Development of Numerical Modeling for Finding Reflection and Transmission Coefficients in an Engineering Learning Paradigm of Computational Thinking

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ABSTRACT

In electromagnetics and antenna engineering, reflection and transmission coefficients are important parameters to be evaluated statistically and numerically to obtain effective computer simulations. This paper proposes a pedagogy to understand the development methodology of numerical reflection and transmission coefficient models. The proposed method employs relatively easy mathematics instead of the conventional ways that use difficult mathematics. These are then simulated as computer models. The study in this paper included a medium comprising of air and glass placed in it. Firstly, the region was divided into a gridded structure where the grids were mathematically formulated. Points on the grid were distinguished between different media using electrical permittivity and conductivity. The grids included boundary conditions and electric field wave propagation through them. Both boundary conditions and field equations were numerically modelled and discretized using the finite difference method with a proportional h^2 error. This method provided a system of linear equations, which were then solved linearly to obtain the required reflection and transmission coefficients. An important aspect of the presented work is to provide an example approach for computational thinking (CT), which is now considered an important part of any engineering curriculum.

Keywords: Engineering education, Computational thinking, Computer simulations

1. INTRODUCTION

Learning by computer simulations is an integral part of a learning strategy called computational thinking (CT) (German 2019; Basu 2013; Wing 2008; Magana 2017). However, before computers can be utilized to solve a problem, the problem must be understood very clearly. Computational thinking is a learning technique that helps to achieve this, explicitly solution execution and evaluation. Computer simulations are integral to the engineering curriculum to ease learning (Khuda 2021; Ozdemir 2020; Khuda I. E. 2019; Yoo 2021). These studies show that many complex and difficult concepts have become fairly easy for students to grasp and understand the subject using computer simulation tools. To make computer models, numerical modelling of the underlying topic is required. Numerical analysis involves the study of those algorithms that use numerical approximation for mathematical analysis problems (Sajid 2015; Atkinson 1992; Du 2020). Numerical analysis helps implement the analytical models into a computing machine which helps in the modelling and simulation of engineering problems

(Khuda 2017 'a'; Khuda 2017 'b'; Khuda 2019). Thus they are an integral part of modern data science and data analysis. The inclusive objective of numerical analysis is the design and analysis of procedures to give estimated and approximate but precise solutions to difficult problems.

From another perspective, computer simulations are also very useful in reducing students' stress and anxiety levels during their academic learning. Among different socio-economic factors, academic learning is a leading stress factor among students (Britton 2011; Stassart 2017; Khuda 2020; Yang 2018; Stewart 2020). Being in a state of anxiety during education hurts cognitive functioning and thus degrades learning. Anxiety and stress result in a disorder that can deteriorate a student's integrative progress socially, emotionally, and cognitively. Recent studies have shown that computer simulations serve as effective learning tools and help significantly reduce stress and anxiety among students during their academic learning. Students now have more affinity for computers, tablets, and other electronic gadgets. Educating and learning using such tools have improved the learning process by reducing stress and anxiety about the subject (Randhir Singh 2004; Schön 2014; Rodríguez del Rey 2021; Bayanova 2019).

However, comparatively detailed mathematical modelling is required to develop an appropriate computer simulation. Many topics in the engineering domain require such understanding to make them effective for learning. The calculation of reflection and transmission coefficients is one such area of consideration. The reflection and transmission coefficients help identify the characteristics of the two media based on their electrical parameters, viz permittivity, conductivity, and permeability. Although analytical models are available for calculating reflection and transmission coefficients, mathematicians have also used numerical methods because they help calculate current, voltages, and other electrical circuit parameters

