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## A Generalised Poisson Mishra Distribution

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### ABSTRACT

**Background:** “Mishra distribution” of B. K. Sah (2015) has been obtained in honor of Professor A. Mishra, Department of Statistics, Patna University, Patna (Sah, 2015). A one parameter Poisson-Mishra distribution (PMD) of B. K. Sah (2017) has been obtained by compounding Poisson distribution with Mishra distribution. It has been found that this distribution gives better fit to all the discrete data sets which are negative binomial in nature used by Sankarn (1970) and others. A generalisation of PMD has been obtained by mixing the generalised Poisson distribution of Consul and Jain (1973) with the Mishra distribution.

**Materials and Methods:** It is based on the concept of the generalisations of some continuous mixtures of Poisson distribution.

**Results:** Probability density function and the first four moments about origin of the proposed distribution have been obtained. The estimation of parameters of this distribution has been discussed by using the first moment about origin and the probability mass function at  $x = 0$ . This distribution has been fitted to a number of discrete data-sets to which earlier Poisson-Lindley distribution (PLD) and PMD have been fitted.

**Conclusion:**  $P$ -value of generalised Poisson-Mishra distribution is greater than PLD and PMD. Hence, it provides a better alternative to the PLD of Sankarn and PMD of B. K. Sah.

**Keywords:** Compounding, generalised Poisson distribution, goodness of fit, Mishra distribution, moments, Poisson-Lindley distribution, Poisson-Mishra distribution.

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**INTRODUCTION**

**Mishra-distribution**

A one-parameter Mishra distribution (MD) with parameters  $\varphi$  is defined by its probability density function (pdf):

$$f(x; \varphi) = \frac{\varphi^3 (1+x+x^2)e^{-\varphi x}}{(\varphi^2 + \varphi + 2)} \quad ; \varphi > 0, x > 0 \quad \dots\dots\dots (1)$$

The  $r^{\text{th}}$  moment about origin of this distribution is obtained as

$$\mu'_r = \frac{r! [\varphi^2 + (r+1)\varphi + (r+1)(r+2)]}{\varphi^r (\varphi^2 + \varphi + 2)} \quad \dots\dots\dots (2)$$

Putting the value  $r = 1, 2, 3$  and  $4$  in the expression (2), the first four moments about origin of the Mishra distribution are obtained as

$$\begin{aligned} \mu'_1 &= \frac{1! (\varphi^2 + 2\varphi + 6)}{\varphi (\varphi^2 + \varphi + 2)} ; \mu'_2 = \frac{2! (\varphi^2 + 3\varphi + 12)}{\varphi^2 (\varphi^2 + \varphi + 2)} ; \mu'_3 = \frac{3! (\varphi^2 + 4\varphi + 20)}{\varphi^3 (\varphi^2 + \varphi + 2)} \\ \mu'_4 &= \frac{4! (\varphi^2 + 5\varphi + 30)}{\varphi^4 (\varphi^2 + \varphi + 2)} \quad \dots\dots\dots (3) \end{aligned}$$

Moment generating function of Mishra distribution can be obtained as

$$[M_x(t)] = \int_0^\infty e^{tx} f(x) dx = \frac{\varphi^3}{(\varphi^2 + \varphi + 2)} \left\{ \frac{(\varphi - t)^2 + (\varphi - t) + 2}{(\varphi - t)^3} \right\} \quad \dots\dots\dots (4)$$

Distribution function of the Mishra distribution is obtained as

$$F_x = \int_0^x \frac{\varphi^3}{(\varphi^2 + \varphi + 2)} (1+x+x^2)e^{-\varphi x} dx = 1 - e^{-\varphi x} - \frac{\varphi x(\varphi + \varphi x + 2)e^{-\varphi x}}{(\varphi^2 + \varphi + 2)} \quad \dots\dots (5)$$

