Role of Manipulative Materials in Mathematics Teaching and Learning

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Abstract

This study explores how using manipulative materials affects mathematics teaching and learning in grade seven of basic schools. A qualitative approach with an ethnographic design was used, focusing on three basic schools in Kathmandu. The research included three grade-seven mathematics teachers and six students from three schools, selected through probability sampling. Semi-structured interviews were conducted to gather participants' views, and the validity of the interview guidelines was confirmed through expert review. I observed the mathematics classroom and conducted a focus group discussion with students. Data collection took place during the participants' classroom time and focus group discussion were conducted in students' break time. Manipulative materials enhance problem-solving skills, build confidence, support different learning styles, and help teachers explain complex ideas effectively, making lessons fun and interactive. The findings show that manipulative materials greatly improve student engagement in mathematics, offering valuable insights for teachers and policymakers to enhance basic-level mathematics education.

***Keywords*:** Manipulative materials, students' achievement, qualitative research design, classroom teaching, mathematics learning

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Introduction

In the sixties, the use of mathematics manipulatives was often justified on the basis of the ancient proverb: I hear and I forget, I see and I remember, I do and I understand. The saying "Mathematics manipulatives are useful" is still used in early mathematics teaching, but research is needed to determine their effectiveness and under what conditions (Carbonneau et al., 2013). If not effective, adjustments can be made to improve the situation (Swan & Marshall, 2008). Manipulative materials in mathematics teaching refer to physical objects or tools used to help students understand mathematical concepts through hands-on experience (Strom, 2009). These materials can include items such as counting blocks, geometric shapes, fraction bars, or measuring tools like rulers and protractors (Reys, 1971). By manipulating these objects, students can explore mathematical ideas in a concrete manner, which enhances their understanding of abstract concepts (Bungao-Abarquez, 2020). Manipulative materials encourage active engagement, facilitate visualization of mathematical relationships, and support problem-solving skills by allowing students to experiment and discover mathematical principles through direct interaction with the materials (Capuno & Plando, 2015). This approach is particularly effective in developing conceptual understanding and deepening mathematical reasoning among learners of various ages and abilities.

The significance of manipulative materials in mathematics classrooms lies on their ability to enhance learning in several key ways. Foremost, they provide a concrete representation of abstract mathematical concepts, making these ideas more tangible and understandable for students (Dias et al., 2020). This hands-on approach promotes active engagement and deeper understanding by allowing students to physically manipulate objects to explore mathematical relationships and properties (Peshkin & Sanderson, 1988). Manipulative materials also cater to different learning styles, offering visual and tactile learners a way to grasp concepts that may be challenging through traditional methods alone (Carbonneau et al., 2013). Moreover, these materials support collaborative learning and problem-solving skills as students work together to explore, discuss, and apply mathematical concepts in meaningful contexts (Cain-Caston, 1996).

Overall, manipulative materials play a crucial role in promoting conceptual understanding, mathematical reasoning, and building confidence in students' mathematical abilities. Justo et al. (2022) say that all students can achieve knowledge from manipulatives if their teachers know how to adapt the learning process to meet each student's needs. The National Council of Teachers of

Mathematics (NCTM, 2000) has suggested employing tools in the classroom since educational research and learning theory both support this practice. However, the foundation of mathematics is the creation of materials and artifacts to address real-world issues before formal, logical, and cognitive structures (Joseph, 1997).

Students benefit from manipulatives because they enable them to change from concrete experiences to abstract reasoning, so by manipulating things, students initiate the process of comprehending mathematical processes and procedures. Mathematical concepts can be deeply understood by students through the effective use of manipulatives to help connect ideas and integrate their knowledge (Fadzil & Saat, 2019). Additionally, Namukasa et al. (2009) maintain that with the increased use of computer manipulatives in teaching, there is a need for theoretical discussions on the role of manipulatives. They review theoretical rationales for using manipulatives and illustrate how earlier distinctions of manipulative materials are broadened to include new forms of materials such as virtual manipulatives. Manipulatives are in a broader network of learning tools (Komatsu & Jones, 2019). Manipulative materials support students' learning by providing specific tools that enhance understanding. It demonstrates a complementary relationship between virtual and concrete manipulatives, showing that multiple theories can justify the use of the same material (Fadzil & Saat, 2019). Exploring specific manipulatives can also generate new theoretical insights, which is valuable for designing, selecting, categorizing, and evaluating effective learning tools