Furthermore, they can be readily used at high frequencies where analytical solutions may fail. One-dimensional electromagnetic wave propagation problems can be expressed precisely in integral-differential equations. The numerical solution of this equation is complicated and time taking. Mur (1976) described a method of transforming these equations into integral equations of the Volterra type. This method helped to have the solution of the concluding equation easily and within a short time. Md Shahar Aftanasar (2015) described the Design of Experiment (DOE) implementation for RF and Microwave course. The objective was to demonstrate the use of DOE in investigative laboratory activity for 3rd-year students in the RF and Microwave Course. Rambousky (2015) derived generalized reflection and transmission coefficients from calculating the currents and practical layouts of non-homogeneous transmission lines. Magdalena (2018) investigated the wave reflection from a porous wave absorber analytically and numerically. This paper first derived the analytical solution of the wave reflection coefficient from a porous wave absorber over a flat bottom. The same equations were solved numerically using the finite volume method on a staggered grid, and a comparison was made between the two.

The numerical technique was implemented to develop surface waves that pass through a porous absorber over varied bottom topography. In literature, many numerical methods are available. However, the problem stemmed from a lack of a detailed mathematical explanation of the resultant reflection and transmission coefficients. This lack has created a learning gap for students to understand the underlying phenomenon to reach the outcomes. Hence there is still a dire need to develop textbook-level knowledge for the students to understand the formulation of reflection and transmission coefficients from numerical methods. The finite difference is one of the simplest numerical techniques available. A finite-difference solution to Poisson's or Laplace equation proceeds in three steps, namely (1) dividing the solution region into grids or nodes, (2) approximating the differential equation and boundary conditions by a set of linear algebraic equations, and finally (3) solving this set of algebraic equations.

2. MATERIALS AND METHODS

When a plane wave from medium one is interrupted by medium two, some portion of it is reflected in medium one, and the remaining is transmitted in medium two. A simple experiment can consider a plane wave propagating in the +zdirection and incident on the boundaryz=0 between medium one (for z < 0) and medium two (for z > 0). Let medium one is air/ vacuum and medium two be a fairly denser medium, i.e., glass. The electrical field can be assumed to propagate in the +ve x dimension, considering that the magnetic field is in the +ve y direction. All three, i.e., electric field, magnetic field, and wave propagation, are orthogonal, along xy with an z axis. Reflection and transmission coefficients determine the plane wave's reflection and transmission.

An electromagnetic field problem is generated by considering that an electromagnetic plane wave propagates towards a denser medium two from a rarer medium 1. The denser medium is glass, and the rarer medium is taken as air. It is required to compute reflection and transmission coefficients when a wave enters from the air into a denser glass medium. The glass medium can be considered to have a thickness of 2a. This media makes it an even function. Considering region space is defined by the variablex, the physical constraints are $\epsilon(x),\mu(x)=\mu_o\ \sigma(x)$ and for the glass $-a \le x \le a$. Region outside is air having $\epsilon(x) = \epsilon_o$, $\mu(x)=\mu_o$ and $\sigma(x)=0$ *for* $|x|>a$. The differential equation defines the total electric field as,

$$-\frac{d^2\left(E_z(x)\right)}{dx^2}+\mu_0\left[j\omega\sigma(x)-\omega^2\varepsilon(x)\right]E_z(x)=0 \qquad (1)$$

The technique applied matched the expression of the incoming wave to the numerical solution in the vacuum region. The matching is done outside the scattered region, i.e., in the vacuum region. The advantage is that the behavior of the incoming field is known in the vacuum region. Likewise, the reflected and transmitted waves can be described in the vacuum region. This method can match the discretized interior region to the fields outside two points $b > a$. Hence the matching points are in a vacuum.

Firstly it is required to obtain boundary conditions atx=-b and x=b. Supposedly if the incident field is $E_z^i(x)=E_o^i e^{-jk_o x}$, the reflected wave can be written as $E_z^r(x)=E_o^r e^{jk_o x}$. The transmitted field then becomes as $E_z^t(x)=E_o^t e^{-jk_o x}$ in all these analytical expressions $k_o=\frac{\omega}{c_o}$ and $c_o=1/\sqrt{\varepsilon_o\mu_o}$.

