



Case study on the size of the particle for transmission probability to take place through the barrier

Rajendra Neupane*

neupanerajendra5@gmail.com

Abstract

Diffraction phenomenon occurs when the wavelength of the wave is comparable to the size of the obstacle. Tunneling through a barrier occurs when the particle size is comparable to the width of the barrier. The Transmission probability versus potential energy curve is hyperbolic in nature. This main aim of this article is to show that there is no probability of penetrating the particle through the barrier if barrier width is extremely larger and extremely smaller than the particle size. The condition for tunneling is similar to the necessary and sufficient condition for diffraction to occur. The various examples presented here shows that the penetration of the particle through the barrier is only possible when the size of the particle is comparable to the size of the barrier height. This article shows that probability occurrence, Wave-particle duality and uncertainty of a particle exists only when there is comparable size between the particle and its barrier width.

Key words: *Diffraction, tunneling, potential barrier, Transmission probability.*

Introduction

Wave particle duality, Heisenberg's uncertainty principle and probability have a key role in tunneling phenomenon (Gasiorowicz, 2007). Probability occurrence is independent of time (Griffiths, 2008). During tunneling, the particle shows a wave behavior. A particle shows reflection, refraction, diffraction, interference and polarization phenomenon which is confirmed by Davisson and Germer experiment (Murugesan, 1997). The wave shows particle behavior during photoelectric effect (Zettili, 2009). The Schrodinger time independent equation is

$$-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} u(x) + V(x)u(x) = E(x)u(x)$$

Where, $u(x)$ represents the wave function. $E(x)$

represents the total energy of the particle, $V(x)$ represents the potential energy of the particle (Zettili, 2009). The wave function gives the particle behavior.

According to the classical theory, an alpha particle cannot escape from the potential well due to its insufficient energy. The barrier height is in the order of 30 Mev and the decayed alpha particles have energies only in the range 4 to 9 Mev (Gamow, 1928). This energy range from 4 to 9 Mev is very small, but the half-lives of the radioactive alpha emitters range from $\sim 10^{-7}$ s to 10^{-8} s, while the energy changes by a factor of 2, the change in the half-lives is about 25 orders of magnitude. Classical arguments fail to account for alpha decay, but quantum mechanics

* Mr. Neupane is Lecturer in Physics at Birendra Multiple Campus

provides a straight forward explanation based upon the concept of tunneling where a particle can be found in a classically forbidden region (Gamow, 1928).

Laue and Bragg showed the diffraction of x-rays through a crystal (Fukamachi et.al., 2004). For diffraction phenomenon to take place, the wavelength of the wave should be Comparable to the size of the obstacle (Patterson, 1924). The size of the interatomic distance in a crystal is of the order of 1 \AA . The wavelength of the x-ray varies from 1 \AA to 100 \AA . So Laue used zinc sulphide crystal for the diffraction of x-rays (Patterson, 1939).

The accepted evidence for wave –like behavior is the phenomenon of diffraction (Shvyd'ko et.al., 2006). The rainbow pattern of colours that we see when we look at the surface of a compact disk is caused by light waves diffracting from the regularly spaced bands of shiny material that make up the tracks (Bucksbaum, 2001). This effect can be seen because the wavelength of light, although small, is large enough to be comparable to the spaces between adjacent tracks (Bucksbaum, 2001). It is comparable with the diffraction phenomenon observed in single slit, and double slit. Double slit diffraction is a corner stone of Quantum mechanics (Roger et. al., 2013). It illustrates key features of Quantum mechanics: Interference and the particle wave duality of matter. Richard Feynman presented a thought experiment to show these features (Roger et.al., 2013). The barrier potential energy is distributed to various levels. Similarly, the energy acquired by the electron also varies due to Heisenberg's uncertainty principle and probabilistic distribution of energy (Sarkar and Bhattacharyya, 2008). The various planes in a crystal are compared to the distributed energy levels (energy band) of the barrier according to the quantum model.

Probability density is given by

$$\rho = |\psi|^2 \text{ (Agarwal, 2002)}$$

Transmission probability is given by the ratio of the amplitude of the transmitted wave to the amplitude of the incident wave and is given by

$$\text{Transmission Coefficient } t(T) = \frac{\text{Amplitude of the transmitted wave}}{\text{Amplitude of the incident wave}}$$

The weak nuclear force is the second weakest force, after the force of gravity, and it's the force with the shortest range (Rutherford, 1899). The alpha particle emits from the nucleus even if the alpha particle has less energy than the barrier potential inside the nucleus (Agarwal and Prakash, 2002). Nucleus is considered as the well. The alpha particle bounces back and forth inside the nucleus. The cold emission of electron from the metal surface is also the case of tunneling (Mandel, 2015). Weak field is also responsible for the tunnel effect. Many physical, chemical and biological processes occur due to application of low field (Majumdar, 2011). The case of mutation in biological process is the cause of tunnel effect (Majumdar, 2011). The solar panel works due to the interaction of weak field. Many chemical reactions can be activated due to lower energy. The lower energy is responsible to emit the electrons from the metal surface (in case of solar panel) and the energy required to activate the reaction in case of chemical reaction (Majumdar, 2011).

