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A New Gravitational Radiation from General Relativity

Jose L. Parra[a)]

Department of Physics, Florida International University, Miami, FL 33199, USA.

[a)]*Corresponding author: JLparra@fiu.edu*

Abstract. Actual cosmological and galactic data is showing the possible existence of a non-visible gravitational mass. This mass has been called dark matter, and researchers are looking for an explanation of its physical nature. A new solution to Einstein's general relativity equations is introduced here. This new solution assumes that gravity has density and can produce pressure as any other known field does. Our hypothesis does not create any mathematical contradiction with anything that is known and points to the conclusion that dark matter is a non-linear manifestation of the gravitational field.

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INTRODUCTION

The solution of the General Relativity (GR) theory for spherically symmetrical gravitational fields founded by Schwarzschild [1] has two mathematical inconveniences. Applying Schwarzschild's metric to strong gravitational fields shows imaginary solutions that force any moving energy or particle to concentrate in the center of symmetry. Schwarzschild's metric can also create infinities with alternative signs. Those mathematical properties are not expected in physics. If Schwarzschild's metric is considered valid for weak and medium gravitational fields, then the situations mentioned before should not occur.

The GR equations are solved for specific conditions, such as the space only having a gravitational field. Schwarzschild [1] found that solution under the assumption that the density and pressure of the gravitational field are zero. Another condition is that space has matter. Tolman [2] founded that solution under the assumption that the density and pressure of matter are functions of the physical parameters. This paper examines the possibility of space having density and pressure different from zero. Since matter and gravity are different in their nature, it should not be expected that the last two solutions mentioned would have similar mathematical forms.

This paper will assume the gravitational energy density and pressure of gravitational fields, both of which will play key roles at the time of forming all the cosmological bodies and their relative movements. This idea follows the recommendation made by Einstein in page 80 [3], by specifying "*It must be remembered that besides the energy density of the matter, there must also be given an energy density of the gravitational field, so that there can be no talk of principles of conservation of energy and momentum for matter alone.*"

A natural description of the world must be described with math that does not include any exotic properties. If that result is conducted in this paper, it could be expected that another paper will mathematically justify the existence of a universe without flying away galaxies. That globally static universe with local evolution now looks possible, according to the cosmological information collected with the actual technology. In summary, the main motivation of this paper is to show that it is mathematically possible to describe the cosmos without mentioning any dark matter or energy.

THE MODEL

Walecka's approach [4] will be followed in this paper. There, the reader can check in detail all the steps omitted in this paper for the sake of space. Walecka defined the square of the differential interval ds as

$$ds^2 = A_{(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 - B_{(r)}(Cdt)^2 \qquad (1)$$

where $A_{(r)}$ and $B_{(r)}$ are the functions to be found. Functions $A_{(r)}$ and $B_{(r)}$ will describe the gravitational field properties at the radial and temporal distances, and they will quantify the strength interaction between the gravitational field and the matter enclosed. Note that the relativistic invariant speed C was used instead of the speed of light c under the constraint that c is less than but close to C. Later, it would be checked out if c and C are identical, as it was proposed by [5]. In general, Eq. (2) could be related to a four-by-four matrix with all elements equal to zero except the diagonals

$$\left(g_{rr}, g_{\theta\theta}, g_{\phi\phi}, g_{44}\right) = \left[A_{(r)}, r^2, r^2\sin^2\theta, -B_{(r)}\right] \tag{2}$$

The Christopher's symbols that are different from zero in all the matrix elements of this metric become,

$$\begin{array}{lll} \Gamma^r_{rr} = \frac{A'}{2A} & \Gamma^r_{\theta\theta} = -\frac{r}{A} & \Gamma^r_{\phi\phi} = -\frac{r\sin^2\theta}{A} \\ \Gamma^r_{44} = \frac{B'}{2A} & \Gamma^\theta_{r\theta} = \frac{1}{r} & \Gamma^\theta_{\phi\phi} = -\sin\theta\cos\theta \\ \Gamma^\phi_{r\phi} = \frac{1}{r} & \Gamma^4_{r4} = \frac{B'}{2B} & \Gamma^\phi_{\theta\phi} = \frac{\cos\theta}{\sin\theta} \end{array} \tag{3}$$

