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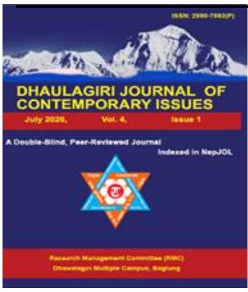


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# Pascal's Triangle and Bernoulli Numbers: Applications in Scientific Computing

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## Abstract

Bernoulli numbers are the key concept in number theory. These numbers first appeared in Jakob Bernoulli's famous book Ars Conjectandi. These are used in the Euler-Maclaurin summation formula. This a formula enables efficient computation of the slowly converging series. Bernoulli numbers and the world-famous Pascal's triangle are closely related. Here, the focus is on their formulations, properties, and interconnections related to information technology, computer science, and scientific computing to explore and highlight how they improve the algorithm efficiency, computational speed, and memory optimization.

**Keywords:** Approximation, Bernoulli Numbers, Numerical Computation, Pascal's Triangle, Recursive Algorithms.

## Introduction

Pascal's Triangle and Bernoulli numbers are fundamental concepts in mathematics, rooted in early combinatorics and number theory. With the rapid evolution of the modern concepts of information technology and scientific computing, their importance has grown far beyond theoretical mathematics. Nowadays, these two are integrated, developing efficient algorithms, exploring efficient numerical methods, cryptographic protocols, error-correcting codes, and various scientific computing. Their mathematical properties enable the optimization of different recursive algorithms, data compression, analysis, and modifications in scientific research and modern engineering.

Pascal's Triangle, studied by Blaise Pascal in the 17th century, formalized earlier ideas from Chinese and Indian mathematics. Its recursive formula, C (n, k) = C (n-1, k-1) + C (n-1, k), is a key in combinatorics, underpinning efficient algorithms in discrete mathematics, dynamic programming, probability, and Computer science (Graham et al., 2018). Bernoulli number, introduced by Jakob Bernoulli in Ars Conjectandi (1713), arose from studying the sum of powers of integers. They now play a crucial role in the Euler-Maclaurin formula, linking sums and integrals, improving error estimates in numerical methods, and supporting algorithms for special functions, numerical approximation, and series in scientific computing (Press et al., 2020).

Generating functions, recurrence relations, and binomial expansions are common links between these two famous numerical structures. Binomial coefficients can also be derived from Pascal's Triangle. The recursive property of Bernoulli numbers is useful in series summation, symbolic computation, and the development of efficient algorithms for constants and special functions (Cohen, 2021). These are studied extensively in the literature; however, their integration into modern technology and optimization is comparatively lacking. Here, our focus is on analyzing the recursive generation of Pascal's Coefficients and Bernoulli numbers. Their computational and symbolic computations, some mathematical deductions, and their applications in various fields are also explored.

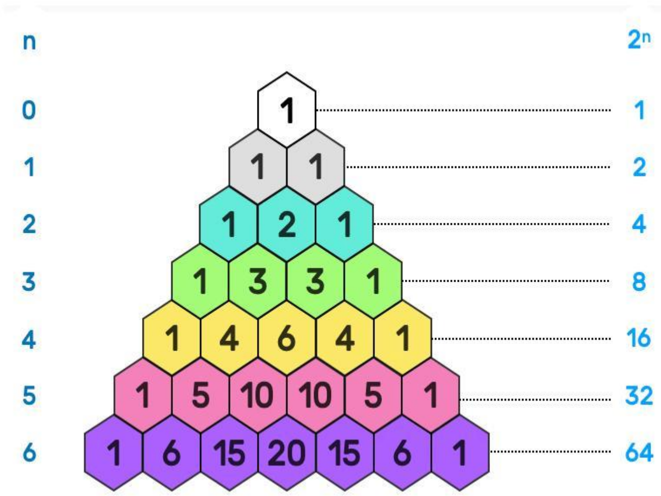
## Basics on Pascal's Triangle and Bernoulli Numbers

Pascal used his arithmetical triangle in 1653; however, his methodology was not documented until 1665. For all integers n$\geq 1$ and 1$\leq k \leq n,$ it can be expressed,

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

In simple form, it can be shown as in Figure 1.