Following Davidson (1968), manipulative materials have been organized into fifteen general categories: blocks, calculators/computers, cards, construction, drawing tools, geoboards, measuring devices, miscellaneous items, models, numerical games, puzzles, shapes and tiles, strategy games, student instructional materials, and teacher resource materials. Each item has been described briefly, and current prices (of course, subject to change) have been quoted. Since such matters as postage, handling charges, and discounts depend on the particular order submitted, no attempt has been made to take these into account.

It is recommended that schools obtain the latest catalogs of the various suppliers. Suggested grades are indicated merely to help with the buying; since most of the materials are very open-ended, they have uses appropriate to other ages as well as those mentioned. While using instructional materials like slideshows and videos in class is standard in 21st-century education, this passive method cannot be regarded as true manipulatives because student participation is still passive (Galvez et al., 2023).

Manipulative materials are widely considered effective in bridging the gap between concrete experiences and abstract mathematical concepts. However, as a problem, there is still a lack of comprehensive understanding of how effectively these materials are integrated into mathematics teaching and the extent of their impact on students' learning outcomes. Therefore, this study aims to investigate the role of manipulative materials in enhancing mathematics instruction and to examine their influence on students' engagement, conceptual understanding, and overall academic achievement in mathematics.

Through the proper transfer of knowledge, they facilitate understanding of concepts, principles, and real-life problem-solving. By encouraging teachers and students to use these resources, mathematical concepts are better comprehended and retained. In Nepal, mathematics achievement remains a challenge for many students, with traditional teaching methods often failing to support deep conceptual understanding and sustained interest in the subject. Although manipulatives have shown promise globally in enhancing student engagement and comprehension, their use and impact within Nepal's classrooms are largely underexplored. This study aims to address this gap by investigating how manipulatives influence students' activities and learning in mathematics in the Nepalese context, providing insights for more effective and contextually relevant teaching strategies.

i. To understand how manipulatives impact student achievement and conceptual development in Nepal's educational context, a targeted study is necessary. Overall, the study sets out to find out how students and teachers perceive the role of manipulative materials in mathematics learning. More specifically, the research was conducted with the following objectives.

ii. To explore how manipulatives enhance students' conceptual understanding of mathematics at the basic level specifically class seven, and

iii. To assess students' motivation and engagement when using manipulatives.

Theoretical Review

According to theoretical viewpoints on how manipulatives affect mathematics learning, practical resources can help students go from concrete to abstract thought, which is consistent with Piaget's phases of cognitive development. According to Piaget's theory, which places an emphasis on active, experiential learning, manipulatives aid students in visualizing and physically interacting with mathematical concepts, leading to a greater comprehension and retention. Furthermore, social development theory emphasizes the function of manipulatives in group learning, where peer relationships and supervised exploration of these resources improve mathematical accomplishment and engagement. On the basis of these theories, I advance the following theoretical review.

Teacher Perceptions of Using Manipulative Materials

Teachers frequently see manipulatives as useful resources for improving their students' comprehension of difficult mathematical ideas by giving abstract concepts a concrete form (Moyer-Packenham, 2016). Many people think that employing manipulatives might help students become more adept at solving problems and become more involved in the learning process (Ojose & Sexton, 2009). However, some educators might also be apprehensive about successfully incorporating these resources into the curriculum, especially if they are untrained or have little experience with them.

Student Engagement and Motivation in Manipulative Materials

By encouraging students to actively engage with mathematical concepts, manipulative materials can greatly increase student engagement by giving abstract ideas a more tangible and approachable form (Meke et al., 2018). Math is frequently more fun for children who engage in hands-on discovery, which also encourages them to experiment, raise questions, and work together to find answers (Morrow, 1982). Students who actively participate in the learning process feel more involved in it and develop greater confidence in their skills through manipulatives, which promotes greater understanding of mathematics (Moyer-Packenham & Westenskow, 2013).

