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Research Management Cell,
Tribhuvan University, Sanothimi Campus, Sanothimi, Bhaktapur

# Innovative Pedagogies in Realistic Mathematics Education: What, Why and How of Design-Based Classroom Research

*Tika Ram Pokhrel*
Teaching Assistant, Department of Mathematics Education
Tribhuvan University, Sanothimi Campus, Bhaktapur, Nepal
Institute for Innovative Learning, Mahidol University, Salaya, Thailand
*tikarampokhrel@gmail.com*
ORCID iD: *https://orcid.org/0000-0002-1612-2761*

## Abstract

*Developing a mathematical mindset involves understanding the underlying cognitive processes. Mathematization, a key in realistic mathematics education (RME), is an activity of organizing a subject from reality or a mathematical subject. This literature-based paper intends to provide guiding principles for conducting design research in mathematics classrooms as guided by RME. Guided re-invention of mathematics and didactical phenomenology are two key philosophical underpinnings of RME. The activity, reality, level, intertwinement, interactivity, and guidance principles are central to how RME fosters mathematical understanding and problem-solving skills. The innovative plans in design-based research are developed by using design heuristics that follow the framework of the theory of realistic mathematics education. The main methodological intention of design-based research (DBR) is to actively influence and improve educational situations through designed interventions. Interventionist, theory-generative, prospective, and reflective analysis, iterative, and ecologically valid and practice-oriented are the key five characteristics of DBR. DBR is a process-oriented investigation focusing on how learning occurs, consisting of three successive, flexible, and iterative phases: Preliminary design, the teaching experiment, and retrospective analysis. Designing an innovation in mathematics education and finalizing for better learning can follow the sequence of RME principles to DBR phases.*

**Keywords**:*Argumentative grammar, design-based research, mathematization, preliminary design Realistic mathematics education*

## Background

It can be argued that the creation of mathematics would stem from counting. Counting by correspondence of sheep and pebbles can be considered as human-created mathematics. Since mathematics is essentially a human creation (Boaler, 2022), it is inherently tentative, transient, and always open to additional development (Tasić, 2001). The development of mathematics can be linked with the development trends of the world. This can be linked with the development of the application area of mathematics. Mathematics can be considered as an economic, social, and political entity. There are many different mathematical applications that are interwoven with different socioeconomic frameworks (Chen et al., 2025). Mathematical thoughts and applications are shaped by sociopolitical dimensions and historical contexts (Valero, 2004). Mathematics is viewed as a dynamic human endeavor. It is intricately entwined with historical developments and societal structures, necessitating ongoing reevaluation and ethical consideration (Chen et al., 2025). Thus, it can rather be considered as a purely objective or absolute field. The nature of mathematics is linked with mathematics learning and applying it in different fields.

The field of mathematics education is dynamic and always changing to meet people's professional, social, and cognitive needs in a world that is changing quickly (Kaviya & Suryadharani, 2025). Determining what type of mathematics is to be taught and developing successful teaching strategies requires an understanding of the philosophical foundations of mathematics education (Ernest, 2012). Instead of concentrating only on performance, the cognitive science of mathematics examines how mathematical concepts are inferentially organized. The development of a mathematical mindset is necessary for learning mathematics (Boaler, 2019). Changing from a passive, memorization-focused approach to an active, inquiry-driven approach to mathematics, accepting mistakes, appreciating creativity, and using computational thinking abilities (Megawanti et al., 2024).

Developing a mathematical mindset involves understanding the underlying cognitive processes. Three components of mathematics education theory propose situational mathematics, verbalized mathematics, and symbolic mathematics as three parallel and independent components (Megawanti et al., 2024). Situational mathematics focuses on real-life problems and scenarios, making mathematics relatable (Kaviya & Suryadharani, 2025). Mathematical knowledge expressed in natural language fosters semantic understanding (Lange, 2013). Symbolic mathematics consists of traditional abstract symbols, formulas, computational procedures, etc. All three components of mathematics should be core learning objectives and mutually transformative and interrelated. The realistic approach of mathematics education brings mathematics and real-life situations together to facilitate better learning of mathematics.

Designs are used in different research, such as action research, experimental research, and classroom research. How to conduct design-based classroom research following the philosophical guidelines of realistic mathematics education is the main issue that is dealt with in the paper. So, this paper will deal with what, how, and why to use DBR in Mathematics Education.