**Poisson-Mishra Distribution (PMD)**

It has been obtained by mixing Poisson distribution with the Mishra distribution (1). Suppose that the parameter  $\lambda$  of Poisson distribution follows Mishra distribution (1). Its probability mass function (pmf) has been obtained as

$$\text{PMD}(x; \varphi) = \int_0^\infty \left[ \sum_{x=0}^\infty \frac{e^{-\lambda} \lambda^x}{x!} \right] \frac{\varphi^3 (1 + \lambda + \lambda^2) e^{-\varphi \lambda}}{(\varphi^2 + \varphi + 2)} d\lambda \dots\dots\dots (6)$$

$$\text{PMD}(x; \varphi) = \frac{\varphi^3}{(\varphi^2 + \varphi + 2)} \cdot \frac{[(1 + \varphi)(2 + \varphi + x) + (1 + x)(2 + x)]}{(1 + \varphi)^{x+3}}; x = 0, 1, 2, \dots; \varphi > 0 \dots\dots\dots (7)$$

We name this distribution as 'Poisson-Mishra distribution (PMD)'. The expression (7) is the probability mass function of PMD.

The  $r^{\text{th}}$  moment about origin of the PMD (7) can be obtained as

$$\mu'_r = E[E(X^r / \lambda)] = \frac{\varphi^3}{(\varphi^2 + \varphi + 2)} \int_0^\infty \left[ \sum_{x=0}^\infty x^r \frac{e^{-\lambda} \lambda^x}{x!} \right] (1 + \lambda + \lambda^2) d\lambda \dots\dots\dots (8)$$

Obviously, the expression under bracket is the  $r^{\text{th}}$  moment about origin of the Poisson distribution. So, the first four moments about origin of the PMD can be obtained as

$$\mu'_1 = \frac{\varphi^3}{(\varphi^2 + \varphi + 2)} \int_0^\infty \lambda(1 + \lambda + \lambda^2) e^{-\varphi \lambda} d\lambda = \frac{[\varphi(\varphi + 2) + 6]}{\varphi(\varphi^2 + \varphi + 2)} \dots\dots\dots (9)$$

Taking  $r = 2$  in (8) and using the second moment about origin of the Poisson distribution, the second moment about origin of the PMD is obtained as

$$\mu'_2 = \frac{\varphi^3}{(\varphi^2 + \varphi + 2)} \int_0^\infty (\lambda + \lambda^2)(1 + \lambda + \lambda^2) e^{-\varphi \lambda} d\lambda = \frac{[\varphi(\varphi + 2) + 6]}{\varphi(\varphi^2 + \varphi + 2)} + \frac{[2\varphi(\varphi + 3) + 24]}{\varphi^2(\varphi^2 + \varphi + 2)} \dots (10)$$

Similarly, taking  $r = 3$  and 4 in (8) and using the respective moments of the Poisson distribution, we get finally, after a little simplification, the third and fourth moments about origin of the one parameter

$$\text{PMD as } \mu'_3 = \frac{[\varphi(\varphi + 2) + 6]}{\varphi(\varphi^2 + \varphi + 2)} + \frac{[6\varphi(\varphi + 3) + 24]}{\varphi^2(\varphi^2 + \varphi + 2)} + \frac{[6\varphi(\varphi + 4) + 120]}{\varphi^3(\varphi^2 + \varphi + 2)} \dots\dots\dots (11)$$

$$\mu'_4 = \frac{[\varphi(\varphi + 2) + 6]}{\varphi(\varphi^2 + \varphi + 2)} + \left[ \frac{14(\varphi(\varphi + 3) + 12)}{\varphi^2(\varphi^2 + \varphi + 2)} \right] + \frac{[36\{\varphi(\varphi + 4) + 20\}]}{\varphi^3(\varphi^2 + \varphi + 2)} + \frac{[24\{\varphi(\varphi + 5) + 30\}]}{\varphi^4(\varphi^2 + \varphi + 2)} \dots(12)$$

Consul and Jain (1973) obtained a two-parameter generalised Poisson distribution (GPD) given

by its probability function 
$$P(x) = \frac{\lambda(\lambda + x\theta)^{x-1} e^{-(\lambda+x\theta)}}{x!} \dots\dots\dots (13)$$

where  $x = 0, 1, 2, \dots; \lambda > 0; |\theta| < 1$  (Consul and Jain, 1973).

In this paper, generalization of the PMD (7) has been obtained by mixing the GPD (13) with the MD (1).

