



Effect of Superthermal Species on Critical Mach Number and Small-Amplitude Solitary Waves in Dusty Plasmas

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Abstract. Dusty plasmas are plasmas that contain charged dust particles, in addition to the usual ions and electrons, and are found in space, industry, and laboratory experiments. Due to their omnipresence, studying them helps us understand natural phenomena such as planetary rings, comet tails, and interstellar clouds, as well as improve plasma-based technologies on Earth. In this work, we investigate the critical Mach number M_{cr} and small-amplitude solitary wave structures in an unmagnetized dust-acoustic waves (DAWs) dusty plasma with superthermal kappa-distributed electrons κ_e and ions κ_i . Using the Sagdeev pseudopotential approach, we analyze the effects of superthermal species, the ratio of dust to electron temperature σ_d , and the concentration of ion density δ_i . The results show that the critical Mach number is affected by superthermal species and the ratio of dust to electron temperature. The amplitude and width of small-amplitude solitary waves are also affected by the Mach number M and the concentration of the ion density. These findings are relevant for understanding nonlinear wave dynamics in space and astrophysical dusty plasma environments.

Received: August 10, 2025; **Revised:** October 21, 2025; **Accepted:** October 29, 2025

Keywords: Superthermal species, small amplitude, Sagdeev potential

1. INTRODUCTION

Dusty plasmas are plasmas that contain charged dust particles, in addition to the usual ions and electrons, and are found in space, industry, and laboratory experiments. Due to their omnipresence, studying them helps us understand natural phenomena such as planetary rings, comet tails, and interstellar clouds [1, 2], as well as improve plasma-based technologies on Earth [3]. Several studies have been dedicated to the study of dust particles in different fields [4, 5, 6, 7]. Dust particulates are found in both nature and the man-made system. The presence of dust particulates leads to the modification of plasma dielectric properties [8, 9, 10]. The negativity of the dust grains is caused by the higher thermal velocity of electrons compared to that of ions [11]. The presence of massive and highly charged dust grains modified the basic properties of plasma known as dusty plasma, compared to those of ordinary plasma [12].

Dust Acoustic (DA) waves are one of the basic waves in plasma that have been studied theoretically [13, 14, 15]. and experimentally [16, 17]. The properties and behavior of DAWs have been studied through research

that helps to understand dust dynamics in plasma systems. The two critical factors that directly lead to the appearance of dust acoustic waves (DAWs) are the presence of massive, charged dust grains, which provide the inertia needed for the low-frequency wave motion, and the thermal pressure of electrons and ions, which provides the restoring force that drives the wave. This combination produces unique dynamics and characteristics of DAWs. The phase velocity of the DAWs is greater than the thermal velocity of charged dust grains but less than that of electrons and ions, i.e., $v_{td} < \omega/k < v_{ti}, v_{te}$ [13]. The verification of ions of these types of wave is also experimentally confirmed [16].

Many studies have focused on the linear and nonlinear theories of dusty plasmas, yet research shows that they can also generate large-amplitude waves. These waves highlight the significance of nonlinear effects. Unmagnetized DAWs using the Sagdeev potential approach for variable dust charges and two different temperatures have been studied [18]. They report that the expansion of the Sagdeev potential led to the existence of both compressive and rarefactive solitary waves. Sagdeev's pseudopotential approach has been used to study nonlinear periodic, soli-

tary, and double-layer excitations in plasmas. It is found that positive energy solitons are observed with a subsonic Mach number [19]. Unmagnetized dusty plasma consisting of kappa-distributed electrons for DAWs has been studied using Sagdeev's potential approach for analyzing concentrations and various sizes of dust grains [20]. Theoretical analysis in magnetized dusty plasmas using the Sagdeev pseudopotential method has established that the amplitude of DAWs has a direct relation with Mach number [21]. It is found that both compressive and rarefactive potentials can coexist, and the ion density ratio plays a crucial role in this coexistence.