**Figure 1**
*Pascal coefficients*




The triangle is set up as shown in the following patterns.

| Index | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | | | | | | |
| 1 | 1 | 1 | | | | | |
| 2 | 1 | 2 | 1 | | | | |
| 3 | 1 | 3 | 3 | 1 | | | |
| 4 | 1 | 4 | 6 | 4 | 1 | | |
| 5 | 1 | 5 | 10 | 10 | 5 | 1 | |
| 6 | 1 | 6 | 15 | 20 | 15 | 6 | 1 |

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The relationship between Pascal's Triangle and Bernoulli numbers emerges through generating functions, binomial expansions, and recurrence relations. Binomial coefficients derived from Pascal's Triangle are used to construct efficient recursive algorithms for computing Bernoulli numbers. With the growth of digital computation, such structured approaches became valuable for reducing computational cost, improving numerical stability, and optimizing memory usage in algorithm design (Cornen et al., 2009).
To illustrate this, we shall now consider the generating function.

$$e^t = \frac{1}{e^t - 1} \tag{1}$$

Taking advantage of the familiar expansion

$$e^t = 1 + \frac{t}{1!} + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \qquad (2)$$

We can write

$$f(t) = \frac{t}{1 + \frac{t}{1!} + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \cdots - 1} = \frac{1}{\frac{t}{1!} + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \cdots} \qquad (3)$$

From this, the function f(t) is expanded in a power series about t = 0 for the sake of convenience in subsequent. Computations, we represent this series as

$$\frac{t}{e^t - 1} = \sum_0^{\infty} \frac{B_n}{n!} \; t^n \qquad (4)$$

And the first few numbers are

$B_0 = 1$; $B_1 = -\frac{1}{2}$ ; $B_2 = 1/6$ ; $B_4 = -1/30$; $B_6 = 1/42$ ; $B_8 = -1/30$; ... … …

For all odd integers, $n \geq 3$ then $B_n = 0$.

This created an overlapping structure and some sub problems in which $B_2$ depends on $B_0$ , $B_1$, and $B_3$ depends on $B_0$, $B_1$ , $B_2$, and so on. Such a structure is the central concept of dynamic programming. Here, the Bernoulli number problem becomes like computing a structure where each state depends upon the previously solved steps that exactly the dynamic programming paradigm.

A periodic extension of $B_n(t)$ can be defined as

$$B_n(x) = \begin{cases} B_n(t) & 0 \leq t < 1 \\ B_n(t-1) & t \geq 1 \end{cases}$$

where, $B_0 = f(0) = 1$.

To determine the other coefficients, $B_n$ ($n$ = 1, 2, 3,…) of the Expansion, which are called Bernoulli numbers, we make use of the identity

$$\sum_{n=0}^{\infty} \frac{t^n}{(n+1)!} \sum_0^{\infty} \frac{B_n}{n!} \; t^n = 1 \qquad (5)$$

Multiplying together the power series and equating to zero the coefficient of the positive power of the variable t, we obtain an infinite system of linear equations:

$$\frac{B_n}{n!} \cdot \frac{1}{1!} + \frac{B_{n-1}}{(n-1)!} \cdot \frac{1}{2!} + \cdots + \frac{B_0}{(0)!} \cdot \frac{1}{(n+1)!} = 0.$$

Or, multiplying by $(n+1)!$ and noting that

$$\sum_{r=0}^{n} B_{n-r} \, C_{n+r}^{n-r} = \sum_{r=0}^{n} B_{n-r} \, \frac{(n+r)!}{(n-r)!\,(r+1)!} \qquad (6)$$

If we agree to set $B_r = B^r$ then the last formula may be compactly written in the symbolic form like, $(B+1)^{n+1} - B^{n+1} = 0$