From these analytical expressions, the entire field is the superposition of the incident and reflected field in the region defined by $x < -a$ or the left boundary defined by $x = -b$.

Similarly, for a region $x > a$, the entire field is equal to the transmitted field of the region. This will serve as the right boundary condition defined by $x = b$. The mathematical descriptions are shown as follows:

The Left Boundary, *x* = - *b*

The total field at the left boundary is

$$E_z^t(x)=E_z^i(x)+E_z^r(x) \qquad (2)$$

$$E_z^t(x)=E_o^i e^{-jk_o x}+E_o^r e^{jk_o x} \qquad (3)$$

Where,

$$k_o=\frac{\omega}{c_o}$$

and

$$c_o=1\Big/\sqrt{\varepsilon_o\mu_o}$$

Taking the derivative of (1) w.r.t x gives,

$$\frac{d\left(E_z^t\left(x\right)\right)}{dx} = -jk_o E_o e^{-jk_o x} + jk_o E_o e^{jk_o x} \tag{4}$$

$$\frac{d\left(E_z^t\left(x\right)\right)}{dx} = -jk_o E_z^i\left(x\right) + jk_o E_z^r\left(x\right) \tag{5}$$

There are two unknown parameters from (2) to (5). One is the transmitted field, i.e. $E_z^t\left(x\right)$, and the other is the reflected field, i.e. $E_z^r\left(x\right)$. The incident field, i.e. $E_z^i\left(x\right)$, is known. In order to evaluate the transmitted wave/ field at the left boundary, (1) is multiplied by jk_o and subtracted (4) from it. It gives the boundary condition as follows,

$$jk_o E_z^t\left(x\right) - \frac{d\left(E_z^t\left(x\right)\right)}{dx} = 2\, jk_o E_o\, e^{-jk_o x} \tag{6}$$

The Left Boundary, $x = b$

The electric field at the right boundary consists only of the transmitted field. i.e.

$$E_z\left(x\right) = E_z^t\left(x\right) \tag{7}$$

$$E_z^t\left(x\right) = E_t e^{-jk_o x} \tag{8}$$

The right boundary is evaluated by taking the derivative of (8) w.r.t.x as follows,

$$\frac{d\left(E_z^t\left(x\right)\right)}{dx} = -jk_o E_t e^{-jk_o x} = -jk_o E_z^t\left(x\right) \tag{9}$$

Multiply (8) with jk_o and add to (9), as

$$jk_o E_z^t\left(x\right) + \frac{d\left(E_z^t\left(x\right)\right)}{dx} = 0 \tag{10}$$

The total electric field satisfies the differential equation (1). It is discretized on the grid using the finite differential approximation as,

$$\left.\frac{d^2\left(E_z\left(x\right)\right)}{dx^2}\right|_{x=x_o} \approx \frac{E_z\left(x_{n+1}\right) - 2E_z\left(x_n\right) + E_z\left(x_{n-1}\right)}{\Delta x^2} = \frac{\zeta_{n+1} - 2\zeta_n + \zeta_{n-1}}{\Delta x^2} \tag{11}$$

The formulation in (11) is the second-order derivative, and $E_z\left(x\right)\big|_{x=x_o} = \zeta_n$. This latter denotes the unknowns on the grid. On substituting (11) in (10) provided,

$$-\zeta_{n-1} + \zeta_n\left[2 + \Delta x^2 \mu_o\left[j\omega\sigma\left(x\right) - \omega^2\varepsilon\left(x\right)\right]\right] - \zeta_{n+1} = 0 \tag{12}$$

The grid points are modelled, so the material interfaces are defined by the fall between the grid points. The grid points are chosen as follows,

$$x_n = \left(n + \frac{1}{2}\right)\Delta x \tag{13}$$

Where $\Delta x = a / N$. Here N is an integer defined by $N \geq 2$ and $n = -2N - 1, -2N, ...-2N$. In this case, the material interface $x = \pm a$ falls between the grid points. Hence the derivative between the grid points can be obtained using the finite difference approximation. The derivative is approximated using the second-order in h, i.e. $O(h^2)$, in Taylor expansion, as shown in the following equation below.

$$\frac{d}{dx}E_z\left(x\right)\Big|_{x=x_n+\frac{h}{2}} \approx \frac{E_z\left(x_{n+1}\right)-E_z\left(x_n\right)}{\Delta x} \qquad (14)$$

Where h is the distance between the grid points?