Likewise, Piaget's Theory of Cognitive Development emphasizes that children learn best when they are actively involved in the learning process, and manipulatives support this by providing concrete experiences that help students transition from concrete to abstract thinking. This theory considers learners' diverse perspectives and identifies critical features that address specific learning gaps, with objects of learning defined comprehensively to encompass academic knowledge typically taught in schools through manipulatives (Hanfsting et al., 2019). Piaget's theory suggests that children develop scientific and abstract thinking during the formal operational stage, requiring equilibration and lessons that build on prior knowledge with the help of manipulative materials, especially in mathematics classes (Oogarah-Pratap et al., 2020). The implications of Jean Piaget's theory for studying the effects of manipulative materials on student achievement and mathematics learning in the Nepalese context emphasize the importance of developmentally appropriate instructional strategies. According to Piaget, children progress through stages of cognitive development, and manipulatives can be particularly effective during the concrete operational stage, where students learn best through hands-on activities and concrete representations of abstract concepts (Sevinç, 2019). In Nepal, incorporating manipulatives into mathematics education can help bridge the gap between concrete and formal operational thinking, enabling students to gradually move towards more abstract mathematical reasoning and enhancing their engagement in the subject.

Teachers can enhance learning by tailoring experiences to each child's cognitive development stage, ensuring that activities are developmentally appropriate and engaging, and applying manipulative materials (Larbi & Mavis, 2016). By encouraging children to test their own hypotheses and ideas, they adopt an environment where students can independently form conclusions and opinions, which strengthens critical thinking skills. Learning is viewed as a lively process involving constant restructuring and adaptation of cognitive processes, allowing students to expand their understanding through exploration (Babakr et al., 2019; Sanghvi, 2020). Through manipulative materials for learning methods, teachers can further support intellectual development by providing hands-on, practical experiences that make abstract concepts more accessible, meaningful, and concrete.

Methods

In this study, a qualitative research design grounded in ethnography was applied to explore the effects of manipulatives on classroom practices and student learning. The study was based on the ontological perspective of social multiple reality, aiming to subjectivity to understand student outcomes without researcher influence (subjectivity as epistemology, value-laden axiology. The research included three mathematics teachers (one from each school) and six students (two from each school) from grade seven, selected through probability sampling from community schools in Kathmandu. Data collection tools were carefully prepared, aligning them with grade seven. A convenience sampling procedure was employed to select schools, with the entire community school population in the district considered the population for generalizability.

To facilitate data collection, I met with head teachers of each sampled school, explaining the research purpose, emphasizing the voluntary nature, and outlining its contribution to educational knowledge. The head teachers organized and supervised the data collection, ensuring that both teachers and students participated autonomously without pressure. Privacy was prioritized, with respondent names kept confidential to encourage honest and open responses. This approach helped maintain ethical standards in the research, reinforcing its reliability and the integrity of the findings. For a deeper analysis, I conducted classroom observations while mathematics teachers were delivering their lessons, with particular attention to their use of manipulative materials. In addition, I engaged selected students in in-depth discussions by asking targeted questions to explore their experiences, perceptions, and usage of these materials in learning mathematics. The observation of mathematics teaching in grade seven classrooms was guided by the research questions, and the students' perspectives were further explored through detailed interviews.

Results and Discussion

The results and discussion reveal the significant impact of manipulative materials on student learning, with a particular focus on three key themes. First, enhancing conceptual understanding through manipulatives allows students to engage with abstract mathematical concepts in a tangible way, deepening their comprehension. Second, the use of manipulatives facilitates the transition from concrete to abstract learning, enabling students to visualize and grasp complex mathematical ideas more effectively. Lastly, incorporating hands-on materials encourages the development of problem-solving and critical thinking skills, as students actively engage in exploring and analyzing mathematical challenges. These findings underscore the importance of integrating manipulatives in the classroom to support comprehensive mathematical learning.