**MATERIALS AND METHODS**

It is based on the concept of the generalisations of some continuous mixtures of Poisson distribution. Probability mass function of the generalized Poisson-Mishra distribution (GPMD) has been obtained by compounding the generalised Poisson distribution of Consul and Jain with Mishra distribution. The first four moments of GPMD have been obtained. Parameters of this distribution have been obtained by using the first moment about origin and probability of  $x$  at zero. GPMD has been fitted to two discrete data-sets which are negative binomial in nature.

**RESULTS**

**A generalised Poisson-Mishra distribution (GPMD)**

Suppose that the parameter  $\lambda$  in the GPD (13) is a random variable and follows the Mishra distribution (1) with parameter  $\phi$ . We have thus the Mishra mixture of the GPD is obtained as

$$\begin{aligned}
 P(x; \phi, \theta) &= \int_0^\infty \frac{\lambda(\lambda + \theta x)^{x-1} e^{-(\lambda + x\theta)}}{\Gamma(x+1)} \cdot \frac{\phi^3 (1+x+x^2) e^{-\phi x}}{(\phi^2 + \phi + 2)} dx \dots\dots\dots (14) \\
 &= \int_0^\infty \frac{\lambda(\lambda + \theta x)^{x-1} e^{-(\lambda + x\theta)}}{\Gamma(x+1)} \cdot \frac{\phi^3 (1+\lambda + \lambda^2) e^{-\phi \lambda}}{(\phi^2 + \phi + 2)} d\lambda \\
 &= \frac{\phi^3 e^{-\theta x}}{\Gamma(x+1)(\phi^2 + \phi + 2)} \int_0^\infty \lambda^x \left(1 + \frac{\theta x}{\lambda}\right)^{x-1} (1 + \lambda + \lambda^2) e^{-\lambda(1+\phi)} d\lambda
 \end{aligned}$$

After a little simplification, we get

$$P(x; \phi, \theta) = \frac{\phi^3 e^{-\theta x}}{(\phi^2 + \phi + 2)x!} \sum_{i=0}^{x-1} \frac{(x-1)!(\theta x)^i}{i!(x-i-1)!} \left[ \frac{\Gamma(x-i+1)}{(1+\phi)^{x-i+1}} + \frac{\Gamma(x-i+2)}{(1+\phi)^{x-i+2}} + \frac{\Gamma(x-i+3)}{(1+\phi)^{x-i+3}} \right]$$

After a little simplification and arrangement of terms, we get

$$\begin{aligned}
 P(x; \phi, \theta) &= \frac{\phi^3 e^{-\theta x}}{(\phi^2 + \phi + 2)(1+\phi)^{x+3}} \left[ \left\{ (1+\phi)^2 + (1+\phi)(1+x) + (2+x)(1+x) \right\} + \right. \\
 &\quad \left. \sum_{i=1}^{x-1} \frac{\theta^i x^{i-1} (x-i) \left\{ (1+\phi)^2 + (1+\phi)(x-i+1) + (x-i+2)(x-i+1) \right\}}{i!(1+\phi)^{-i}} \right] \\
 \dots & (15)
 \end{aligned}$$

The expression (15) is the pmf of Mishra mixture of GPD. It can be seen that at  $\theta=0$ , it reduces to the PMD (7) and hence it may be termed as ‘generalized Poisson-Mishra distribution (GPMD)’. It can also be noted that  $P(X = 0)$  in the GPMD becomes independent of  $\theta$ , the additional parameter introduced in the distribution, and its role starts from  $P(X=1)$  onwards.

**Moments of GPMD**

The  $r^{\text{th}}$  moment about origin of the GPMD (15) can be obtained as  $\mu'_r = E[E(X^r / \lambda)] \dots$  (16)

Thus, we thus get

$$\mu'_r = \int_0^\infty \left[ \sum_{x=0}^\infty \frac{x^r \lambda (\lambda + x\theta)^{x-1} e^{-(\lambda + \theta x)}}{\Gamma(x+1)} \right] \frac{\varphi^3 (1 + \lambda + \lambda^2) e^{-\varphi \lambda}}{(\varphi^2 + \varphi + 2)} d\lambda \dots\dots\dots (17)$$

Obviously the expression under bracket is the  $r^{\text{th}}$  moment about origin of the GPD (13).