The E-field at the half grid can be evaluated from the neighboring grid points of its value, as shown in the following equation.

$$\frac{d}{dx}E_z\left(x\right)\Big|_{x=x_n+\frac{h}{2}} \approx \frac{E_z\left(x_{n+1}\right)-E_z\left(x_n\right)}{2} \qquad (15)$$

The left boundary $x = -b$ can be solved by substituting (14) and (15) into (6) as,

$$\zeta_1\left(jk_o\Delta x+2\right)+\zeta_2\left(jk_o\Delta x-2\right)=4\,jk_o\Delta x E_o e^{-jk_o(-b)} \qquad (16)$$

The right boundaryx=-b can be solved by substituting (14) and (15) into (10) as,

$$\zeta_{N_{gp-1}}\left(jk_o\Delta x-2\right)+\zeta_{N_{gp}}\left(jk_o\Delta x+2\right)=0 \qquad (17)$$

Where, N_{gp} is the number of grid points.

The boundary conditions and differential equation provided a linear equation system of the form $Az = b$, where $z = \left[\zeta_1,\ \zeta_2,.....\zeta_{N_{gp}}\right]^T$. This discretisation created ten grid points placed from left to right. The first row in the grid matrix showed the left boundary condition. The wave equation was from the second to the ninth row, and the last row corresponded to the right boundary condition. The matrix A of size 10 ×10 is reproduced here as follows,

$jko\ \Delta x+2$	$jko\ \Delta x-2$	0 … 0		0	0
-1	$2+\Delta x^2\ \mu_o\ [j\omega\sigma(x_2)-\omega^2\ \varepsilon(x_2)]$	-1… 0		0	0
…..	……….	……..	…….	……	…….
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
0	0	0	-1	$2+\Delta x^2\ \mu_o\ [j\omega(x_2)-\omega^2\ \varepsilon(x_2)]$-1	
0	0	0	0	$jk_o\ \Delta x - 2$	$jk_o\ \Delta x + 2$

The matrix b of size 10×1 is shown as,

$$\begin{matrix} 4jk_o \Delta x E_o\, e^{-jko\,(-b)} \\ 0 \\ 0 \\ . \\ . \\ . \\ 0 \end{matrix}$$

The reflection and transmission coefficients can be calculated analytically by (18) and (19), respectively.

$$R = \frac{k_o^2 - k_1^2}{\Delta} e^{ajk_o\left(e^{jak_1} - 1\right)} \qquad (18)$$

$$T = \frac{k_o k_1}{\Delta} 4e^{j2a\left(k_o + k_1\right)} \qquad (18)$$

Where

$$\Delta = \left(k_o + k_1\right)^2 e^{j4ak_1} - \left(k_o - k_1\right)^2, k_o = {}^{\omega}\!/_{c_o} \text{ and } \quad k_1 = \sqrt{\varepsilon_r k_o^2 - j\omega\mu_o\sigma}$$

Putting the above definitions in (18) and (19) gives the following (20) and (21).

$$R = \frac{E_z\left(x\right) - E_o e^{-jk_o(-a)}}{e^{jk_o(-a)}} \qquad (20)$$

$$T = \frac{E_z^t\left(x\right)}{E_z^i\left(x\right)} = \frac{E_z\left(x\right)}{E_o e^{-jk_o(a)}} \qquad (21)$$

Where $E_z\,(x)$ is the average value of the material interface at $x < \alpha$ and $x > \alpha$ and $E_o = 1$.