We have studied the critical Mach number and small-amplitude solitary wave structures in an unmagnetized dusty plasma containing superthermal kappa-distributed electrons and ions. Employing the Sagdeev pseudopotential method, we explore the influence of superthermal particles, the ratio of dust to electron temperature, and ion density concentration. The results provide insight into nonlinear wave behavior in space and astrophysical dusty plasma environments. This article is organized into four sections. The first section includes the introduction, the second section presents our assumptions and the governing equations, the third section interprets the results and discussion, and the fourth section presents the conclusion of our research work.

2. BASIC EQUATIONS

We consider an unmagnetized dusty plasma consisting of electrons and ions having kappa distributions and negatively charged dust grains. The propagation of DAWs in one dimension, having low-frequency waves in a plasma containing warm dust particles ($\gamma = 3$), is governed by the following continuity, momentum, and poisson's equations,

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d v_d) = 0, \quad (1)$$

$$m_d n_d \left(\frac{\partial v_d}{\partial t} + v_d \frac{\partial v_d}{\partial x} \right) = -q_d n_d \frac{\partial \phi}{\partial x} - \frac{\partial p_d}{\partial x}, \quad (2)$$

and

$$\frac{\partial^2 \phi}{\partial x^2} = -\frac{1}{\epsilon_0} (en_e - en_i - q_d n_d), \quad (3)$$

where v_d , and m_d , are the fluid velocity and mass of dust particles, respectively, n_s , is the number density of species $s (= i, e, d)$, $q_d = -Z_{d0}e$ is the stationary charge dust grains, e is the electronic charge, and ϕ is the electrostatic potential. The dust partial pressure is $p_d = T_d n_d^\gamma / n_0^{\gamma-1}$. Here, x and t are the space and time variables. In Eq. 3,

the electron and ion number densities are given by the κ distributions [22],

$$n_e = n_{e0} \left(1 - \frac{e\phi}{T_e(\kappa_e - 3/2)} \right)^{-\kappa_e + 1/2}, \quad (4)$$

and

$$n_i = n_{i0} \left(1 + \frac{e\phi}{T_i(\kappa_i - 3/2)} \right)^{-\kappa_i + 1/2}, \quad (5)$$

where T_s is the temperature of the species s .

Let us introduce the dimensionless quantities as

$$X = x/\lambda_D, \Psi = e\phi/T_e, N_s = n_s/n_{s0}, u_d = v_d/C_d, \\ t = t/\omega_{pd},$$

where $\lambda_D = (\epsilon_0 T_e / Z_{d0} n_{d0} e^2)$ is the length of the Debye dust, $C_d = (Z_{d0} T_e / m_d)^{1/2}$ is the acoustic speed of the dust and $\omega_{pd} = (Z_{d0}^2 n_{d0} e^2 / \epsilon_0 m_d)^{1/2}$ is the plasma frequency of the dust.

Using dimensionless quantities, the normalized form of Eqs. (1 - 5) yield

$$\frac{\partial N_d}{\partial t} + \frac{\partial}{\partial X}(N_d u_d) = 0, \quad (6)$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial X} = \frac{\partial \Psi}{\partial X} - 3\sigma_d N_d \frac{\partial N_d}{\partial X}, \quad (7)$$

$$\frac{\partial^2 \Psi}{\partial X^2} = N_e \delta_e - N_i \delta_i + N_d, \quad (8)$$

$$N_e = \left(1 - \frac{\Psi}{\kappa_e - 3/2} \right)^{-\kappa_e + 1/2}, \quad (9)$$

and

$$N_i = \left(1 + \frac{\Psi}{\sigma_i (\kappa_i - 3/2)} \right)^{-\kappa_i + 1/2}, \quad (10)$$

where $\sigma_d = T_d/T_e$, $\delta_e = n_{e0}/Z_{d0}n_{d0}$, and $\delta_i = n_{i0}/Z_{d0}n_{d0}$.

