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ABSTRACT

In this study, we investigate the Klein-Nishina differential cross-section equation for total cross-section and extend it to calculate the total molecular cross-section for compounds Al$_2$O$_3$, PbO, and Fe$_2$O$_3$. Our findings reveal that the total molecular cross-section of these compounds is significantly larger, with values 5 times greater than the total atomic cross-section. Furthermore, we determine that the molecular cross-section of Al$_2$O$_3$, PbO, and Fe$_2$O$_3$ is 73, 132, and 118 times greater than the total electronic cross-section, respectively, while the atomic cross-section of these compounds is 15, 66, and 24 times greater. At low energy levels ranging from 1-5 MeV, the entire molecular, atomic, and electronic cross-section dominates due to Compton scattering. However, as the photon energy increases, Compton scattering becomes negligible, and a slight contribution from pair production scattering is observed. We also establish the adequate atomic numbers for Al$_2$O$_3$, PbO, and Fe$_2$O$_3$, which are determined to be 15, 66, and 24, respectively. These results highlight the significance of mass attenuation, cross-section, and adequate atomic number in the selection of radiation shielding materials for various protection purposes. The findings from this study provide valuable insights into the properties and behavior of these compounds, enabling informed decisions in radiation shielding applications.

Keywords: Radiation shielding material, Cross-section, Compton scattering, Compound, Mass attenuation.

INTRODUCTION

The Mass attenuation coefficients (MAC) of silver(Ag)/copper(Cu)/zinc(Zn) alloy with Ag 14.80%, Cu 57.61%, and Zn 27.59% weight fraction determined in the energy range 220 to 662 keV with gamma rays has good results agree with the theoretical values with error less than 1%. However, MAC decreased with increasing gamma ray’s energies because photons interacted with Ag/Cu/Zn alloy [1]. Gamma attenuation behavior on commercial stainless steels and boron steels was observed by [2] using XCOM computer code and found that Theoretical and experimental MAC are closely related. Shield material selections based on the energy of photon and Lead is a highly shielding material with a suitable density of 11.35g/cm$^3$, high atomic number, and inexpensive. Furthermore, MAC, MEAC, and kerma relative to air and kerma values differed between Fe–Ce and Fe-Ni alloys due to photoelectric cross-sections that vary with an atomic number [3]. Several composite materials assigned are d for X-ray and gamma photon interactions as shielding materials. The shielding material thickness for the authors satisfies $2 \leq \ln \left( \frac{\mu}{\rho} \right) \leq 4$ with transmission $0.5 \geq T \geq 0.25$. MAC for materials composed of various elements, mixture rule, is necessary and expressed by Morabadi and Kerur in 2010 as

$$\mu_{\text{comp}} = \sum w_i \mu_i \quad \text{........................................ (1)}$$
The probability of a photon interacting with the material per unit path length is called LAC. Photon attenuation coefficients depend on the photon energy and the material density, which is important for radiation shielding [4]. Here $w_t (\mu_i)$ denoted as weight fraction and MAC of $i$th element and material composed of multi-elements is expressed as,

$$w_t = \frac{n_i A_i}{\sum n_i A_i}$$

(2)

Here $A_i$, $n_i$ represent the atomic weight of the $i$th element and the number of formula units. Now, the total molecular cross-section ($\sigma^m_t$) it can be written as

$$\sigma^m_t = \frac{1}{N_A} \sum_i \left( \mu_i \right) n_i A_i$$

(3)

Also, the total atomic cross-section ($\sigma^a_t$) for the element is expressed [5] as

$$\sigma^a_t = \frac{\sigma^m_t}{\sum n_i}$$

(4)

Also, the total electronic cross-section ($\sigma^e_t$) for the element is expressed as

$$\sigma^e_t = \frac{1}{N_A} \sum_i f_i A_i \left( \frac{\mu_i}{\rho_i} \right), f_i = \left( \frac{n_i}{\sum n_i} \right)$$

(5)

Here $f_i$, $Z_i$ called a fractional abundance of the element $i$th and atomic number number of, the Effective Atomic number of the compounds represented [6] as $Z_{eff} = \frac{\sigma^a_t}{\sigma^e_t}$

$$\left( \frac{d\sigma}{d\Omega} \right)_a = \frac{Zr^2}{2} \left( \frac{1}{1+\alpha(1-\cos\theta)} \right)^2 \left( 1 + \cos^2 \theta \right) + \frac{\alpha^2(1-\cos\theta)^2}{1+\alpha(1-\cos\theta)}$$

(7)

This equation is a differential atomic cross-sectional area equation for K-N. Also, the total K-N cross-section per atom is written as,

$$\sigma^a_t = 2\pi \int_0^\pi \left( \frac{d\sigma}{d\Omega} \right)_a \sin\theta d\theta$$