**Realistic Mathematics Education**

Realistic mathematics education (RME) is rooted in the philosophical idea of Hans Freudenthal (1905-1990) that mathematics should be viewed as a human activity (Gravemeijer & Terwel, 2000). This perspective sees that mathematics is dynamic rather than static and an active process of mathematizing reality rather than passively receiving a closed system of rules and definitions. Mathematization is an activity of organizing a subject from reality or a mathematical subject (Jupri et al., 2021). Horizontal mathematization is the process of transforming real-life problems into mathematical problems. Students use different mathematical tools to understand and solve the problems situated in real-life scenarios (Acharya et al., 2025). The term 'realistic' in RME is extended beyond real-world situations in which students can imagine or realize the situation in their minds. This can be explained by the term 'experientially real'(Van den Heuvel-Panhuizen & Drijvers, 2020), which is the lived or felt reality. Students express and define mathematics within a context. Then they visualize and formalize problems in different ways and establish relationships within real-life problems. Finally, students transform them into mathematical forms. Vertical mathematization is the process of reorganizing within the mathematical system itself by using connections between concepts and strategies. Students move into the abstract world of symbols. Students relate and organize their findings during horizontal mathematization. Students express relationships in formulae, internalize and adapt examples, combine examples, and generalize from the examples, which leads to digging deeper into the mathematical understanding (Laurens et al., 2017). Thus, in general, horizontal mathematization precedes vertical one.

Guided re-invention of mathematics and didactical phenomenology are two key philosophical underpinnings of RME (Gravemeijer, 2008). Guided re-invention is to empower students to rediscover mathematical concepts, tools, and insights by themselves. The guided re-invention process suggests the proactive role of teachers and a carefully designed educational program to facilitate a shift in students' understanding. The goal of learning is not just for the concept to be discovered, but for the discovery to serve as a tool to bridge informal knowledge with formal knowledge (Inci et al., 2023). The didactical phenomenology refers to the method of describing mathematical concepts, structures, and ideas in relation to real-world phenomena. This tries to find real-life situations so that it helps in the generalization of the mathematical concepts and processes.

### Problems and Objectives

RME provides philosophical guidelines for learning mathematics. Designing lessons and innovations for better learning is the main purpose of RME. How the core of design principles and guidelines is used in design-based research is the main concern in classroom research of mathematics education. The concerns regarding the design of lessons and/or innovations and the implementation of these ideas are guided by the core principles. There is literature separately discussed for RME and Design-based research, but there is limited research to bring these ideas together. The objective of the paper is to discuss the what, how, and why of DBR in mathematics education as guided by the core principles and guidelines of RME.

## Method and Procedure

The literature-based research methodology was used in the analysis and generation of this paper. The researcher used Google Scholar for searching the literature. First, the researcher used the keywords: realistic mathematics education, Design-based research in mathematics, were used. The review article from 2015- 2025 was used in searching for realistic mathematics education. There were 17,900 resources found. Then the researcher used realistic mathematics education with mathematization and found 155 resources. The researcher selected 20 articles and book chapters based on the abstracts of these resources. Similarly, the researcher used design-based research in mathematics education and found that there were 8400 resources available. The researcher found 55 resources when the keywords design-based research in mathematics education with mathematization were used. The researcher selected 18 of these articles and book chapters to review. The paper, developed by reviewing 38 literature, presents core guiding principles of RME and then what, how, and why of DBR in developing and finalizing innovations in mathematics education.

## Results and Analysis

### Core Principles and Design Guidelines

How to design a lesson and facilitate learning mathematics as guided by RME is shaded by its core principles and design guidelines (Van den Heuvel-Panhuizen & Drijvers, 2020). These principles are central to how RME fosters mathematical understanding and problem-solving skills.

