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Unpacking the Dynamics of Curvature and Torsion Using JavaScript

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Abstract

Understanding the curvature and torsion is a fundamental concept in differential geometry, which is often a struggle for students due to its only symbolic computation. This Phenomenography study explores how interactive digital tools, specifically JavaScript-based interactive simulations, enhance students understanding on the dynamics of curvature and torsion. TPACK framework and connectivism is used as theoretical framework in this study to answer the research question" How do students understand the dynamics of curvature and torsion using JavaScript based mathematical simulation". The finding of this study is based on thirteen master's students at Tribhuvan University, Nepal during 2024 academic year. This study finds that the interactive tools like JavaScript significantly improve students' conceptual clarity on the dynamics of curvature and torsion, integrating concept definition with concept image: "curvature is bending along straight line, torsion is distortion along the plane".

Keywords: *curvature, torsion, JavaScript, TPACK, phenomenography*

Introduction

Higher mathematics is inherently abstract. This fundamental characteristic is widely acknowledged in academic circles, as evidenced by research from numerous scholars (eg, Durand-Guerrier (2016), Lockwood, Ellis, & Lynch (2016), Zhen, Weber, & Mejia-Ramos (2016), and Dawkins & Roh (2016), as cited in (Dhakal, 2019). So, there are challenges for students to fully grasp abstract mathematical concepts.

Various approaches have been explored to make mathematical abstraction more tangible. Researchers have attempted to materialize these ideas through examples, graphical representations, analogies, and metaphors, which is also highlighted in a research work (Dhakal, 2019). Furthermore, these studies have investigated the role of different types of reasoning-including conceptual, ideational, metalinguistic, procedural, syntactic, semantic, cognitive, and metacognitive in making abstract mathematics more accessible (Dhakal, 2019).

As in other content area of mathematics education, a core challenge in geometry is similar, for instance, to precisely defining and differentiating geometric quantities. For example, line segments are uniquely identified by their lengths, circles by their radii, and triangles by axioms like side-angle-side. Similarly, a space curve is uniquely defined by two scalar quantities: its curvature and torsion. However, based on teaching experiences in a university-level Differential Geometry course, I see that students often lack the expected mathematical competency in these areas, particularly to distinguish the interpretation of curvature and torsion.

In this era of growing digital technology, there's a good opportunity to leverage digital tools to enhance students' comprehension of mathematical concepts. Therefore, this study aims to contribute to this understanding by exploring whether mathematical concepts can indeed be materialized effectively through the pedagogical application of digital tools, like JavaScript based simulations.

In differential geometry of three-dimensional curves, the curvature of the curve at the point is the measure of bending on the curve along tangent direction. The torsion also characterizes the bending, but it is about how much the osculating plane (the plane of curvature) twists along the curve. In essence, the absolute value of torsion quantifies the rate at which the osculating plane rotates around the normal vector of the curve (Huang, 2018).

It is mentioned that "the Serret Frenet formula is a matrix equation representing the fundamental unit vectors along with their derivatives to represent curvature and torsion" (Dhakal, 2025), it is written as

$$\begin{bmatrix} \vec{t}' \\ \vec{n}' \\ \vec{b}' \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} \vec{t} \\ \vec{n} \\ \vec{b} \end{bmatrix}$$

It is to noted that the coefficient matrix appearing on the right is skew-symmetric, also it is notifying that the curvature κ is nonnegative scalar whereas the torsion is a signed scalar (Koirala & Dhakal, 2024; Pundir et al., 2021).

Problem Statement

Students frequently struggle with the abstract nature of mathematical concepts, often preferring to memorize formulas rather than grasping their underlying meaning. Due to these foundational gaps, students often find it challenging to visualize abstract concepts, such as the geometric interpretation of curvature and torsion. For example, while teaching at master students at university, the researcher has realized that students incorrectly assumed the meaning of curvature and torsion in 3D space curve. To address this issue, JavaScript based interactive simulation can influence and support students' learning for meaningful interpretation, which can further help student towards a more accurate understanding.

Research Question

The study investigates students understanding on mathematical concept images as they interact with dynamic visualizations through JavaScript. The central inquiry guiding this study is How do students understand the dynamics of curvature and torsion using JavaScript-based mathematical simulations?

Theoretical Framework

This study is grounded on the theory of "Technological Pedagogical Content Knowledge (TPACK) framework" (Herring et al., 2014; Koehler et al., 2014; Mishra et al., 2007), which provided a lens to examine how students integrated their understanding of mathematical content with digital tools and pedagogical strategies. TPACK emphasizes the interconnectedness of three domains: technological knowledge (TK), pedagogical knowledge (PK), and content knowledge (CK) (Mishra & Koehler 2016; Mishra et al., 2007). Additionally, the study used connectivism, a learning theory for the digital age, which highlights how knowledge is distributed across a digital tools and how learning occurs by navigating and forming connections among these tools (Mishra & Koehler 2016; Mukhlis et al., 2024).