(8)

Here $\theta$ is scattering angle overall photons. Now from (7) and (8), we get

$$\sigma^a_t = 2\pi \int_0^\pi \frac{Zr^2}{2} \left( \frac{1}{1+\alpha(1-\cos\theta)} \right)^2 \left( 1 + \cos^2 \theta \right) + \frac{\alpha^2(1-\cos\theta)^2}{1+\alpha(1-\cos\theta)} \sin\theta d\theta$$

(9)

On solving the total KN cross-section per atom is obtained as,

$$\sigma^a_t = \frac{Z2\pi r_0^2 \left( \frac{1+\alpha}{\alpha^2} \left[ \frac{2(1+\alpha)}{1+2\alpha} \ln \left( 1+2\alpha \right) \right] + \frac{\ln \left( 1+2\alpha \right)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^2} \right)}{\ln(1+2\alpha)}$$

Since Klein-Nishina atomic cross-sections were obtained by multiplying electronic cross-sections with charge number $Z$ of each element that is $\sigma_a = Z. \sigma_e$, therefore from equation (9), the electronic cross-sectional area for KN is
\[ \sigma_{t}^{e} = 2\pi r_{0}^{2} \left( \frac{1+\alpha}{a^{2}} \left[ \frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha} \right] + \frac{\ln(1+2\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^{2}} \right) \] ..................................................(10)

Where, \( r_{0} = 2.818 \times 10^{-13} m \) is the classical electron radius, \( Z \) is the nuclear charge of the target molecule, and \( \alpha = \frac{E}{m_{e}c^{2}} = \frac{hf}{0.511 MeV} \) [7]. On putting the value of \( \sigma_{e} \) in \( \frac{\mu}{\rho} = \sigma_{e}Z_{N}^{e} \frac{A}{A} \) we get,

\[ \frac{\mu}{\rho} = 2\pi r_{0}^{2} \left( \frac{Z_{N}^{e} A}{A} \right) \left( \frac{1+\alpha}{a^{2}} \left[ \frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha} \right] + \frac{\ln(1+2\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^{2}} \right) \] ..................................................(11)

Therefore, this equation gives mass attenuation coefficient in terms of KN parameters and known as Compton mass attenuation coefficient is provided by using, \( \frac{\mu}{\rho} = N_{A}Z_{N}^{e} \frac{A}{A} \). Where \( N_{A} \) is the Avogadro's number \((6.02 \times 10^{23} \text{ atom/mol})\), \( Z \) is the atomic number, and \( A \) is the material atomic mass [8].

**Shielding Properties of compound \( Al_{2}O_{3} \)**

Now from equation (11) for \( Al_{2}O_{3} \), we have mass attenuation coefficient is obtained as,

\[ \left( \frac{\mu}{\rho} \right)_{Al_{2}O_{3}} = 0.4995 N_{A} 2\pi r_{0}^{2} \left( \frac{1+\alpha}{a^{2}} \left[ \frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha} \right] + \frac{\ln(1+2\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^{2}} \right) \] ..................................................(12)

The total molecular cross-section of \( Al_{2}O_{3} \) from equation (3) and (12) is obtained as

\[ (\sigma_{t}^{m})_{Al_{2}O_{3}} = 2491.0194 \times 10^{-26} \left( \frac{1+\alpha}{a^{2}} \left[ \frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha} \right] + \frac{\ln(1+2\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^{2}} \right) \] ..................................................(13)

The total atomic cross-section of \( Al_{2}O_{3} \) from equation (4) and (12) is obtained as

\[ (\sigma_{t}^{a})_{Al_{2}O_{3}} = 498.20 \times 10^{-26} \left( \frac{1+\alpha}{a^{2}} \left[ \frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha} \right] + \frac{\ln(1+2\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^{2}} \right) \] ..................................................(14)

The electronic cross-section area of \( Al_{2}O_{3} \) from (5) and (12) calculated as

\[ (\sigma_{t}^{e})_{Al_{2}O_{3}} = 33.99 \times 10^{-26} \left( \frac{1+\alpha}{a^{2}} \left[ \frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha} \right] + \frac{\ln(1+2\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^{2}} \right) \] ..................................................(15)

Now, adequate atomic numbers for \( Al_{2}O_{3} \) are obtained as \( Z_{eff} \) \( Al_{2}O_{3} = \frac{(\sigma_{t}^{m})_{Al_{2}O_{3}}}{(\sigma_{t}^{e})_{Al_{2}O_{3}}} \approx 15 \).