where apostrophe means derivative with respect to the radius. The Ricci's tensor for spherical symmetry is,

$$\begin{aligned} R_{rr} &= -\frac{B''}{2B} + \frac{B'}{4B}\left(\frac{A'}{A} + \frac{B'}{B}\right) + \frac{A'}{rA} \\ R_{\theta\theta} &= 1 - \frac{1}{A} + \frac{r}{2A}\left(\frac{A'}{A} - \frac{B'}{B}\right) \\ R_{44} &= \frac{B''}{2A} - \frac{B'}{4A}\left(\frac{A'}{A} + \frac{B'}{B}\right) + \frac{B'}{rA} \\ R_{\phi\phi} &= R_{\theta\theta}\mathrm{Sin}^2\theta \end{aligned} \tag{4}$$

The energy and momentum density tensor satisfy

$$T_{rr} - \tfrac{1}{2}T g_{rr} = \tfrac{1}{2}\left[\rho_{(r)}C^2 - P_{(r)}\right]A_{(r)}$$

$$T_{\theta\theta} - \tfrac{1}{2}T g_{\theta\theta} = \tfrac{1}{2}\left[\rho_{(r)}C^2 - P_{(r)}\right]r^2$$

$$T_{44} - \tfrac{1}{2}T g_{44} = \tfrac{1}{2}\left[\rho_{(r)}C^2 + 3P_{(r)}\right]B_{(r)}$$

$$T_{\phi\phi} - \tfrac{1}{2}T g_{\phi\phi} = \tfrac{1}{2}\left[\rho_{(r)}C^2 - P_{(r)}\right]r^2\mathrm{Sin}^2\theta$$

where the gravitational energy density and pressure are $\rho(r)$ *and* $P(r)$. The Einstein's field equation [4] expressed as,

$$R_{\mu\nu} = \frac{8\pi G}{C^4}\left(T_{\mu\nu} - \frac{1}{2}T g_{\mu\nu}\right) \tag{5}$$

yield the Einstein's field equations for rr, $\theta\theta$, 44 and $\phi\phi$

$$\begin{gathered} r: -\frac{B''}{2B_r} + \frac{B'}{4B_r}\left(\frac{A'}{A_r} + \frac{B'}{B_r}\right) + \frac{A'}{rA_r} = \frac{4\pi G}{C^4}(\rho_r C^2 - P_r)A_r \\ \theta\theta: 1 - \frac{1}{A_r} + \frac{r}{2A_r}\left(\frac{A'}{A_r} - \frac{B'}{B_r}\right) = \frac{4\pi G}{C^4}(\rho_r C^2 - P_r)r^2 \\ 44: \frac{B''}{2A_r} - \frac{B'}{4A_r}\left(\frac{A'}{A_r} + \frac{B'}{B_r}\right) + \frac{B'}{rA_r} = \frac{4\pi G}{C^4}(\rho_r C^2 + 3P_r)B_r \\ \phi\phi: \{R_{\theta\theta}\sin^2\theta = \frac{8\pi G}{C^4}\left(T_{\phi\phi} - \tfrac{1}{2}T g_{\phi\phi}\right) sin^2\theta \end{gathered} \tag{6}$$

The pressure can be eliminated from the above set of equations by doing $\frac{rr}{2A} + \frac{\theta\theta}{r^2} + \frac{44}{2B}$. The algebraically output become,

$$\frac{d}{dr}\left[\frac{r}{A_{(r)}}\right] = 1 - \frac{8\pi G}{C^4}\rho_{(r)}C^2 r^2 \tag{7}$$

The solution of Eq. (7) cannot be founded analytically at this stage because the gravitational energy density function $\rho(r)$ is unknown. The solution of Eq. (7) should hold the solution for intermediate (Schwarzschild's) and weak (Newtonian's) gravitational fields. The exponential function can help because $\exp x \approx 1 + x + 0.5x^2 \approx \frac{1}{1-x}$. That is, by imposing this physical constriction, the function A_r will satisfy,

$$A_r = \exp\left[\frac{2G(M_R + m_r)}{C^2 r}\right] \tag{8}$$

where M_R is any gravitational seed mass of radius R and m_r is the mass of the gravitational field from the radius R to the radius r. At this time, it could be useful to clarify that the gravitational field will not be counted twice in Eq. (8) because the approximate calculation of Eq. (8) will give us twice the Schwarzschild's potentials if the gravitational field were counting twice, which is not the case.