Enhancing Conceptual Understanding through Manipulatives

With permission from the head teacher and the class teacher, I observed a mathematics lesson or classroom focusing on algebra. The teacher entered the classroom without any manipulative materials. The topic of the lesson was the expansion of $(a+b)^2$. However, the teacher explained the concept theoretically, stating that $(a+b)^2 = a^2 + 2ab + b^2$, and asked students to copy the formula in their notebooks. No practical demonstration or use of the manipulative materials was incorporated to illustrate the expansion $(a+b)^2 = a^2 + 2ab + b^2$ without demonstrating the concept visually or interactively using the manipulatives. When the teacher posed a problem, such as finding the value of $(x+y)^2$, the students struggled to solve it, indicating a gap in conceptual understanding. This situation shows that simply introducing the algebraic expansion is insufficient; teachers need to actively engage students with the manipulatives to build a deeper understanding of abstract mathematical concepts. It seems that most of the students faced difficulty in expanding $(x+y)^2$, while a few talented students managed to do so.

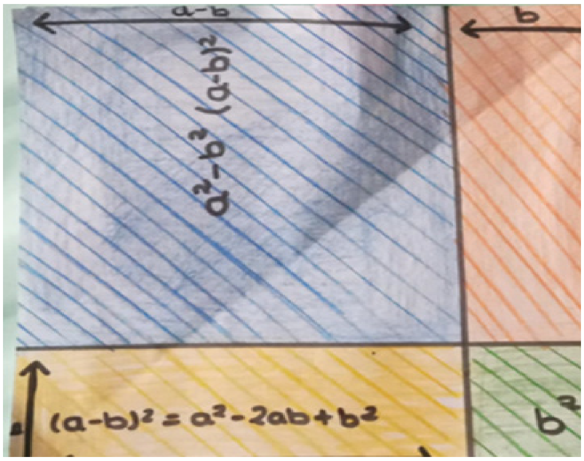

Figure 1

Teacher's Visual Aid (I)

After the lesson, I asked the math teacher how much the students understood the lesson he taught. I also asked why he taught only theoretically and if there were no teaching materials available. I suggested he use a binomial method or show a cardboard model of $(a+b)^2$ on the board. The teacher admitted that he did not use teaching materials during classroom teaching. He mentioned that the students lacked prior knowledge and had little desire to study. He also said that the students had a habit of memorizing instead of understanding, no matter how much they were taught. Meanwhile, I requested the math teacher to use manipulative materials in his teaching the next day. I discussed with him how these materials could help make the lesson more engaging and improve the students' understanding. After explaining the importance of incorporating hands-on tools, I planned to observe his teaching with manipulative materials the following day.

The next day, I arrived at the school on time and received permission to observe the same mathematics teacher's class. I sat at the back of the classroom and noticed that the teacher was using a teaching approach different from the one he used the previous day. The lesson involved creating categories of a+b on a cardboard, which engaged the students and encouraged them to ask questions. The teacher also tested their understanding by asking questions like, "What is the square of or a+b?" to which the students responded correctly. The use of manipulative materials seemed to help the students grasp the basic concepts of squares and square roots effectively.

After the lesson, I used the probability sampling method to select four students, two boys and two girls, for a group discussion. I asked them questions like what difference they noticed between teaching theoretically the previous day and using manipulative materials that day, and which method they found easier. The students said they understood the lesson faster with manipulative materials and noted that the teacher does not always use them. Later, I discussed with the teacher how manipulative materials can help make difficult concepts easier to teach and understand. I suggested that the teacher not create all the materials himself, but involve the students in making them as well. This way, the students' knowledge would increase, and their learning would become deeper.

From the above classroom observation, I found that manipulative materials make learning mathematics more interesting and enjoyable for students. These materials encourage students to actively participate in the lessons, making the learning process more engaging. Instead of just listening to the teacher, students can touch, move, and explore the materials, which helps them focus better and stay curious about the concepts being taught. For example, using shapes, colored cards, or counters keeps students actively involved in the lesson. This interactive approach makes it easier for students to understand complex topics and builds their confidence in learning mathematics. It also fosters collaboration among students, as they often work together in groups while using these materials, making the classroom environment more fun and engaging.

Manipulative materials not only make learning enjoyable but also enhance the quality of teaching and learning. They promote active involvement of both teachers and students, which is essential for effective mathematics instruction (Istiandaru et al., 2017). When students actively participate and interact with materials, they gain a deeper understanding of mathematical concepts (Bungao-Abarquez, 2020). This method helps teachers explain difficult topics clearly and complete the curriculum on time. As a result, students perform better in their studies and feel more confident about their math skills. By integrating manipulative materials into lessons, teachers can create a positive and interactive learning environment that increases students' interest, participation, and overall achievement in mathematics (Ojose & Sexton, 2009).