To study the existence of an arbitrary wave amplitude of the dust particle from Eqs. (6 - 10). Let us introduce the stationary frame $\xi = X - Vt$, V being the velocity of the waves, and using the boundary conditions $\xi \rightarrow \infty$, $u_d \rightarrow u_0$, $N_d \rightarrow 1$, $\Psi \rightarrow 0$, and integrate and combine the equations. (6), and (7), yield

$$3\sigma_d N_d^4 - N_d^2 (M^2 + 3\sigma_d + 2\Psi) + M^2 = 0, \quad (11)$$

where $M = V - u_0$, being the Mach number in the rest of the frame. This equation is in the quadratic form of N_d^2 and can be solved as

$$N_d^2 = \frac{(M^2 + 3\sigma_d + 2\Psi) \pm \sqrt{(M^2 + 3\sigma_d + 2\Psi)^2 - 12\sigma_d M^2}}{6\sigma_d}. \quad (12)$$

Assume

$$N_d = \sqrt{p} \pm \sqrt{q}. \quad (13)$$

Squaring Eq. (13) and comparing with Eq. (12), yield

$$N_d = \frac{[(M + 3\sigma_d)^2 + 2\Psi]^{1/2} - [(M - 3\sigma_d)^2 + 2\Psi]^{1/2}}{2\sqrt{3}\sigma_d}. \quad (14)$$

The negative branch of the dust density has been taken in Eq. (14) to satisfy the boundary conditions. As a result, the simple algebraic form of the RHS of Eq. (12) has been obtained. Using Eqs. (9), (10), and (14) into Eq. (8) yield

$$\begin{aligned} \frac{\partial^2 \Psi}{\partial \xi^2} = & \delta_e \left(1 - \frac{\Psi}{\kappa_e - 3/2}\right)^{-\kappa_e + 1/2} - \\ & \delta_i \left(1 + \frac{\Psi}{\sigma_i (\kappa_e - 3/2)}\right)^{-\kappa_e + 1/2} + \\ & \frac{1}{2\sqrt{3}\sigma_d} \left[[(M + 3\sigma_d)^2 + 2\Psi]^{1/2} - [(M - 3\sigma_d)^2 + 2\Psi]^{1/2} \right]. \end{aligned} \quad (15)$$

Now, multiply Eq. (15) by Ψ' on both sides and integrate, yielding the "energy integral"

$$\frac{1}{2} \left(\frac{d\Psi}{d\xi} \right)^2 + V(\Psi) = 0, \quad (16)$$

where the Sagdeev potential or the pseudopotential $V(\Psi)$ of the plasma system with "coordinate" Ψ (pseudo position) and ξ the "time" given by

$$V(\Psi) = \beta_1 + \beta_2 - \frac{1}{6\sqrt{3}\sigma_d} (\beta_3 + \beta_4), \quad (17)$$

where $\beta_1 = -\delta_e \left(1 - \frac{\Psi}{\kappa_e - 3/2}\right)^{-\kappa_e + 3/2} + \delta_e$,

$\beta_2 = -\delta_i \sigma_i \left(1 + \frac{\Psi}{\kappa_i - 3/2}\right)^{-\kappa_i + 3/2} + \delta_i \sigma_i$,

$\beta_3 = [(M + \sqrt{3}\sigma_d)^2 + 2\Psi]^{3/2} - (M + \sqrt{3}\sigma_d)^3$,

and $\beta_4 = -[(M - \sqrt{3}\sigma_d)^2 + 2\Psi]^{3/2} + (M - \sqrt{3}\sigma_d)^3$.