Methods

The main purpose of this study is to explore how interactive JavaScript-based visual tools influence students' conceptual understanding of mathematical ideas, which is the dynamic of curvature and torsion. To achieve this, the study employed a qualitative phenomenographic methodology, a branch of phenomenology (Creswell et al., 2018; Denzin & Lincoln, 2018; Steffe & Nesher, 2009), to investigate how master students interpret and make sense of mathematical phenomena, the JavaScript simulation.

Phenomenography is a research approach to understand how individuals experience learning activities. In this study, that scope is extended to include engagement with interactive digital tools, as mentioned in a literature by Steffe & Nesher (Steffe & Nesher, 2009). In the context of mathematical learning, this means that a student does not engage with a mathematical concept in isolation, but rather with how they personally perceive and experience that concept.

A key principle of phenomenographic research is the understanding that different learners may interact with the same instructional content—a text, a problem, a lecture, or in this case, a digital applet—and come away with different interpretations and levels of understanding (Steffe & Nesher, 2009). These variations are not random, but patterned and meaningful, shaped by the learners' prior knowledge, perspectives, and context. This understanding informed the decision to use phenomenography as a methodological tool in this research.

The study is conducted with a group of thirteen undergraduate students (5 female, 8 male) enrolled in a "Differential Geometry" course during the 2024 academic year at Tribhuvan University, this were a complete census type study, because there were 13 students studying durin these academic years in CDED. For the data collection, multiple forms of data collection were utilized—including student interviews, classroom observations, and recordings of student interactions with the simulation environment to support data triangulation and strengthen the reliability of findings as suggested in number of research books (Creswell et al., 2018; Denzin & Lincoln, 2018).

Results

In this study, the visualization of JavaScript simulation of curvature and torsion are based on the symbolic computation as mentioned by authors (Koirala & Dhakal, 2024; Pundir et al., 2021), which is given as

$$\kappa = |\vec{r}' \times \vec{r}''|, \tau = \frac{[\vec{r}', \vec{r}'', \vec{r}''']}{|\vec{r}' \times \vec{r}''|^2} \text{ and } [\vec{r}', \vec{r}'', \vec{r}'''] = \kappa^2 \tau$$

and

$$\kappa = \frac{|\dot{\vec{r}}\times\ddot{\vec{r}}|}{|\dot{\vec{r}}|^3},\ \tau = \frac{[\dot{\vec{r}},\ddot{\vec{r}},\dddot{\vec{r}}]}{|\dot{\vec{r}}\times\ddot{\vec{r}}|^2} \text{ and } \left[\dot{\vec{r}},\ddot{\vec{r}},\dddot{\vec{r}}\right] = \kappa^2\tau\dot{s}^6$$

Initially, many students' concept images of curvature and torsion often remained abstract or solely based on algebraic formulas. As mentioned in textbook (Koirala & Dhakal, 2024; Pundir et al., 2021). For example, the definition is given by "let C : $\vec{r} = \vec{r}$ (s) be a space curve and P be a point on it, then curvature at P is defined as the rate of rotation of tangent (change in the direction

of tangent) at P. The magnitude of curvature is denoted by κ (kappa) and defined by

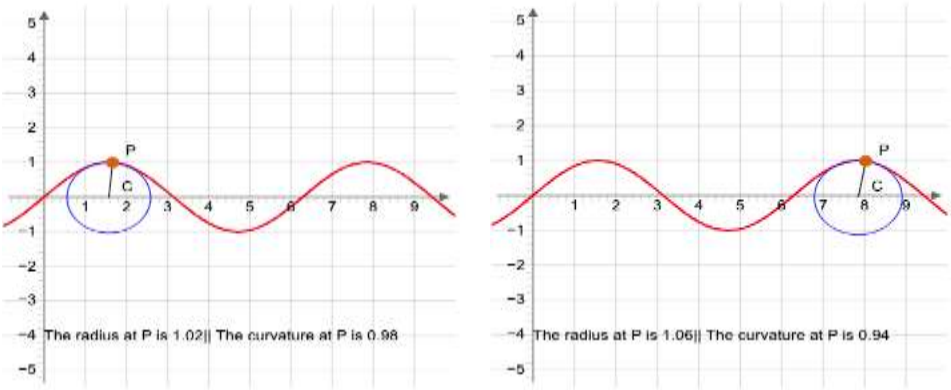
$$\kappa = \lim_{\delta s\to 0}\frac{\delta\theta}{\delta s} = \frac{d\theta}{ds}$$

where δθ is angle between tangents at two neighboring points P and Q on

the curve along arc length δs as δs→0" (Koirala & Dhakal, 2024; Pundir et al., 2021).