The density of the field in Eq. (7) will be forced to preserve the math. Then, by plugging Eq. (8) into Eq. (7), the density of the field must be,

$$\rho_{(r)} = \left\{1 - \frac{1 + \frac{2G(M_R + m_r)}{C^2 r}}{\exp\left[\frac{2G(M_R + m_r)}{C^2 r}\right]}\right\}\frac{C^2}{8\pi G r^2} \tag{9}$$

The mass of the gravitational field at a distance r from the center [4] can be calculated numerically with

$$dm_{(r)} = \rho_{(r)}\sqrt{A_r}4\pi r^2 dr \tag{10}$$

and inserting Eq. (9) into the above definition, it is founded the equation to calculate numerically the increment of gravitational mass with the radius,

$$dm_{(r)} = \left\{exp\left[\frac{G(M_R + m_r)}{C^2 r}\right] - \frac{1 + \frac{2G(M_R + m_r)}{C^2 r}}{exp\left[\frac{G(M_R + m_r)}{C^2 r}\right]}\right\}\frac{C^2}{2G}dr \tag{11}$$

At this point, the reader can check out that the functions for the density and the increment of mass have zero values when the exponential functions are close to one.

The missing function $B_{(r)}$ can be founded by doing $\frac{rr}{2A} + \frac{44}{2B}$,

$$B_{(r)} = \frac{e^{\frac{8\pi G}{C^4}\int_{R_c}^{r} A_{(r)}\left[\rho_{(r)}C^2 + P_{(r)}\right]r dr}}{A_{(r)}} \approx \frac{1}{A_{(r)}} \tag{12}$$

The pressure, according to $\frac{44}{2B} - \frac{rr}{2A}$, satisfies

$$P_{(r)} = \frac{C^4}{8\pi G}\left[\frac{B''_{(r)}}{2} + \frac{B'_{(r)}}{r}\right] \tag{13}$$

and it can by calculated numerically with

$$P_{(r)} = \frac{G\left\{\frac{(M_R+m_r)^2}{r^2} + \left[m'_r - \frac{2(M_R+m_r)}{r}\right] m'_r\right\}}{4\pi r^2 \exp\left[\frac{2G(M_R+m_r)}{C^2 r}\right]} - \frac{rC^2 m''_r}{2G} \tag{14}$$

The covariant derivative of the energy-momentum tensor, Equation 10.43, page 192 [4], is

$$T^{\mu\nu}{}_{;\nu} = \left\{\begin{array}{c} \frac{1}{\sqrt{-g}}\frac{\partial}{\partial q^\nu}\left[\sqrt{-g}\left(\rho_{r,t}C^2 + 3P_{r,t}\right)\frac{u^\mu u^\nu}{C^2}\right] \\ g^{\mu\nu}\frac{\partial P_{r,t}}{\partial q^\nu} + +\Gamma^\mu_{\lambda\nu}\left(\rho_{r,t}C^2 + 3P_{r,t}\right)\frac{u^\lambda u^\nu}{C^2} \end{array}\right\} = 0 \tag{15}$$

where each adding term is

$$g^{\mu\nu}\frac{\partial P_{r,t}}{\partial q^\nu} = g^{rr}\frac{\partial P_{r,t}}{\partial r} + g^{\theta\theta}\frac{\partial P_{r,t}}{\partial \theta} + g^{\phi\phi}\frac{\partial P_{r,t}}{\partial \phi} + g^{44}\frac{\partial P_{r,t}}{\partial t} \tag{16}$$

it is

$$g^{\mu\nu}\frac{\partial P_{r,t}}{\partial q^\nu}\bigg|_{order\,zero} \approx g^{rr}\frac{\partial P_{r,t}}{\partial r} + g^{44}\frac{\partial P_{r,t}}{\partial t} \tag{17}$$