**Shielding Properties of compound \( PbO \)**

Now from equation (11) for \( PbO \), we have mass attenuation coefficient is obtained as,

\[ \left( \frac{\mu}{\rho} \right)_{PbO} = 0.38 N_{A} 2\pi r_{0}^{2} \left( \frac{1+\alpha}{a^{2}} \left[ \frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha} \right] + \frac{\ln(1+2\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^{2}} \right) \] ..................................................(16)

The total molecular cross-section of \( PbO \) from equations (3) and (16) obtained as

\[ a(\sigma_{t}^{m})_{PbO} = 4473.36 \times 10^{-26} \left( \frac{1+\alpha}{a^{2}} \left[ \frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha} \right] + \frac{\ln(1+2\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^{2}} \right) \] ..................................................(17)

The total atomic cross-section of \( PbO \) from equations (4) and (16) obtained as

\[ (\sigma_{t}^{a})_{PbO} = 2236.68 \times 10^{-26} \left( \frac{1+\alpha}{a^{2}} \left[ \frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha} \right] + \frac{\ln(1+2\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^{2}} \right) \] ..................................................(18)

The electronic cross-section area of \( PbO \) from (5) and (16) calculated as

\[ (\sigma_{t}^{e})_{PbO} = 33.99 \times 10^{-26} \left( \frac{1+\alpha}{a^{2}} \left[ \frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha} \right] + \frac{\ln(1+2\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^{2}} \right) \] ..................................................(19)
Radiation Shielding Properties of Oxides (Al$_2$O$_3$, PbO, and Fe$_2$O$_3$) based on Klein-Nishina Cross-section

Now, adequate atomic number for PbO is accommodating as, \( Z_{\text{eff}}^{\text{PbO}} = \frac{\langle \sigma_f \rangle_{\text{PbO}}}{\langle \sigma_f \rangle_{\text{PbO}}} \approx 66 \).

### Shielding Properties of compound Fe$_2$O$_3$

Now from equation (11) for Fe$_2$O$_3$ we have mass attenuation coefficient is obtained as,

\[
\left( \mu \rho \right)_{\text{Fe}_2\text{O}_3} = 0.50N_A 2\pi r_d^2 \left\{ \frac{1+\alpha}{\alpha^2} \left[ \frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{2\alpha} \right] + \frac{\ln(1+\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^2} \right\} \tag{20}
\]

The total molecular cross-section of Fe$_2$O$_3$ from equation (3) and (20) is obtained as

\[
\langle \sigma_m \rangle_{\text{Fe}_2\text{O}_3} = 3989.62 \times 10^{-26} \left\{ \frac{1+\alpha}{\alpha^2} \left[ \frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{2\alpha} \right] + \frac{\ln(1+\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^2} \right\} \tag{21}
\]

The total atomic cross-section of Fe$_2$O$_3$ from equation (4) and (20) is obtained as

\[
\langle \sigma_a \rangle_{\text{Fe}_2\text{O}_3} = 797.92 \times 10^{-26} \left\{ \frac{1+\alpha}{\alpha^2} \left[ \frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{2\alpha} \right] + \frac{\ln(1+\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^2} \right\} \tag{22}
\]

The total electronic cross-section area of Fe$_2$O$_3$ from (5) and (20) is obtained as

\[
\langle \sigma_e \rangle_{\text{Fe}_2\text{O}_3} = 33.99 \times 10^{-26} \left\{ \frac{1+\alpha}{\alpha^2} \left[ \frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{2\alpha} \right] + \frac{\ln(1+\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^2} \right\}
\]

Now, adequate atomic numbers for Fe$_2$O$_3$ are obtained as \( Z_{\text{eff}}^{\text{Fe}_2\text{O}_3} = \frac{\langle \sigma_e \rangle_{\text{Fe}_2\text{O}_3}}{\langle \sigma_e \rangle_{\text{Fe}_2\text{O}_3}} \approx 24 \).

### RESULT AND DISCUSSION

The adequate atomic number of compounds Al$_2$O$_3$, PbO and Fe$_2$O$_3$ was obtained as 15, 66, and 24, respectively. The cross-section formula calculation depends upon the Klein-Nishina. Table 1 shows the comparison of the entire molecular, atomic, and electronic cross-sections. The comparison shows that the total molecular cross-section is greater than the whole atomic and electronic cross-section; the nuclear cross-section is also more significant than the electronic cross-section. To calculate the cross-section area, 1MeV to 400MeV energy of incidence photon for compound Al$_2$O$_3$, PbO, and Fe$_2$O$_3$.