Or, replacing (n+1) by n,

$$(B+1)^n - B^n = 0 \qquad (7)$$

Putting n = 2, 3, 4,….in the last formula, we obtain an infinite system of equations connected with Bernoulli numbers and Pascal's triangle, with the help of the above Equations (4), (5), (6), and (7), we get,

$$1B_0 + 2B_1 = 0$$
$$1B_0 + 3B_1 + 3B_2 = 0$$
$$1B_0 + 4B_1 + 6B_2 + 4B_3 = 0$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots..$$

Moreover, let us multiply the terms in Equation (4) on both sides by $e^t - 1$, we get,

$$t = \sum_{0}^{\infty} \frac{B_n}{n!}\ t^n \ \sum_{m=1}^{\infty} \frac{t^m}{m!} \tag{8}$$

Then, by comparing the coefficients, we get $\sum_{k=0}^{\infty} \binom{n+1}{k}\ B_k\ = 0$ for $n \geq 1$. These relations establish a strong mathematical and numerical relationship between the Bernoulli coefficients and Pascal's triangle.

## Applications

Traditionally, Pascal's Triangle and Bernoulli numbers were discussed in pure mathematics, especially in number theory. Nowadays, these are the areas of interest for computer scientists, mathematical optimizers, and algorithmic designers. These are the basic tools for computational and combinatorial programming, dynamic programming, and backtracking. To the same degree, these are significant in software development, data analysis, soft computing, cryptography, error correction, probabilistic modeling, logic circuits, numerical differentiation, approximation techniques, algorithmic computations, stable numerical calculations, minimization of memory demands, information technology, etc. (Crandall & Pomerance, 2018; Sedgewick & Wayne, 2023; Beck & Robins, 2015; Higham, 2019; Borwein & Bailey, 2015; Butzer & Hauss, 2016).

Some of their major applications can be summarized as follows:

**Algorithmic Computation of Bernoulli Numbers.** By generating function, the Bernoulli number as in Equation (4) is expressed as,

$$\frac{t}{e^t - 1} = \sum_{0}^{\infty} \frac{B_n}{n!}\ t^n$$

Likewise, for the recursive computation, $\sum_{k=0}^{n} \binom{n+1}{k} B_k = 0$ for n≥ 1. Then

$$B_n = \frac{-1}{n+1} \sum_{k=0}^{n} \binom{n+1}{k} B_k$$

It is the basis of many algorithmic computation systems and is applicable.

**Euler–Maclaurin Series Approximation**. The Euler-Maclaurin formula connects the discrete sums and continuous integrals in the form,

$$\sum_{a}^{b} \boldsymbol{f(K)} = \int_{a}^{b} \boldsymbol{f(x)dx} + \frac{\boldsymbol{f(a)+fb)}}{\boldsymbol{2}} + \sum_{\boldsymbol{m-1}}^{\boldsymbol{p}} \frac{\boldsymbol{B_{2m}}}{\boldsymbol{(2m)!}} (\boldsymbol{f^{2m-1}(b)} - \boldsymbol{f^{2n-1}(a)}) + \boldsymbol{R_p}$$

It applies to numerical integration, asymptotic analysis, approximations, simulations, and scientific computing.

**Polynomial Expansion and Power Series.** Bernoulli's number appears in polynomial expansions and power series forms of trigonometric and exponential functions. For example,

$$\tan x = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} 2^{2n-1}}{(2n)!} B_{2n} x^{2n-1}$$

It applies to computer algebra systems, signal processing, Fourier series, approximations, etc.

**Symbolic Computation in Mathematical Software.** Mathematical software like MATLAB and Mathematica. Maple and SageMath also used Bernoulli's number in their algorithms. For example, the Riemann zeta function is

$$\zeta(2\mathrm{n}) = \frac{(-1)^{\mathrm{n}+1} \, (2\pi)^{2\mathrm{n}} \, \mathrm{B}_{2\mathrm{n}}}{2 \, (2\mathrm{n})!}$$

**Dynamic Programming and Combinatorial Algorithms**. The dynamic programming stores intermediate results and avoids recomputation. It is structured like,

$$\mathrm{D_n} = \frac{-1}{\mathrm{n}+1} \sum_{\mathrm{k}=0}^{\mathrm{n}-1} \binom{\mathrm{n}+1}{\mathrm{k}} \mathrm{D_k}$$

The main philosophy of such computation is to break a complex computational problem into iterative subproblems.