To have a solitary wave solution, the pseudopotential must satisfy the condition,

$$V(0) = 0 = \frac{\partial V(0)}{\partial \Psi}, \quad \frac{\partial^2 V(0)}{\partial \Psi^2} < 0. \quad (18)$$

Thus, the physical solution of Eq. (16) holds if

$$\delta_e \frac{\kappa_e - 1/2}{\kappa_e - 3/2} - \frac{\delta_i}{\sigma_i} \frac{\kappa_i - 1/2}{\kappa_i - 3/2} + \frac{1}{M^2 - 3\sigma_d} < 0, \quad (19)$$

i.e.,

$$M > M_{cr} \simeq \sqrt{3\sigma_d + \frac{1}{\frac{\delta_i}{\sigma_i} \frac{\kappa_i - 1/2}{\kappa_i - 3/2} - \delta_e \frac{\kappa_e - 1/2}{\kappa_e - 3/2}}}. \quad (20)$$

Eq. (20), satisfies the lower limit of the Mach number, M , and provides the necessary condition for the existence of SWs. Eq. (20) depicted that the lower limit of M depends on the concentration of dust temperature, the concentration of ion density δ_i , the concentration of electron density δ_e , and the superthermal species for electrons κ_e and ions κ_i . Again, the series expansion of $V(\Psi)$ is carried out about the origin ($\Psi = 0$), to study the small amplitude solitary wave structures. We adopt the method followed by [23] for the investigations of small-amplitude solitons, and consider

$$\frac{1}{2} \left(\frac{d\Psi}{d\xi} \right)^2 + A\Psi^2 + B\Psi^3 = 0, \quad (21)$$

where

$$A = -\frac{\delta_e}{2} \frac{\kappa_e - 1/2}{\kappa_e - 3/2} - \frac{\delta_i}{2\sigma_i} \frac{\kappa_i - 1/2}{\kappa_i - 3/2} - \frac{3}{M^2 - 3\sigma_d}, \quad (22)$$

$$\begin{aligned} B = & \frac{\delta_e}{6} \frac{(\kappa_e - 1/2)(\kappa_e + 1/2)}{(\kappa_e - 3/2)^2} + \frac{(\kappa_i - 1/2)(\kappa_i + 1/2)}{(\kappa_i - 3/2)^2} \times \\ & \frac{\delta_i}{6\sigma_i^2} + \frac{3\sqrt{3}}{2} \left(\frac{1}{(M - \sqrt{3}\sigma_d)^3} - \frac{1}{(M + \sqrt{3}\sigma_d)^3} \right). \end{aligned} \quad (23)$$

The analytical solution of Eq. (21) gives the usual Korteweg-de Vries (KdV)-type solution

$$\Psi(\xi) = -\Psi_{\max} \text{sech}^2(\xi/\Delta_w), \quad (24)$$

where the maximum amplitude or potential of the soliton (Ψ_{\max}), and the width of the soliton Δ_w yield

$$\Psi_{\max} = A/B, \quad (25)$$

$$\Delta_w = \sqrt{-2/A}. \quad (26)$$

The potential (Ψ) may be positive or negative, depending on the value of B . If $B > 0$ ($B < 0$), Ψ is positive (negative). In other words, the sign of the potential of the small-amplitude solitons included in the plasma model is determined by the sign of the coefficient of Ψ^3 in the Taylor expansion of $V(\Psi)$ about $\Psi = 0$.

3. RESULTS AND DISCUSSION

To perform the numerical calculation, we used parameters relevant to the photoassociation region that separates HII regions from dense molecular clouds [14, 24]. The critical Mach number M_{cr} as a function of the concentration of ion temperature, $\sigma_i = T_i/T_e$ for different values of the concentration of dust temperature $\sigma_d = T_d/T_e$ is shown in Fig. 1. It is found that as the dust concentration

increases, the critical Mach number also increases. The physical reason behind this is that σ_d has a direct relation with M_{cr} as seen in Eq. (20) which causes an increase in M_{cr} as an increase in σ_d . The profiles of the critical Mach number M_{cr} as a function of the concentration of ion temperature σ_i for the different values of the superthermal species κ_i and κ_e are depicted in Figs. 2 (a) and 2 (b), respectively. On increasing the superthermal species κ_i and κ_e , the critical Mach number is found to decrease as both have an inverse relation with M_{cr} seen from Eq. (20). The qualitative similarity plots for κ_i and κ_e are that the enhanced superthermal ion and electron components associated with a κ -distribution with low κ have only a quantitative effect. The values that support solitons have different critical Mach numbers in the two cases.