(The Eq. (17) is called order zero or normal mode because it was assumed $P^\theta = P^\phi = 0$, where the Greek symbol means derivative respect to the corresponding variable.)

$$\begin{array}{l} \frac{1}{\sqrt{-g}}\frac{\partial}{C\partial t}\left[\sqrt{-g}\left(\rho_{r,t}C^2 + 3P_{r,t}\right)\frac{u^4u^4}{C^2}\right] \\ \approx \frac{1}{B}\frac{\partial}{C\partial t}\left(\rho_{r,t}C^2 + 3P_{r,t}\right) \end{array} \tag{18}$$

and

$$\begin{array}{l} \Gamma^\mu_{\lambda\nu}\left(\rho_r C^2 + 3P_r\right)\frac{u^\lambda u^\nu}{C^2} = 0^i_{\lambda 4} + \Gamma^4_{44}\left(\rho_r C^2 + 3P_r\right)\frac{u^4u^4}{C^2} \\ \Gamma^\mu_{\lambda\nu}\left(\rho_r C^2 + 3P_r\right)\frac{u^\lambda u^\nu}{C^2} = \frac{B'}{2A}\left(\rho_r C^2 + 3P_r\right)\frac{1}{B} \end{array} \tag{19}$$

After some algebra,

$$\left\{\begin{array}{c} 2e^{\frac{4G(M_R+m_r)}{C^2 r}}\frac{\partial\left(2P_{r,t} + \rho_{r,t}C^2\right)}{C\partial t} + 2\frac{\partial P_{r,t}}{\partial r} + \\ \left[\left(\frac{8\pi G}{C^4}P_{r,t}r + \frac{1}{r}\right)e^{\frac{2G(M_R+m_r)}{C^2 r}} - \frac{1}{r}\right]\left(3P_{r,t} + \rho_{r,t}C^2\right) \end{array}\right\} = 0 \tag{20}$$

The full solution of the above equation should exclude any division by zero and should include the known physics for extremely weak gravitational fields. The solutions are:

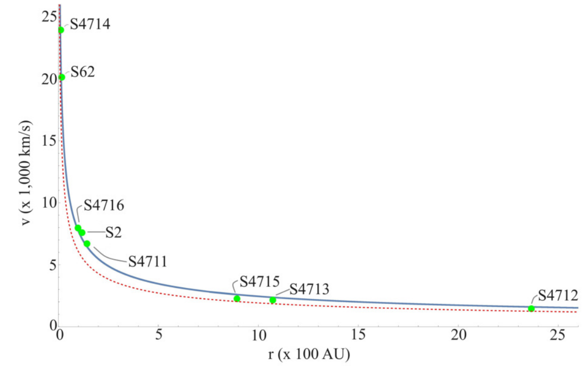

FIGURE 1. The experimental star velocities shown here are only related to the periapsis position of the stars. Error bars were not included because they would not lead to any useful visual information. The blue curve is the representation of Eq. (29), and the red dashed curve is from the accepted Schwarzschild's solution.

$$\begin{array}{l} P_{r,t} = P_r - \left[P\mathrm{Sin}(\omega t + kr)\right]\hat{r}_\leftrightarrow - \left[P\mathrm{Cos}(\omega t + kr)\right]\hat{r}_\updownarrow \\ \rho_{r,t} = \rho_r + \left[\rho\mathrm{Sin}(\omega t + kr)\right]\hat{r}_\leftrightarrow + \left[\rho\mathrm{Cos}(\omega t + kr)\right]\hat{r}_\updownarrow \\ where\ \mathrm{k} = \frac{2\pi}{\lambda}\ and\ \omega = 2\pi f \end{array} \tag{21}$$

The hat r symbols are unitary vectors, and the double arrow shows the direction in which that solution is valid. The horizontal double arrows are valid only in the radial direction, while the vertical double arrows are valid only in any direction perpendicular to the radial direction. To clarify, they are vectors with orthogonal conditions that avoid division by zero. It is interesting that the coherence of light has comparable properties.