First, the activity principle posits that students are active participants in their learning. Learning by doing is best for learning mathematics. When students are actively participating in the learning process, they construct mathematical concepts themselves. Second, the reality principle emphasizes that mathematics learning should start from the problem situations that are meaningful and experientially real to students (Laurens et al., 2017). It is believed that rich contexts enable students to attach meaning to the mathematical concepts that they develop during the learning process. Third, the level principle acknowledges that the student's progression in learning mathematics involves various levels of understanding. The typical progress can be described from informal, context-related solutions, through creating shortcuts, and finally moves to schematizations. The schematization can also be an insight into models of how concepts and strategies are related. Thus, progressive schematization fosters a deeper understanding of concepts, algorithms, and strategies. Fourth, the intertwinement principle suggests that different mathematical contents, such as numbers, geometry, measurement, and data handling, are to be integrated so that this is helpful in a holistic understanding of mathematics. This will show how different concepts and strategies are connected across and within the domains. This intertwinement is necessary for addressing the challenge of isolated learning and fast forgetting, as suggested by Freudenthal. Fifth, the interactivity principle views mathematics learning as both an individual and a social activity. Thus, it is advisable to organize whole-class discussions, group work, etc., along with individual work. The interactions should allow students to gain ideas for improving their strategies, inventions with peers, and resolutions of misconceptions. Sixth, the guidance principle is linked with the guided re-invention. The teacher's proactive role should encourage students to rediscover concepts themselves. This will shift in thinking mathematically in students so that they can work as mathematicians.

RME emphasizes developing computational skills along with the $21^{st}$-century skills such as creative thinking, reasoning skills, problem-solving abilities, and adaptability (Vaezi et al., 2019; Van den Heuvel-Panhuizen & Drijvers, 2020). By the use of the activity and reality principle, RME leads to a more in-depth understanding rather than rote memorization. The use of level principles facilitates a gradual and profound grasp of mathematical concepts, algorithms, and strategies. Guidance principle encourages students to rediscover mathematical concepts themselves, which leads students to a deeper understanding of mathematics as well as working as mathematicians. By the use of the interactivity principle, it can be said that students learn from peers and reflect on their mathematics learning. As mathematization is a key process, it helps to empower students' learning from real-life scenarios. Finally, as RME is known to "increase students' logical, critical, and creative thinking abilities (Kusmaryono & Maharani, 2021), it helps to develop higher-order thinking skills. RME is considered a long-term and ongoing process of development (Inci et al., 2023) rather than a fixed and finished theory of learning. Flexibility is an important feature; RME allows researchers and developers to have different emphasis and accents. The adaptability feature enables RME to continuously evolve and address contemporary educational needs.

Design thinking in Mathematics is viewed as one of the approaches of RME. Though Design is primarily related to the engineering field, it can also be used in education. Design is more appropriate when we test innovative pedagogies in real-time situations and refine them for further applications. An innovative plan in design-based research is developed by using design heuristics that follow the framework of the theory of realistic mathematics education (Doorman et al., 2013).

**Design-Based Research**

The goals of innovations in mathematics education are to address inequality, enhance learning outcomes, and get students ready for the sophisticated world (Agbata et al., 2024). Together, these developments seek to change mathematics education from a specialized, methodical field that filters students to one that is open, innovative, and available to all students. A method and strategy for innovation aimed at gaining a competitive edge is design thinking (Martin, 2009). The iterative nature of strategy design and implementation is highlighted by design thinking (Beckman & Barry, 2007). Thus, observing, learning, designing, and validating are the four essential design thinking steps that are also incorporated into the strategy design process.

In recent years, design has become a popular approach for conceiving and creating educational innovations and technologies (Hall, 2020). Design-based research (DBR) is considered a prominent research methodology in education, especially in mathematics education (Hall, 2020; Prediger et al., 2015). The rise of DBR is the result of the gap between the theoretical research and its implications in the practices of mathematics education (Getenet, 2019). The relationship among educational research, policies, and practices has been considered as a complex and often characterized by a significant divide (Puntambekar, 2018). In other words, there is a divide between basic research and applied research. This divide is particularly evident in mathematics education. For example, research is about the curriculum development in an ideal setting, which is disseminated without being effectively implemented in school settings. Educational scholars crafted new theories and methodologies to bridge the gap between the researchers and practitioners. DBR has emerged as a research methodology to address the divide between theory and practice in education (Hall, 2020). Design thinking approaches such as design-research, design-experiment, and developmental research carry a similar notion to DBR, though each of them has a different foci (Vaezi et al., 2019).

DBR focusing on the learning process is informed by constructivist learning theories such as Piaget, Vygotsky, and Dewey (Pardjono, 2016). Students are considered as epistemic agents, a longitudinal study of change, and interconnected thought and action are three broad assumptions about learning. Accordingly, Students are active constructors of knowledge by utilizing their unique experiences and resources (Schuh, 2003). DBR is typically conducted over extended periods to examine gradual or dramatic changes as students learn substantial ideas, often focusing on conceptual learning (Doorman, 2019). Thought and action are intricately linked and influence each other. The main focus of design is usually to address two interconnected questions (Hall, 2020): is it possible to improve education, and if so, how?