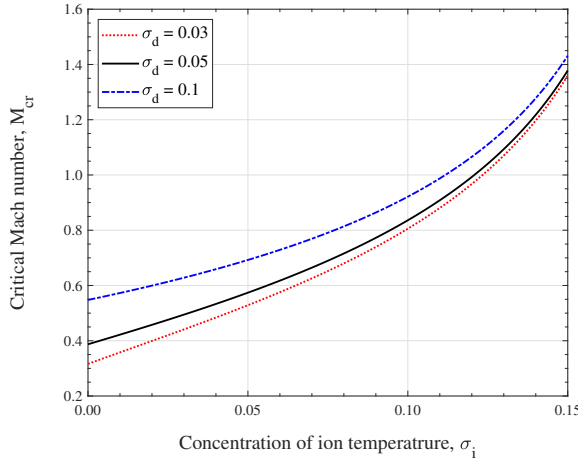


FIGURE 1: Profiles of the critical Mach number M_{cr} as a function of the concentration of the ion temperature ($\sigma_i = T_i/T_e$) for different values of concentration of dust temperature $\sigma_d = T_d/T_e$ with $\kappa_i = 2$, $\kappa_e = 5$, the concentration of ion density $\delta_i = 5$.

The critical Mach number M_{cr} as a function of the concentration of ion temperature, $\sigma_i = T_i/T_e$ for different values of the concentration of dust temperature $\sigma_d = T_d/T_e$ is shown in Fig. 1. It is found that as the dust concentration increases, the critical Mach number also increases. The physical reason behind this is that σ_d has a direct relation with M_{cr} as seen in Eq. (20) which causes an increase in M_{cr} as an increase in σ_d . The profiles of the critical Mach number M_{cr} as a function of the concentration of ion temperature σ_i for the different values of the superthermal species κ_i and κ_e are depicted in Figs. 2 (a) and 2 (b), respectively. On increasing the superthermal species κ_i and κ_e , the critical Mach number is found to decrease as both have an inverse relation with M_{cr} seen from Eq. (20). The qualitative similarity plots for κ_i and κ_e are that the enhanced superthermal ion and electron components associated with a κ -distribution with low κ have only a