Using manipulatives in algebra lessons greatly enhanced students' understanding compared to a purely theoretical approach. At first, many students found algebraic expansion difficult due to reliance on memorization (Strom, 2009). However, when hands-on materials were introduced, students became more engaged, asked insightful questions, and developed a stronger grasp of the concepts. Actively involving students in creating their own manipulatives further deepened their comprehension (Meke et al., 2018). This aligns with Piaget's cognitive development theory and cognitivism, as hands-on experiences help learners construct knowledge by moving from concrete to abstract thinking. Regular use of manipulatives fosters active exploration and meaningful learning, reinforcing deeper mathematical understanding (Istiandaru et al., 2017).

Facilitating the Concrete to Abstract Learning Transition

I visited the second school as planned and met the principal. I explained to him the objectives, procedures, and confidentiality of my research clearly. I assured the principal that the process would remain confidential. I requested permission to observe a mathematics teacher's class, as this would contribute to my research. The principal called the math teacher and explained the purpose of my visit. After discussing it with both the principal and the teacher, we agreed on the observation. The math teacher then guided me to the classroom, and the observation began smoothly.

In this lesson, the mathematics teacher introduced the topic $(a-b)^2$ in algebra. He began by writing the formula $(a-b)^2 = a^2 - 2ab + b^2$ on the whiteboard and briefly explained its components. To make the concept engaging and interactive, he incorporated manipulative materials placed near the whiteboard. He selected two students from the class and asked them to pick and toss one of the materials. The students eagerly participated, creating a lively and engaging start to the lesson.

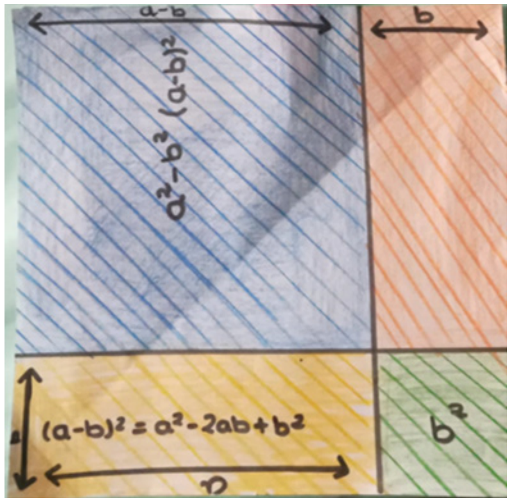

Figure 2

Teacher's Visual Aid (II)

Using the manipulative materials as a visual aid, the teacher connected the physical demonstration to the algebraic concept. He explained that the square of $(a-b)$ involves expanding the expression $(a-b) \times (a-b)$ and breaking it into its parts: a^2, $-2ab$, and b^2. By referring to the materials, he illustrated how the interaction between a and b results in the middle term, $-2ab$ while the individual squares, a^2 and b^2, represent the squared components. The students were attentive and actively participated by asking questions and solving examples. This approach helped the class visualize and better understand the concept, making the algebraic formula more relatable and easier to grasp.

After the mathematics class, I randomly selected four students (two boys and two girls) to discuss the use of manipulative materials in their mathematics lessons. The students shared that while the teacher used manipulatives during some lessons, it was not a regular practice. They mentioned that the teacher incorporated these materials when it was possible and relevant to the main core curriculum. According to the students, the focus of the lessons was usually on completing the required syllabus, so the use of manipulatives depended on the topic being taught. They appreciated the use of materials when included, as it made the lessons more engaging and easier to understand, but they also noted that such activities were occasional rather than frequent.

The figures representing the algebraic identity $(a-b)^2 = a^2 - 2ab + b^2$ are excellent tools for teaching basic-level mathematics. These visual aids help students connect abstract algebraic concepts with tangible geometric representations, making it easier for them to grasp the formula. Teachers find that such diagrams enhance students' understanding by breaking down complex ideas into simpler, more visual components. The colorful and engaging nature of these figures captures students' attention, making the learning process interactive and enjoyable. Furthermore, involving students in creating or analyzing these diagrams fosters critical thinking and helps them develop a deeper understanding of mathematical concepts. Teachers can also use these tools to demonstrate the practical derivation of formulas, promoting conceptual learning over rote memorization. Overall, these visual representations serve as a bridge between theoretical knowledge and practical application, creating a more meaningful learning experience for students.