The main methodological intention of DBR is to actively influence and improve educational situations through designed interventions. Thus, DBR is inherently interventionist (Alias, 2025; Mygdanis, 2025). To intervene in the educational situation, innovative interventions are created, investigated, and redesigned through successive iterations (McKenney & Reeves, 2013). DBR requires a cyclic process of design and research of innovative situations. This DBR has pragmatics as well as theoretical goals. Pragmatically, DBR involves investigating and improving an innovation for supportive learning. Theoretically, DBR involves developing, testing, and revisiting conjectures about the learning process as well as means of supporting that learning process (Doorman, 2019). DBR is regarded as a research methodology that is both principled and participatory. Educational designers, innovators, policymakers, and technologists get potential support to bridge practice and theory (Hall, 2020).

## Key Characteristics of DBR

The characteristics of DBR can be explored by the three Is: interventional, innovative, and iterative (Hall, 2020). DBR must inevitably entail taking action to improve and modify a learning environment or experience. Usually, it entails doing something new or innovative. Something at the cutting edge of electronic education or learning and teaching. DBR must be iterative. Typically, there are a number of interconnected cycles that each include the internal processes of conception, design, implementation, and evaluation. Since each cycle of DBR seeks to build upon, enhance, and expand the preceding cycle or cycles, it is accretive. Polit studies can be the first of at least three consecutive design meso-cycles, followed by a second cycle, which could involve mainstreaming or scaling up the design, and a final, capstone iteration that should serve to confirm the improvements made to the design process overall as well as identify areas for additional research and development (Hall, 2020).

Interventionist, theory-generative, prospective, and reflective analysis, iterative, and ecologically valid and practice-oriented (Collective, 2003; Haagen-Schützenhöfer & Hopf, 2020; Prediger et al., 2015; Vaezi et al., 2019) are the key five characteristics among different characteristics of DBR. Design-based research is highly interventionist in nature (Design Based Research (Design-Based Research Collective, 2003). The core intent is to investigate possibilities for improvement in education by considering innovation as the solution. For example, the goal is to create and study a new form of instruction rather than simply observing classroom instructional practices. DBR aims to develop a theory that comprises substantiated conjectures about both processes of learning and the meaning of supporting that learning (Hall, 2020). The goal of interventions in DBR is to develop and refine theories, not just to test them in a narrow sense and produce theories that effectively inform the prospective design. The design of innovation is informed by theory prospectively and is further developed through retrospective reflection on any deviation between expected and observed teaching-learning process (Haagen-Schützenhöfer & Hopf, 2020). DBR involves iterative cycles of design and analysis (Cobb et al., 2015). There are micro and macro design cycles. When researchers adapt instructional activities and underlying theory based on ongoing analysis within a design experiment, it follows the micro-design cycles. When design experiments are repeated, the researchers build instructional activities upon one or more preceding studies; macro-design iterations help gain knowledge. The DBR are of pragmatic roots as the emphasis on ecological validity and practical orientation(Doorman, 2019). The research done in real classroom setting so that the research represents the complexity of actual practice. The theories generated are closely tied with the activities of teachers and students, tested locally, and repeatedly revised.

## Phases in DBR

Design approaches typically make two contributions to education: proximal and distal to the integrative model of educational design research (Hall, 2020). In the proximal contribution, DBR focuses on making real-world improvements in the naturalistic learning environment, such as in a classroom. In the distal contribution, design-based approaches also focus on developing new conceptualizations of learning that other educators, educational designers, and technologists can use in their own unique educational contexts, depending on local demands, limitations, and specifications. DBR is a process-oriented investigation focusing on how learning occurs, consisting of three successive, flexible, and iterative phases: Preliminary design, the teaching experiment, and retrospective analysis (Cobb et al., 2015; Doorman, 2019).

### *Preliminary Design*

This phase is the preparation of the design process, explicit choices, and expectations from the intervention for the students' learning, as well as teachers' learning. Design research typically aims to investigate students' learning by specifying carefully those mathematical reasoning that constitute the learning goals. It is expected to have evidence for instructional starting points. This can be done by identifying aspects of students' current reasoning or teachers' current practices upon which instruction can be built. The instructional starting point can be determined by creating and administering assessments such as one-on-one interviews, written assessments, class quizzes, and classroom observations. Next, the task is to delineate a hypothetical learning trajectory (HTL), which comprises testable conjectures on developing students' reasoning and mathematical learning. It is the conjectured learning route to be tested and revised during the study.