Putting  $r = 1$  in (17) and using the mean of the GPD), the mean of the GPMD is obtained as

$$\mu'_1 = \frac{\varphi^3}{(\varphi^2 + \varphi + 2)} \int_0^\infty \frac{\lambda}{(1-\theta)} (1 + \lambda + \lambda^2) e^{-\varphi \lambda} d\lambda = \frac{(\varphi^2 + 2\varphi + 6)}{\varphi(\varphi^2 + \varphi + 2)(1-\theta)} \dots\dots\dots (18)$$

Putting  $r = 2$  in (17) and using the second moment about origin of the GPD, the second moment about origin of the GPMD is obtained as

$$\mu'_2 = \frac{\varphi^3}{(\varphi^2 + \varphi + 2)} \int_0^\infty \left[ \frac{\lambda}{(1-\theta)^3} + \frac{\lambda^2}{(1-\theta)^2} \right] (1 + \lambda + \lambda^2) e^{-\varphi \lambda} d\lambda$$

which after a little simplification gives

$$\mu'_2 = \frac{(\varphi^2 + 2\varphi + 6)}{\varphi(\varphi^2 + \varphi + 2)(1-\theta)^3} + \frac{(2\varphi^2 + 6\varphi + 24)}{\varphi^2(\varphi^2 + \varphi + 2)(1-\theta)^2} \dots\dots\dots (19)$$

Putting  $r = 3$  in (17) and using the third moment about origin of the GPD, the third moment about origin of the GPMD is obtained as

$$\mu'_3 = \frac{\varphi^3}{(\varphi^2 + \varphi + 2)} \int_0^\infty \left[ \frac{\lambda(1 + 2\theta)}{(1-\theta)^5} + \frac{3\lambda^2}{(1-\theta)^4} + \frac{\lambda^3}{(1-\theta)^3} \right] (1 + \lambda + \lambda^2) e^{-\varphi \lambda} d\lambda$$

which after a little simplification gives

$$\mu'_3 = \frac{(1+2\theta)(\varphi^2+2\varphi+6)}{\varphi(\varphi^2+\varphi+2)(1-\theta)^5} + \frac{3(2\varphi^2+6\varphi+24)}{\varphi^2(\varphi^2+\varphi+2)(1-\theta)^4} + \frac{6(\varphi^2+4\varphi+20)}{\varphi^3(\varphi^2+\varphi+2)(1-\theta)^3} \dots (20)$$

Putting  $r = 4$  in (17) and using the fourth moment about origin of the GPD, the fourth moment about origin of the GPMD is obtained as

$$\mu'_4 = \frac{\varphi^3}{(\varphi^2+\varphi+2)} \int_0^\infty \left[ \frac{(1+8\theta+6\theta^2)\lambda}{(1-\theta)^7} + \frac{(7+8\theta)\lambda^2}{(1-\theta)^6} + \frac{6\lambda^3}{(1-\theta)^5} + \frac{\lambda^4}{(1-\theta)^4} \right] (1+\lambda+\lambda^2)e^{-\varphi\lambda} d\lambda$$

which after a little simplification gives

$$\begin{aligned} \mu'_4 &= \frac{(1+8\theta+6\theta^2)(\varphi^2+2\varphi+6)}{\varphi(\varphi^2+\varphi+2)(1-\theta)^7} + \frac{(7+8\theta)(2\varphi^2+6\varphi+24)}{\varphi^2(\varphi^2+\varphi+2)(1-\theta)^6} + \frac{6(6\varphi^2+24\varphi+120)}{\varphi^3(\varphi^2+\varphi+2)(1-\theta)^5} + \\ &\frac{(24\varphi^2+120\varphi+720)}{\varphi^4(\varphi^2+\varphi+2)(1-\theta)^4} \dots (21) \end{aligned}$$

It can easily be seen that at  $\theta = 0$ , these moments reduce to the respective moments of the PMD (7)

**Estimation of parameters**

The GPMD have two parameters  $\varphi$  and  $\theta$ . Here, we have obtained the estimates of these parameters by using  $P(x=0)$  and the first moment about origin of the GPMD.