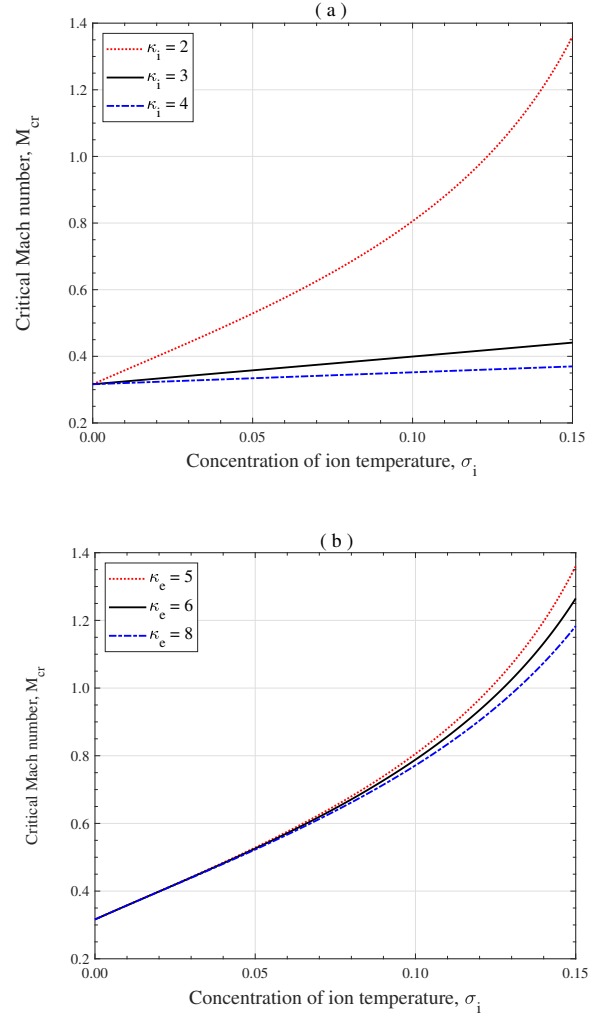


FIGURE 2: Profiles of the critical Mach number M_{cr} as a function of the concentration of the ion temperature ($\sigma_i = T_i/T_e$) for different values of superthermal species for (a) ion and (b) electron, with concentration of dust temperature $\sigma_d = 0.03$, and the concentration of ion density $\delta_i = 5$.

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Figs. 3, 4, and 5 correspond to Eq. (24) for the study of small-amplitude solitary wave structures. The normalized perturbed potential as a function of the normalized distance for different Mach numbers is depicted in Fig. 3. The Mach number (M) is chosen in such a way that its value should be greater than the critical Mach number (M_{cr}). The critical Mach number M_{cr} corresponding to $\sigma_i = 0.15$ and $\sigma_d = 0.03$ is found to be 1.36. It is found that with an increase in Mach number M , the solitary amplitude decreases, and its width increases. The solitary amplitude decreases from 0.0994 to 0.0991, while

its width increases from 0.1899 to 0.1903. The normalized perturbed potential as a function of normalized distance for different values of κ_i is shown in Fig. 4. From Eqs. (22), and (23), it is found that on increasing κ_i , the nonlinear coefficient A remains the same while the dispersion coefficient B decreases, which leads to the increment of normalized perturbed potential as observed from Eq. (25). Further, the increment of the normalized perturbed potential leads to the increment of the width [Seen in Fig. 4]. The result obtained indicated that on increasing superthermal indices κ_i , their amplitude and width increase. The maximum amplitude and width for $\kappa_i = 4$ are found to be 0.49 and 0.64, respectively, while their minimum amplitude and width for $\kappa_i = 2$ are 0.13 and 0.51, respectively. Profiles of the normalized perturbed potential as a function of the normalized distance for different values of ion density concentration, as in the legend, are depicted in Fig. 5. On increasing the concentration of ion density δ_i , the nonlinear coefficient A and the dispersion coefficient B both decrease. As a result, the maximum amplitude decreases and the soliton width increases [Seen in Eqs. (25) and (26), respectively]. The maximum amplitude and width corresponding to $\delta_i = 1.7$ are 0.11 and 0.32, respectively, while the minimum amplitude and width corresponding to $\delta_i = 5$ are 0.10 and 0.19, respectively.

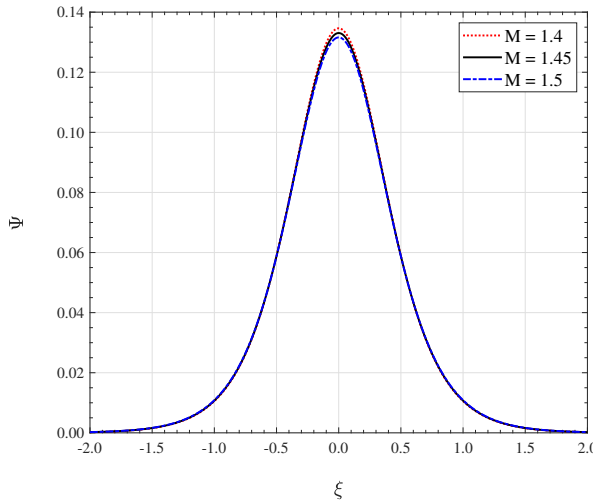


FIGURE 3: Profiles of the normalized wave amplitude Ψ as a function of the normalized distance ξ for different values of the Mach number with concentration of ion temperature $\sigma_i = 0.15$, concentration of dust temperature $\sigma_d = 0.03$, and concentration of ion density $\delta_i = 5$.