#### Table 1: Comparison of Cross Section of Compound

<table>
<thead>
<tr>
<th>Comparison of total molecular and atomic Cross section ratio</th>
<th>Comparison of total atomic and electronic Cross section ratio</th>
<th>Comparison of total molecular and electronic cross-section ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \langle \sigma_m \rangle_{\text{Al}_2\text{O}<em>3} \approx 5 \times \langle \sigma_a \rangle</em>{\text{Al}_2\text{O}_3} )</td>
<td>( \langle \sigma_m \rangle_{\text{Al}_2\text{O}<em>3} \approx 73 \times \langle \sigma_e \rangle</em>{\text{Al}_2\text{O}_3} )</td>
<td>( \langle \sigma_m \rangle_{\text{Al}_2\text{O}<em>3} \approx 15 \times \langle \sigma_e \rangle</em>{\text{Al}_2\text{O}_3} )</td>
</tr>
<tr>
<td>( \langle \sigma_m \rangle_{\text{PbO}} \approx 2 \times \langle \sigma_a \rangle_{\text{PbO}} )</td>
<td>( \langle \sigma_m \rangle_{\text{PbO}} \approx 132 \times \langle \sigma_e \rangle_{\text{PbO}} )</td>
<td>( \langle \sigma_m \rangle_{\text{PbO}} \approx 66 \times \langle \sigma_e \rangle_{\text{PbO}} )</td>
</tr>
<tr>
<td>( \langle \sigma_m \rangle_{\text{Fe}_2\text{O}<em>3} \approx 5 \times \langle \sigma_a \rangle</em>{\text{Fe}_2\text{O}_3} )</td>
<td>( \langle \sigma_m \rangle_{\text{Fe}_2\text{O}<em>3} \approx 118 \times \langle \sigma_e \rangle</em>{\text{Fe}_2\text{O}_3} )</td>
<td>( \langle \sigma_m \rangle_{\text{Fe}_2\text{O}<em>3} \approx 24 \times \langle \sigma_e \rangle</em>{\text{Fe}_2\text{O}_3} )</td>
</tr>
</tbody>
</table>

Since we are considering cross-section, which depends upon the scattering, the energy of radiation decreases after interaction with the compound, and hence reducing radiation by combination protects us from radiation. The higher cross-section during diffusion with a mix shows better for shielding radiation.

### Total Mass Attenuation Coefficient of compound Al$_2$O$_3$, PbO, and Fe$_2$O$_3$

AC (Attenuation Coefficient) quantifies the extent of radiation penetration through a material. A higher penetration of radiation corresponds to a lower MAC (Mass Attenuation Coefficient) value, whereas a higher MAC value indicates a greater reduction in radiation penetration. Therefore, when selecting a shield material for radiation, the MAC value can be used as a determining factor. Figure 1 illustrates that the MAC values of Al$_2$O$_3$ and Fe$_2$O$_3$ compounds are nearly equal, whereas PbO exhibits a different MAC value compared to the other...
materials considered. In this study, Fe$_2$O$_3$ exhibits a higher MAC value than the other compounds, indicating that it provides superior radiation shielding compared to the other materials under consideration.

**Total Cross-section Coefficient of compound Al$_2$O$_3$, PbO, and Fe$_2$O$_3$**

The Klein-Nishina cross-section equation is utilized in Figure 2 to visualize the entire molecular and atomic interaction. The calculations reveal that compound PbO exhibits a lower molecular cross-section compared to others at the same incident photon energy. The total cross-section, represented as log(σ), demonstrates that the molecular cross-sections of PbO and Fe$_2$O$_3$ are nearly equal. However, a significant shift is observed in the total atomic cross-section, particularly for the Fe$_2$O$_3$ compound. Since the cross-section is influenced by the energy of the incident photons, the power of the photons decreases as they pass through PbO, resulting in a higher cross-section for PbO. This reduction in energy offers radiation protection when PbO is used as a shielding material. Notably, the total electronic cross-section for all compounds is equivalent and exhibits dependence on the incident photon energy.

**CONCLUSION**

The calculation reveals that the entire molecular cross-section of each compound is more significant than the atomic and electronic cross-sections. As the incidence energy of photons increases, the cross-section decreases due to the occurrence of Compton scattering at low energy levels. Additionally, with higher photon power, pair production takes place in small quantities, while Compton scattering remains dominant at energies below 1-5MeV. Consequently, Compton scattering is the primary factor in these energy ranges, leading to a high cross-section. The appropriate atomic numbers for Al$_2$O$_3$, PbO, and Fe$_2$O$_3$ compounds are determined as 15, 66, and 24, respectively. Consequently, by calculating the mass attenuation, cross-section, and appropriate atomic number, one can select the optimal radiation shielding material for various protection purposes.
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REFERENCES