$$P(X = 0) = \frac{\varphi^3 \{ (1+\varphi)^2 + (1+\varphi) + 2 \}}{(1+\varphi)^3 (\varphi^2 + \varphi + 2)} = k(\text{say}) \dots (22)$$

$$\text{or } P(X = 0) = \varphi^3 \{ (1+\varphi)^2 + (1+\varphi) + 2 \} - k(1+\varphi)^3 (\varphi^2 + \varphi + 2) = 0 \dots (23)$$

The expression (23) is the polynomial equation in  $\varphi$  which can be solved by the Newton-Raphson or Regula-Falsi method. The first population moment is replaced by respective sample moment and putting the value of  $\varphi$  in expression (18) an estimate of  $\theta$  can be obtained by as

$$(1-\theta) = \frac{(\varphi^2+2\varphi+6)}{\varphi(\varphi^2+\varphi+2)\mu'_1} \dots (24)$$

where  $|\theta| < 1$

**Goodness of fit**

The generalised Poisson-Mishra distribution (GPMD) has been fitted to a number of discrete data-sets to which earlier Poisson-Lindley distribution (PLD) and PMD have been fitted. Here the fittings of the GPMD to two data-sets have been presented in the following tables. The first data is the Student's historic data Hemocytometer counts of yeast cell, used by Borah (1984) for fitting the Gegenbauer distribution (Borah, 1984) and the second is due to Kemp and Kemp (1965) regarding the distribution of mistakes in copying groups of random digits (Kemp & Kemp, 1965).

**Table 1.** Hemocytometer counts of yeast cell.

Number of Yeast Cell	Observed Frequency	Expected Frequency of PLD	Expected Frequency of Two-Parameter PLD	Expected Frequency of PMD	Expected Frequency of GPMD
0	213	234.4	227.6	234.3	213.0
1	128	99.3	101.5	99.3	128.3
2	37	40.4	43.6	40.6	38.8
3	18	16.0	17.9	16.0	14.7
4	3	6.2	6.8	6.1	3.4
5	1	2.4	2.2	2.3	1.2
6	0	1.3	0.6	1.4	0.6
Total	400	400	400	400	400
$\mu'_1$	0.6825		/		
$\mu'_2$	1.2775				
$\hat{\varphi}$		1.9602	3.6728	2.3914	2.0736
$\hat{\alpha}$		-	-0.0916	-	-
$\hat{\theta}$					-0.2191
$\chi^2$		14.3	12.25	14.23	0.33
df		4	3	4	3
P-value		0.0068	0.00698	0.007	>0.95

**Table 2.** Distribution of mistakes in copying groups of random digits.

Number of Errors per Group	Observed Frequency	Expected Frequency of PLD	Expected Frequency of Two-Parameter PLD	Expected Frequency of PMD	Expected Frequency of GPMD
0	35	33.1	32.4	32.9	35
1	11	15.3	15.8	15.3	13.2
2	8	6.8	7.0	6.8	6.2
3	4	2.9	2.9	3.6	3.0
4	2	1.2	1.9	1.4	2.6
Total	60	59.3	60	60	60
$\mu'_1$	0.7833				
$\mu'_2$	1.8500				
$\hat{\varphi}$		1.7434	1.9997	2.1655	2.3750
$\hat{\alpha}$		-	-0.3829	-	-
$\hat{\theta}$					-0.1204
$\chi^2$		2.20	2.11	1.72	0.776
<i>P</i> -value		0.48	0.28	0.58	0.67

## CONCLUSION

It has been observed that the generalized Poisson-Mishra distribution (GPMD) gives better fit to all the discrete data-sets (negative binomial in nature) than the one parameter PLD of Sankarn (1970) (Sankaran, 1970), two-parameter PLD of R. Shanker and A. Mishra (2014) (Shankar & Mishra, 2014) and one parameter PMD of B. K. Sah (2017) (Sah, 2017) because of *P*-value. Hence, it provides a better alternative to the above mentioned distributions.

## CONFLICT OF INTEREST

The author declared that there is no conflict of interest.



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