The design of instructional activities is often guided by specific principles: Guided reinvention, didactical phenomenology, and emergent models (Cobb et al., 2009). The reconstruction of a way of developing a mathematical concept from a problem situation is guided by reinvention. This is often linked with students' reasoning in a meaningful context. Didactical phenomenology is a theoretical as well as methodological approach to identify and analyse how students might experience and build certain mathematical concepts. Thus, didactical phenomenology guides in designing tasks that are grounded in students' experience and way of thinking. The emergent models are those models developed through students' learning activities and thinking, starting from a familiar context, and gradually evolving into more formal mathematical concepts. The innovative plan in design-based research is developed by using design heuristics that follow the framework of the theory of realistic mathematics education (Doorman et al., 2013). Accordingly, the emergent modelling is the design heuristic of design-based research in mathematics education. In learning mathematics, students are supported in recreating or reinventing mathematics. The emergent modelling design heuristic is meant to provide such a reinvention process so that students can think and work as mathematicians.

### *The Teaching Experiment*

This phase is the experimentation to support learning and test the innovative plan developed in the preliminary design phase. The activities and expectations embedded in the hypothetical learning trajectory are confirmed with classroom reality. The main aim is to improve the envisioned trajectory by testing and revising conjectures about the learning process and supports (Doorman et al., 2013). Both the evidence of students' learning and evolving learning environment are collected by using pre-and post-interviews, written assessments, video recordings of the classroom sessions, copies of students' written work, and field notes of the researcher.

### *Retrospective Analysis*

The systematic and reflective process of examining the dataset, generated during the teaching experiment phase, to decide how and why learning occurred or didn't occur. The retrospective analysis, the post-hoc analysis after the teaching experiment and learning processes, aims at identifying patterns, testing theoretical assumptions, and refining the intervention or instructional design and theory (Cobb et al., 2009). Throughout the implementation process, continuous analysis is typically conducted with a direct link to the immediate practical goal of supporting students' learning. The retrospective analysis further seeks to place this learning and the means of learning in a broader theoretical context (Cobb et al., 2015).

The argumentative grammar (Kelly, 2016) is the underpinned methodology that links research questions to data, data to analysis, and analysis to the final claims and assertions. The techniques of argumentative grammar, which lead the concerns about the warrant for claims (Cobb et al., 2015), can be used in retrospective analysis. The argumentative grammar mainly tries to confirm ‘demonstrating impact and generalizability’(Barab & Squire, 2016). The demonstrating impact analyses the learning gains and conceptual change, whereas demonstrating generalizability refers to the transferable principles. Typically, argumentative grammar is a structured language and logic to make claims, provide evidence, offer warrants, acknowledge counterevidence, and draw a conclusion (Bakker, 2018). Thus, the first goal of argumentative grammar is to demonstrate that the students have developed the documented forms of mathematical reasoning in participating in the design study. The second is to demonstrate that the findings are potentially generalizable for the further references. Finally, documenting how each subsequent form of reasoning developed as a reorganization of earlier forms of reasoning and identifying the elements of the classroom learning environment that facilitated the students' development of these successive forms of reasoning are two of the main concerns when performing a retrospective analysis of the entire data corpus (Cobb et al., 2015).

## Discussion and Conclusion

Designing mathematics lessons as guided by the RME principles of horizontal and vertical mathematizations. While designing innovations, there are certain guiding principles to check and confirm. We can prepare a checklist as given in Table 1 and check whether these are addressed or not.

Activity, reality, levels, intertwinement, interactivity, and guidance are the core principles of RME. These principles collectively foster mathematical understanding and problem-solving skills. As RME promotes an active, contextual, and interactive learning environment, it helps students in the mathematization process. When real-world problems are turned into mathematical ones, it is the horizontal mathematization, and when students recognize the problem within the mathematical system to find the solution, then it is vertical mathematization. Mathematization helps to develop students’ logical, critical, and creative thinking. Teachers act as creative and innovative guides. RME environment motivates learning mathematics as well as helps to develop self-confidence. The emergent modelling is the design heuristic of design-based research in mathematics education. In learning mathematics, students are supported in recreating or reinventing mathematics. The emergent modelling design heuristic is meant to provide such a reinvention process so that students can think and work as mathematicians.