The de – Broglie wavelength of the emitted wave is given by

$$\text{Wavelength } (\lambda) = \frac{h}{\sqrt{3mKT}}$$

Where h is Planck's constant, T is the temperature of the specimen, K is Boltzmann's constant (Zettili, 2009).

An electron beam with energy of 600 eV, which corresponds to a de – Broglie wavelength of 50 pm, was generated with a thermionic tungsten filament and several electrostatic lenses (Roger, et.al., 2013). The wavelength of the emitted radiation depends upon the Temperature of the body (Singhal et.al., 1998). When the body is heated, it emits radiations. The intensity and the frequency of the emitted radiation depend upon the temperature of the heating body (Zettili, 2009). The color depends upon the temperature of the body (Zettili, 2009). Color of the radiation emitted by the heating body depends upon its wavelength.

Methods: The data used is arbitrary. The variation of transmission probability with E/V had been calculated earlier. The computer programming used here is Excel. In this work, the energy of the particle has kept constant. The potential energy is varied. The width of the barrier is also constant. The transmission probability for same particle with identical energy has calculated. The equation of transmission probability is used, which is obtained from the case of the square potential barrier, when the width of the barrier is $2a$ and potential energy is of height ' V '. The square potential barrier is the particular case of the smooth potential barrier. The plot of transmission probability versus potential energy (barrier potential) is observed in two dimensional forms.

Discussion and results: The general equation for the transmission probability is

$$T = \frac{4E(V - E)}{4E(V - E) + V^2 \sinh^2 Kl}$$

$$T \approx \frac{16E(V - E)}{V^2} e^{-2Kl}$$

As $Kl \rightarrow \infty$ the transmission coefficient tends to zero which is in agreement with the classical result.

Similarly, when potential is infinite, the transmission probability is zero which is also in agreement with the classical result.

The transmission probability goes on decreasing when the width of the barrier goes on increasing. The slope of the curve goes on decreasing. The curve changes from discrete to continuous when the value of potential goes on increasing from its minimum to maximum value. Thus, the curve changes from quantum nature to classical nature if the value of potential keeps on increasing which is in accordance with Bohr's correspondence principle. In the graph the discontinuous line falling downward represents the quantum behavior while the continuous line represents the classical nature.

Table that indicates values of energy of the quantum particle, potential, barrier width and transmission probability.

| Energy (eV) | Energy(J) | Potential (eV) | Potential (J) | Width (a)m | Ka | Sinh(Ka) | #DIV/0! | Transmission Probability (T) |
|-------------|-----------|----------------|---------------|------------|----------|----------|-------------|------------------------------|
| 0.25 | 4E-20 | 0.25 | 4E-20 | 4E-10 | 0 | 0 | #DIV/0! | #DIV/0! |
| 0.25 | 4E-20 | 0.5 | 8E-20 | 4E-10 | 1.027866 | 1.218662 | 1 | 0.402392248 |
| 0.25 | 4E-20 | 0.75 | 1.2E-19 | 4E-10 | 1.453622 | 2.02243 | 1.125 | 0.178523601 |
| 0.25 | 4E-20 | 1 | 1.6E-19 | 4E-10 | 1.780316 | 2.881572 | 1.333333333 | 0.082841294 |
| 0.25 | 4E-20 | 1.25 | 2E-19 | 4E-10 | 2.055731 | 3.842276 | 1.5625 | 0.041550129 |
| 0.25 | 4E-20 | 1.5 | 2.4E-19 | 4E-10 | 2.298378 | 4.928796 | 1.8 | 0.022357633 |
| 0.25 | 4E-20 | 1.75 | 2.8E-19 | 4E-10 | 2.517747 | 6.15999 | 2.041666667 | 0.012743397 |
| 0.25 | 4E-20 | 2 | 3.2E-19 | 4E-10 | 2.719477 | 7.553238 | 2.285714286 | 0.007610163 |
| 0.25 | 4E-20 | 2.25 | 3.6E-19 | 4E-10 | 2.907243 | 9.125819 | 2.53125 | 0.004721347 |
| 0.25 | 4E-20 | 2.5 | 4E-19 | 4E-10 | 3.083597 | 10.89551 | 2.777777778 | 0.003023378 |
| 0.25 | 4E-20 | 2.75 | 4.4E-19 | 4E-10 | 3.250397 | 12.88091 | 3.025 | 0.001988462 |
| 0.25 | 4E-20 | 3 | 4.8E-19 | 4E-10 | 3.409045 | 15.10164 | 3.272727273 | 0.001338013 |
| 0.25 | 4E-20 | 3.25 | 5.2E-19 | 4E-10 | 3.560631 | 17.57849 | 3.520833333 | 0.000918316 |
| 0.25 | 4E-20 | 3.5 | 5.6E-19 | 4E-10 | 3.706023 | 20.33353 | 3.769230769 | 0.000641273 |
| 0.25 | 4E-20 | 3.75 | 6E-19 | 4E-10 | 3.845921 | 23.39021 | 4.017857143 | 0.000454715 |
| 0.25 | 4E-20 | 4 | 6.4E-19 | 4E-10 | 3.980907 | 26.77346 | 4.266666667 | 0.000326859 |
| 0.25 | 4E-20 | 4.25 | 6.8E-19 | 4E-10 | 4.111463 | 30.50978 | 4.515625 | 0.000237849 |
| 0.25 | 4E-20 | 4.5 | 7.2E-19 | 4E-10 | 4.237999 | 34.62733 | 4.764705882 | 0.000175005 |
| 0.25 | 4E-20 | 4.75 | 7.6E-19 | 4E-10 | 4.360865 | 39.15604 | 5.013888889 | 0.000130068 |
| 0.25 | 4E-20 | 5 | 8E-19 | 4E-10 | 4.480363 | 44.12768 | 5.263157895 | 9.75639E-05 |