The use of manipulative materials in teaching algebra supports Piaget's cognitive development theory and cognitivism by making abstract concepts more concrete and engaging for students (Swan & Marshall, 2008). According to Piaget, learners in the Concrete Operational Stage need hands-on experiences to develop logical thinking before transitioning to abstract reasoning in the Formal Operational Stage. Observations of a lesson on $(a-b)^2$ showed that interactive activities helped students grasp the concept more intuitively, reinforcing cognitivist principles that emphasize active mental processing (Sevinç, 2019). However, students noted that manipulatives were used infrequently due to curriculum constraints. Regular integration of hands-on learning, along with student involvement in creating materials, can enhance conceptual understanding and critical thinking (Davidson, 1968). A balanced approach that incorporates manipulatives without disrupting syllabus completion fosters meaningful learning and supports cognitive development in mathematics education (Morrow, 1982).

Developing Problem-Solving and Critical Thinking Skills

I visited the third school as planned and met with the principal. I explained to him the purpose, steps, and confidentiality of my research clearly. I assured the principal that everything would stay confidential. I asked for permission to observe a math teacher's class to help with my research. The principal called the math teacher and explained why I was there. After talking with both the principal and the teacher, they allowed me to observe the class. The math teacher then took me to the classroom, and the observation started without any issues.

During my classroom observation, the math teacher was teaching the concept that the sum of the three angles in any triangle equals 180 degrees. The teacher began by introducing the topic and explaining that this is a fundamental property of triangles in geometry. To demonstrate the concept, the teacher drew different types of triangles on the board, such as acute, obtuse, and right triangles.

The teacher then used a step-by-step explanation to show how the angles of a triangle always add up to 180 degrees, regardless of its shape. For example, they measured the angles of a triangle drawn on the board and calculated the total. In some cases, the teacher involved the students, asking them to measure the angles using a protractor and add them together. This interactive approach helped students visualize and confirm the concept. By the end of the lesson, the students had a clearer understanding of how this property applies to all triangles.

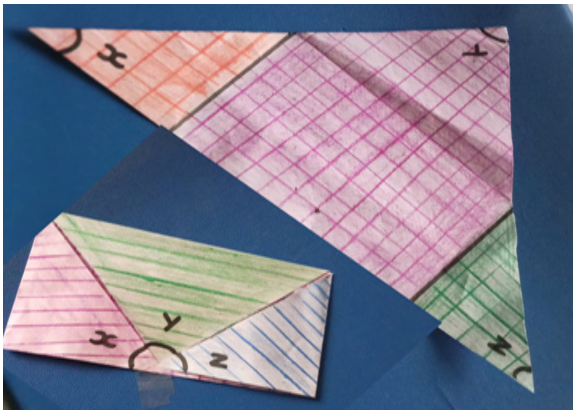

Figure 3

Teacher's Visual Aid (III)

During the lesson, the teacher used a manipulative material to help students understand how the sum of the three angles in a triangle is 180 degrees. The teacher formed a triangle using the material and labeled the three angles as x, y, and z. To demonstrate the concept, the teacher stretched the angles of the triangle outward to form a straight line. While doing this, the teacher asked the students how many degrees a straight line measures. The students confidently answered that a straight line measures 180 degrees. This activity clearly showed how the three angles of a triangle combine to make 180 degrees. It also highlighted the importance of using hand-made materials, as they make learning mathematics more interactive and easier to understand for students.

The figure shown is a creative way to teach the concept that the sum of the angles of a triangle equals 180 degrees. The mathematics teacher can use this visual and hands-on approach to help students understand this fundamental property of triangles. In the figure, the triangle is

divided into three sections representing the three angles: x, y, and z. Each section is marked with different colors, making it easier for students to identify and visualize the angles.

To demonstrate the concept, the teacher can fold or cut the triangle so that the three angles are aligned in a straight line. This shows that the angles combine to form a straight angle, which measures 180^0 degrees. By doing this, students can see and understand the relationship between the angles in a triangle. This method not only makes the learning process interactive but also helps students remember the property more effectively. It encourages exploration and questioning, making mathematics more engaging and meaningful.