Applying the solutions for density and pressure into the earlier differential equation, using the classical GR pressure condition,

$$\frac{\partial P_r}{\partial r} + \frac{1}{2}\left(e^{S_w}\frac{8\pi G}{C^4}P_r r - \frac{1}{r} + \frac{e^{S_w}}{r}\right)\left(\rho_r C^2 + 3P_r\right) = 0, \tag{22}$$

the orthogonality condition, and the definitions $v_g = \frac{\omega}{k}$ and $S_w = \frac{2G(M_R+m_r)}{C^2 r}$; yield

$$\frac{v_g}{C} = \frac{1}{e^{\frac{4G(M_R+m_r)}{C^2 r}}\left(\frac{\rho_\leftrightarrow C^2}{P_\leftrightarrow} - 2\right)} = \frac{1}{e^{\frac{4G(M_R+m_r)}{C^2 r}}\left(\frac{\rho_\updownarrow C^2}{P_\updownarrow} - 2\right)} \tag{23}$$

It is known that electromagnetic radiation has a ratio of three between its energetic density and its pressure. Assuming the same ratio for the the gravitational radiation, both solutions will travel according to

$$\frac{v_g}{C} = \exp\left[-\frac{4G\left(M_R+m_r\right)}{C^2 r}\right] \tag{24}$$

Equation (24) shows that the speed of the radiation will increase after abandoning the source, regardless of the negative sign because the radius is on the denominator side of the fraction. Also, according to Eq. (24), any radiation will never reach the limit speed because space always contains gravitational mass. For example, the existence of a proton per meter cube in the intergalactic space outputs a speed ratio of $\frac{v_g}{C}$ = 1 – 6×10^{-54} in complete agreement with all previous knowledge.

The last fraction is in complete agreement with the energetic photon hypothesis [6, 7]. In other words, Maxwell's classical equations were deduced assuming empty space, which is not a realistic assumption according to the hypothesis introduced in this paper.

DISCUSSION

In this paper, it is assumed that the density and pressure of the gravitational field have values different from zero. They hold an invariant relativistic status without having singularities, and all of this points to a possible reinterpretation of the correlated experimental data. Let's see some examples.

Neutron stars

Any time that the mass of a star reaches 3.18 M_{Sun} in a radius of 12 km, the star density will exceed the $5.7\times10^{17}\ kg\ m^{-3}$ neutron maximum density, according to our hypothesis. Then, the neutrons will be forced to dissolve into the surrounding gravitational soup. According to Eq. (11), now at 20,000 km is a total enclosed mass near to five M_{Sun}. This result matches well with the known minimum of five M_{Sun} that is seen in almost the totality of the known so-called "Black Holes" [8, 9]. In our theoretical result, with no matter available to radiate energy, the star will disappear from observation but is far away from being a "hole." It is the other way around; it will now be a ball full of gravitational energy with no density restriction, and that allows the mass to become colossal.

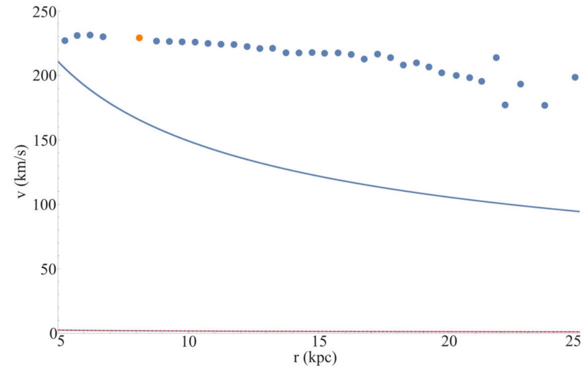

FIGURE 2. The points represent the measured velocities of 35 stars in the Milky Way [13] versus their distance to the center of the galaxy. The two theoretical functions were calculated assuming that a radius of 0.3 AU encloses 6.74 million M_{Sun}. The dashed red curve comes from Schwarzschild's metric, and the continuous one from Eq. (29). The inclusion of the mass of the enclosed stars in the corresponding radius helps to overlap our theoretical function with the experimental data [13]. The orange spot stands for the position of the Sun in the Milky Way. Now the quantity of dark matter necessary to fill up the gap between Schwarzschild's solution and the experimental data is too much, and, according to our hypothesis, something that is not necessary.