**Memory Optimization in Algorithm Design.** Large Bernoulli computations require substantial memory of $O(n^2)$, which can be reduced to $O(n)$ by overwriting the previous rows during iterations. This will increase performance in scientific computing and simulations.

**Numerical Differentiation and Integration.** Bernoulli numbers occur in finite difference approximation, whereas such a Bernoulli correction improves the accuracy in numerical integration techniques like the trapezoidal rule, Simpson's rule, etc.

**Performance Analysis of Algorithms.** The Bernoulli-based asymptotic expansion, like

$$\sum_{k=1}^{p} k^p = \frac{1}{p+1}\sum_{j=0}^{p}(-1)^j \binom{p+1}{j} B_j\, n^{p+1-j}$$

For the quick numerical verification, let p=2, then for $B_2 = \frac{1}{6}$it yields,

$$\sum_{k=1}^{n} k^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{B_2}{2!}\; 2n = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} = \frac{n(n+1)(2n+1)}{6}$$

**Cryptography and Error-Correcting Codes.** Different modular computations are fundamental in cryptography and coding theory.

**Robust Software Development and Scientific Computing.** Bernoulli-based approximations help to design robust numerical computations with numerical stability. To estimate, $e^{0.1}$ one can use,

$$e^x \approx 1 + x + \frac{x^2}{2}$$

Then, $e^{0.1} = 1 + 0.1 + \frac{0.01}{2} = 1.105$ where, $e^{0.1}$ (exact)=1.10517, has numerical stability.

## Conclusion

Pascal’s Triangle and Bernoulli’s numbers are interrelated mathematical components in applied mathematics. They are the backbone of computational mathematics. They are good for combinatorial algorithms, recurrence relations, and dynamic programming. These two are frequently used in algorithm analysis, series expansion, power expansion, scientific computing, simulations, software development, numerical differentiation, numerical integration, numerical approximation, cryptography, error coding, engineering, data analysis, etc. These can be applied to quantum computing, machine learning, advanced cryptography, and numerical optimization strategies. Their integrated and advanced technological development remains challenging.

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## References


Beck, M., & Robins, S. (2015). *Computing the continuous discretely: Integer-point enumeration in polyhedra*. Springer. https://link.springer.com/book/10.1007/978- 1-4939-2969-6

Borwein, J. M., & Bailey, D. H. (2015). *Mathematics by experiment: Plausible reasoning in the 21st century* (2nd ed.). CRC Press. https://doi.org/10.1201/b10704

Butzer, P. L., & Hauss, M. (2016). Applications of Bernoulli numbers in numerical analysis. *Journal of Computational and Applied Mathematics, 305*, 1–15.

Cohen, H. (2021). *Number theory, volume II: Analytic and modern tools*. Springer.

Cormen, T. H., Leiserson, C. E., Rivest, R. L., & Stein, C. (2009). *Introduction to algorithms* (3rd ed.). MIT Press.

Crandall, R., & Pomerance, C. (2018). *Prime numbers: A computational perspective* (2nd ed.). Springer.

Gosper, R. W. (2003). Symbolic summation of series with Bernoulli numbers. *Journal of Symbolic Computation, 35*(4), 385–404.

Higham, N. J. (2019). *Accuracy and stability of numerical algorithms* (2nd ed.). SIAM.

Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. (2020). *Numerical recipes: The art of scientific computing* (3rd ed.). Cambridge University Press. https://dl.acm.org/doi/10.5555/1403886

Sedgewick, R., & Wayne, K. (2023). *Algorithms* (4th ed.). Addison-Wesley.


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