After the classroom teaching, I discussed with the teacher about the use of manipulative materials. I asked him if you used such materials regularly in your class. I also asked several related questions, such as what challenges he faced while preparing these materials during teaching, whether it was always possible to prepare and use them regularly, and how students' understanding differed when teaching with materials compared to without. The teacher self-proclaimed that he could not use manipulative materials regularly due to various reasons. He mentioned a lack of time, the large number of students in the class, and the students' lack of prior knowledge as the main challenges. Despite these difficulties, he acknowledged the benefits of using materials in enhancing students' understanding of mathematical concepts.

In this process, I selected four students (two boys and two girls) from the class using a probability sampling method for a focus group discussion. During the discussion, the students shared that manipulative materials helped both students and the teacher remember key points during the lesson, making the teaching process easier and more organized. They said that using such materials made the lesson more interactive and engaging for them. The teacher also joined the discussion and explained that while teaching geometry, he regularly used manipulative materials for practice and to enhance students' understanding. However, for other chapters, the teacher primarily relied on theoretical teaching, as the nature of those topics did not always lend themselves to the use of manipulatives. The students emphasized that although manipulatives were helpful in geometry, they were not always practical or necessary for other areas of the curriculum.

The use of manipulative materials in teaching triangle angle sums aligns with Piaget's cognitive development theory and cognitivism, as hands-on activities help students transition from concrete to abstract thinking (Capuno & Plando, 2015). In the Concrete Operational Stage, students learn best through visualization and exploration, reinforcing concepts like angle sums through interactive methods (Fadzil & Saat, 2019). A focus group discussion revealed that students found manipulatives engaging and helpful for retention, while the teacher acknowledged their benefits despite challenges like time constraints and large class sizes. However, in the Formal Operational Stage, abstract reasoning becomes more dominant, making manipulatives more useful in geometry than in algebra. This highlights the need for adaptive teaching strategies that align with students' cognitive development for effective mathematics instruction.

Conclusion and Implications

In conclusion, the observations highlight the importance of integrating manipulative materials into mathematics lessons. The use of hands-on tools not only makes the learning process more interactive and engaging for students but also helps in visualizing abstract concepts. In the cases observed, students were able to grasp difficult algebraic and geometric concepts more effectively when manipulative materials were used. The students expressed a stronger understanding and engagement during lessons that incorporated these materials, indicating that they provide an essential bridge between theory and practice. This suggests that manipulative materials can significantly enhance students' comprehension of mathematical concepts, particularly for abstract topics like algebraic expansion and geometric properties.

However, the inconsistent use of manipulative materials in the classroom poses a challenge. While some teachers are open to incorporating these tools, they face difficulties in using them regularly due to constraints such as time, class size, and a lack of prior knowledge among students. Despite these challenges, teachers acknowledged the positive impact of these materials in making abstract mathematical ideas more accessible and easier to understand. The discussions with teachers and students revealed that when manipulative materials are employed, students are more likely to stay engaged and retain information. This reinforces the idea that manipulatives can serve as valuable tools in promoting active learning and deeper conceptual understanding.

Finally, to maximize the benefits of manipulative materials, it is crucial to integrate them consistently into the curriculum. Teachers should be encouraged to find ways to incorporate these tools regularly, particularly for topics that require concrete visualizations. Involving students in the creation of these materials could also foster deeper learning and increase their engagement with the content. Overall, the use of manipulative materials, when employed effectively, can transform the learning environment, making mathematics more enjoyable, interactive, and accessible for students, while helping them transition from concrete to abstract thinking.

The findings emphasize the importance of using manipulative materials to enhance understanding of abstract mathematical concepts. Policymakers should ensure adequate resources, training, and support for teachers to integrate these tools across the curriculum. Professional development programs should focus on equipping teachers with strategies for hands-on, interactive teaching and learning. Additionally, policies should promote a shift from rote memorization to student-centered, experiential learning. Schools and teachers should encourage varied teaching methods, involving students in creating manipulatives to enhance engagement and ownership. This approach can lead to improved academic performance, critical thinking, and a deeper understanding of mathematical concepts.

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