Milky Way

The Lagrangian of a moving star with mass m [4] is

$$L = -mC^2\sqrt{B_{(r)} - \frac{1}{C^2}\left[A_{(r)}\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2\right]} \tag{25}$$

where dot means derivative with respect to time. $\frac{d\phi}{dt}$ is zero because the star stays always in the same plane. A null value for $\frac{dr}{dt}$ can be used by looking at the two solutions where the variation of the length of the radius r is reversed. Those two points are known as periapsis, the nearer point to the focus of rotation, and apoapsis, the farther one to the same focus. Then, the velocity at both points will satisfy

$$r\dot{\theta} = C\sqrt{\frac{1}{2}B_{(r)}r\frac{B'_{(r)}}{B_{(r)}}} \tag{26}$$

Doing $\frac{rr}{2A} + \frac{\theta\theta}{r^2} + \frac{44}{2B}$ comes out

$$\frac{A'_{(r)}}{A^2_{(r)}} = \frac{8\pi G\rho_{(r)}r}{C^2} + \frac{1}{A_{(r)}r} - \frac{1}{r} \tag{27}$$

Then using Eq. (27) in $\frac{rr}{2A} + \frac{44}{2B}$ comes out

$$\frac{B'_{(r)}}{B_{(r)}} = \frac{8\pi G}{C^4}A_{(r)}P_{(r)}r + \frac{A_{(r)}-1}{r} \approx \frac{A_{(r)}-1}{r} \tag{28}$$

The first adding term in Eq. (28) was disregarded because the gravitational pressure decreases extremely fast, far from the gravitational center. Finally, plugging Eq. (28), and Eq. (12) into Eq. (26) comes out,

$$r_{p,a}\dot{\theta} = C\sqrt{\frac{1}{2}\left\{1-\exp\left[-\frac{2G(M_R+m_r)}{C^2 r}\right]\right\}} \tag{29}$$

It has been proposed to include some invisible mass in the study of the speed of the stars in galaxies because the GR standard solution does not overlap with the experimental points. Eq. (29) can describe the speed of stars in the Milky Way [10, 11, 12, 13] if it is assumed that a radius of 11 km encloses 8 M_{Sun}. It is possible to conclude that the mass of so-called dark matter could be the mass of the gravitational field because it does not emit electromagnetic radiation. Figure 1 shows the almost Keplerian velocity of eight stars versus their distances to a common invisible center. The accepted opinion that the center holds a black hole with two variables to be adjusted, its mass and radius, could not be correlated enough to overlap the experimental data. The proposed solution was to assume the contribution of a small quantity of dark matter. Our model does a better job connecting theory with data, making it unnecessary to mention another gravitational influence.

Currently, it is important to mention a new theoretical result. We are accustomed to the spatial-temporal relativistic effect when bodies are moving at velocities close to the speed of light, when strong gravitational fields are present in the region of attention, or in a combination of both situations. Here, a new gravitational consequence appears even where the gravitational potential is Newtonian. The value of Schwarzschild's function is 7.64×10^{-11} at the Sun's position, which, although small, is still producing a big gravitational difference according to our model from Newton's theory because of the accumulation of mass from the gravitational field. Figure 2 is like Figure 1, but now the difference between the accepted theory and our hypothesis is huge. The red-dashed line that represents the accepted theory is so close to the horizontal axis that it is hard to see. Our function, represented as a blue continuous line, is close enough to the experimental data. The mass of the stars within the enclosed sphere can cover the gap, as explained in [13]. The accepted theory proposes to include a correction from an invisible source that plays a bigger numerical role than the observed matter. That is an inconvenient situation in science.

CONCLUSION

The hypothesis introduced in this paper, following Einstein's 1922 [3] recommendation to include gravitational density and pressure into the equations of general relativity, can lead to three useful consequences: (1) the explanation of star velocities around the center of their galaxies without including the so-called dark matter. (2) The possible existence of a new invariant gravitational radiation. (3) and the existence of black balls (called black holes in literature) without singularities. All three consequences make it possible for general relativity and quantum mechanics to become theoretically closer soon.

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