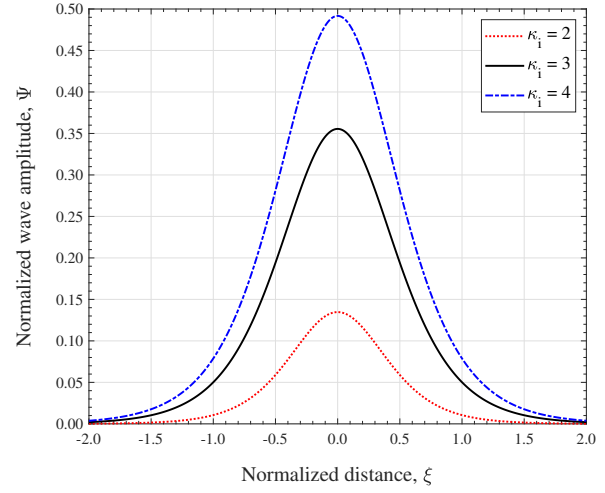


FIGURE 4: Profiles of the normalized wave amplitude Ψ as a function of the normalized distance ξ for different values of κ_i , with Mach number $M = 1.4$, concentration of the ion temperature $\sigma_i = 0.15$, concentration of dust temperature $\sigma_d = 0.03$, and concentration of ion density $\delta_i = 5$.

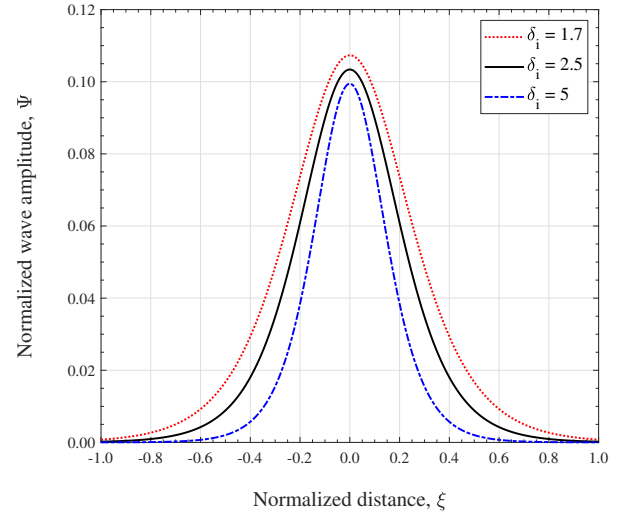


FIGURE 5: Profiles of the normalized wave amplitude Ψ as a function of the normalized distance ξ for different values of the concentration of ion density δ_i , with Mach number $M = 1.4$, concentration of the ion temperature $\sigma_i = 0.15$, and concentration of dust temperature $\sigma_d = 0.03$.

4. CONCLUSION

We have studied the arbitrary amplitude and small amplitude solitary waves in an unmagnetized dusty plasma

with superthermal κ -distributed electrons and ions using the Sagdeev pseudopotential approach. It is found that for arbitrary amplitude, the concentration of dust temperature, superthermal species for ion and electron, i.e., κ_i and κ_e , affects the critical Mach number M_{cr} . Further, for the small amplitude, the Mach number, κ_i , and the concentration of ion density δ_i affect both amplitude and width. However, this model can be extended for future works, including different physical parameters such as magnetic fields and dust charge fluctuations for adiabatic and non-adiabatic dust charge variation systems. These findings provide valuable insight into nonlinear wave dynamics in space and astrophysical dusty plasma environments.

ACKNOWLEDGMENTS

N.P. Acharya acknowledges the University Grants Commission, Bhaktapur, Nepal, for the Ph.D. Fellowship (PhD-78/79-S&T-17).

EDITORS' NOTE

This manuscript was submitted to the Association of Nepali Physicists in America (ANPA) Conference 2025 for publication in the special issue of the Journal of Nepal Physical Society.

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