*Table 1*

**Checklist of Design Principles of RME**

| Core Principle | Fully addressed | Partially addressed | Not addressed | Remarks |
|---|---|---|---|---|
| 1) Innovation is helpful for active learning | | | | |

| | | | | |
|---|---|---|---|---|
| 2) It is with problem-situation and addresses experientially real | | | | |
| 3) It has various levels of learning | | | | |
| 4) It is built based on the intertwinement principle. | | | | |
| 5) It has individual and group activities and interactivity. | | | | |
| 6) It is guided by the re-invention principle so that students can learn as mathematicians do. | | | | |

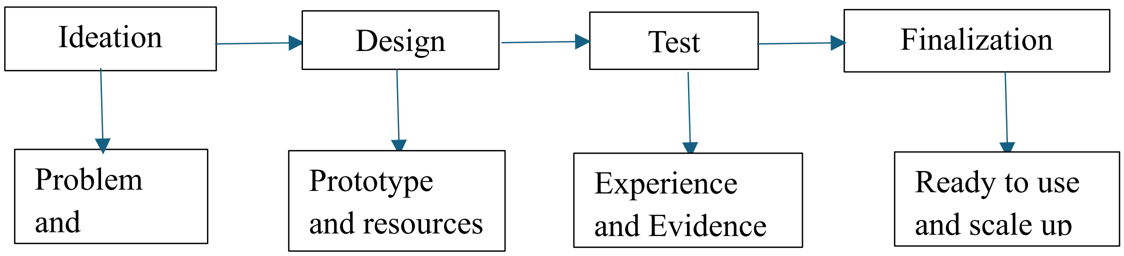
Once innovation is ready for testing, it is finalized by using the key stages of DBR. When we are interested in ways to change or innovate an educational situation for which no solution is at hand yet, something needs to be designed, and the process of teaching and learning that is triggered by this design needs to be investigated, and, in most cases, redesigned. Such a cyclic process of design and research is referred to as design-based research. In design-based research, theory development happens in interaction with experiments, experiments to understand and improve classroom situations (Bakker, Doorman & Drijvers, 2003). Design-based research is highly interventionist in nature. The core intent is to investigate possibilities for improvement in education by considering innovation as the solution. The design of innovation is informed by theory prospectively and is further developed through retrospective reflection on any deviation between expected and observed teaching-learning process (Haagen-Schützenhöfer & Hopf, 2020). DBR involves iterative cycles of design and analysis (Cobb et al., 2015). The pragmatics roots of DBR emphasize ecological validity and practical orientation(Doorman, 2019). The theories generated are closely tied to the activities of teachers and students, tested locally, and repeatedly revised.

DBR is a process-oriented investigation focusing on how learning occurs, consisting of three successive, flexible, and iterative phases: Preliminary design, the teaching experiment, and retrospective analysis (Cobb et al., 2015; Doorman, 2019). These phases can be broken down into four phases: Ideation phase, design phase, test phase, and finalization phase, as shown in Figure 1.

The ideation phase is the phase of generating an innovative solution for a selected problem. The design phase is related to designing a prototype of the proposed solution. The preliminary design is the preparation of the design process, explicit choices, and expectations from the intervention for the students' learning, as well as teachers' learning. To delineate a hypothetical learning trajectory (HTL) is the next task in the preliminary design phase. The test phase is the teaching experiment phase and is the experimentation to support learning and test the innovative plan developed in the preliminary design phase. Different tools are used to collect data and evidence for analysis in the next phase. The main aim of the finalization phase is to improve the envisioned trajectory by testing and revising conjectures about the learning process and supports (Doorman et al., 2013). The retrospective analysis is the final phase and is the systematic and reflective process of examining the dataset generated during the teaching experiment phase to decide how and why learning occurred or didn't occur. The techniques of argumentative grammar, which lead the concerns about the warrant for claims (Cobb et al., 2015), can be used in retrospective analysis. Documenting how each subsequent form of reasoning developed as a reorganization of earlier forms of reasoning and identifying the elements of the classroom learning environment that facilitated the students' development of these successive forms of reasoning are two of the main concerns when performing a retrospective analysis of the entire data corpus.

*Figure 1*

**Phases and Product of Design-Research**




In conclusion, designing an innovation in mathematics education and finalizing for better learning can follow the sequence of RME principles to DBR phases.

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