

Shanker, R.; Sharma, S. and Shanker, R. (2012). A Discrete Two Parameter Poisson Lindley Distribution. *Journal of Ethiopian Statistical Association,* **21**: 22 - 29.

Shanker, R.; Sharma, S. and Shanker, R. (2013). A Two Parameter Lindley Distribution for Modeling Waiting and Survival Times Data. *Applied Mathematics*, **4** (2): 363 - 368

Shanker, R. and Hagos, F. (2015). On Poisson-Lindley distribution and Its Applications to Biological Sciences. *Biometrics & Biostatistics International Journal*, **2** (4): 1-5.

Shanker, R.; Hagos, F.; Sujatha, S. and Abrehe, Y. (2015). On Zero-truncation of Poisson and Poisson-Lindley distributions and Their Applications. *Biometrics & Biostatistics International Journal*, **2** (6): 1-14.

Singh, S. N. and Yadav, R. C. (1971). Trends in rural out-migration at household level, *Rural Demography*, **8**: 53-61.