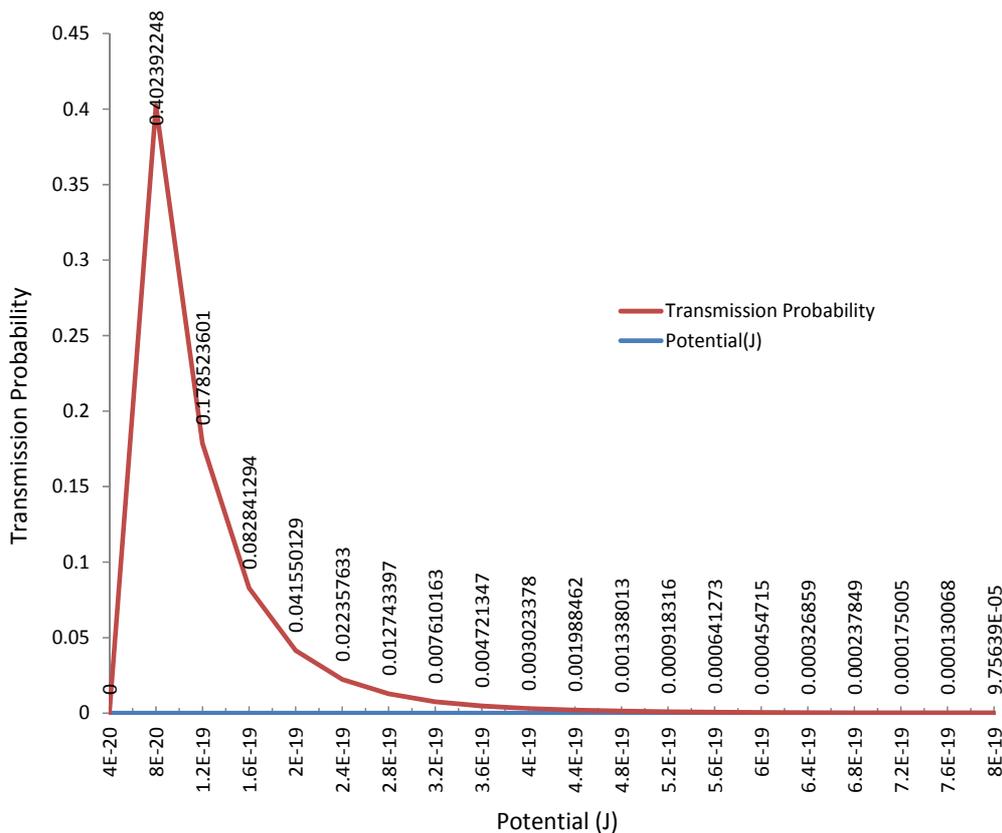


Figure: the variation of the transmission probability with the barrier height

At constant Energy of the particle, the potential energy of the barrier is increased from an arbitrary minimum value to maximum value of 5eV. The transmission probability of the particle through the barrier goes on decreasing as the particle passes through the barrier of increased energy. The variation of transmission probability and potential energy of the barrier shows hyperbolic in nature. The slope of the curve goes on decreasing. The slope indicates the number of oscillations. The decrease in slope indicates the decrease in the number of oscillations. At infinite potential, the particle tends to be at rest so the slope also tends to be zero. The particle also cannot be at rest due to Heisenberg’s uncertainty principle. So the infinite potential is also not possible.

Conclusion: The energy of the particle should be comparable with the barrier potential for the tunnel effect to occur. Diffraction and tunnel effect occurrence requires identical condition in the sense that diffraction phenomenon is the sure case while tunnel effect is only probabilistic.

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