3. RESULTS AND DISCUSSION

Before performing computer simulations, it is noted from (11) that the electric field is written in terms of ζ. The unknown values of electric fields, i.e., ζ can be obtained easily by making a linear solution of matrices A*b*. These matrices are defined above. Once obtained, the reflection and transmission coefficient values can be obtained easily using (20) and (21).

Computer simulations were conducted to compare analytical and numerical models. Using (18) and (19), numerically modelled reflection and transmission coefficients were simulated and plotted for a frequency range of 0.1 GHz to 10 GHz. This is shown in Fig. 1.

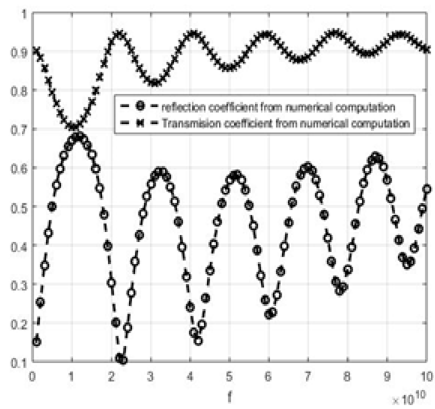

Fig. 1: Reflection and Transmission Coefficients using Numerical Computations

Using (20) and (21), numerically modelled reflection and transmission coefficients were simulated and plotted for a frequency range of 0.1 HHZ to 10 GHz. This is shown in Fig. 2.

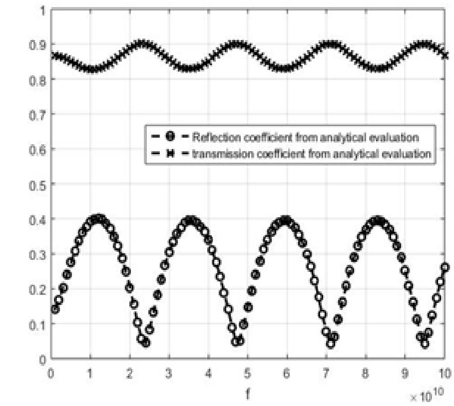

Fig. 2: Reflection and Transmission Coefficients using Analytical Evaluation

An error comparison was also made regarding how error varies for reflection and transmission coefficients. Results show almost the same error for low to medium frequencies, but afterwards, there is deviation. This is shown in Figure 3. The maximum absolute error value for reflection and transmission coefficient obtained is 9.8470×10^{-6} 1.9002×10^{-5} and respectively.

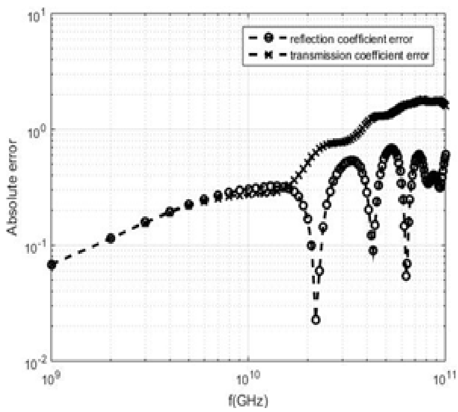

Fig. 3: Reflection and Transmission Coefficients Errors Comparison

4. CONCLUSIONS

This paper presents the mathematical understanding and numerical modelling of reflection and transmission coefficients with an error comparison. The modelling was developed using the finite difference method. This numerical understanding is essential in making complex models where analytical expressions lose their reliability of results. Furthermore, the numerical modelling of reflection and transmission coefficients is required in making complex computer models of transmission lines and antennas.

As mentioned in the abstract, an important learning aspect of the presented work in this paper was to deliver an example computational thinking (CT) method necessary to be part of the engineering curriculum. We have provided an example to fellow academic researchers on developing and inculcating the CT approach in the engineering curriculum. At first, the problem was formulated (abstraction). It was followed by solution expression (automation) and analysis (solution execution and evaluation). Hence we have provided a generic solution as a generalization that can be used to crack many deviations from the preliminary problem.

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