Figure 1:

Local properties of curvature

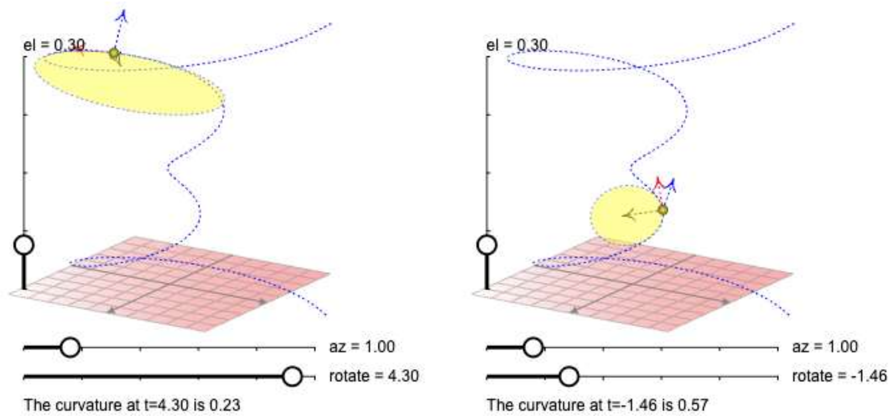


while student used the applet given in Figure 1, students understand that the curvature is measure of deviation of curve from lying in a straight line. It is justified by a quote

> "*curvature is the tendency of a curve to change direction of tangent from point to point along the curve, such as a circle with a small radius, the curvature is relatively large, or equivalently, circle with a large radius, the curvature is small*"-S.

Figure 2:

Local properties of curvature in 3D



This visualization, as shown in Figure 2 helped students to understand that, as a point moved along the curve, the magnitude of the curvature varied. For instance, in a curve like spirals in the applet, the curvature changes with position of the point in the curve. It is justified as student experienced;

> "*when I moved the point along the curve in the simulation, I could see how the bending changed — like, the curvature isn't the same everywhere — it depends on where the point is on the curve*"-S.

This visualization given in Figure 2 helped students to understand that, in a space curve C : $\vec{r} = \vec{r}$ (s), the expression of curvature is $\vec{t}' = \kappa\vec{n}$ where, vector $\vec{t}'$ is called curvature vector, so that magnitude of $\vec{t}'$ is called curvature (Richardson & King, 2002), it is denoted by $|\vec{t}'|$ and defined by $|^{\to}$

$$|\vec{t}'| = \kappa\vec{n}$$

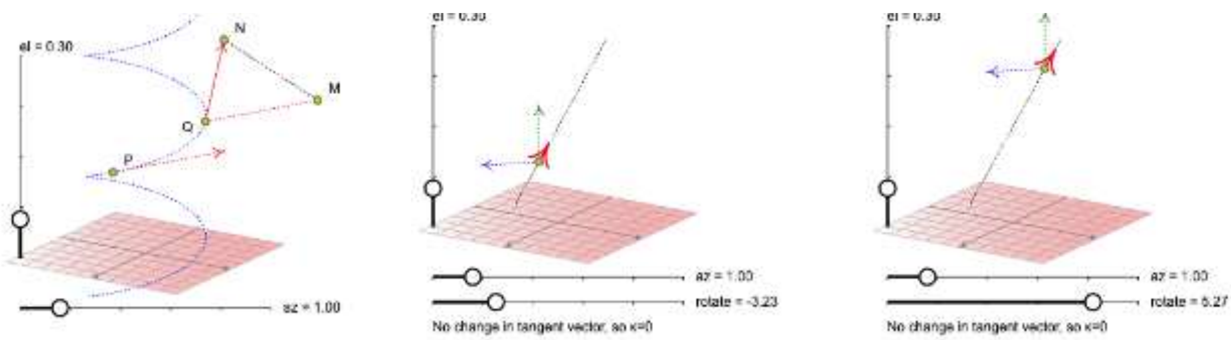
also, the reciprocal of curvature is denoted by

$$\rho = \frac{1}{\kappa}$$

and it is called the radius of curvature.

Figure 3:

Straight line: κ= 0



This visualization given in Figure 3 helped students to understand number of textbook theorems, which is

> necessary and sufficient condition for a curve to be a straight line is that curvature κ= 0 at all points on the curve.

this is justified as a student's response that

> in a straight line, the value of curvature is zero, also a point in which κ= 0 is called a point of inflection

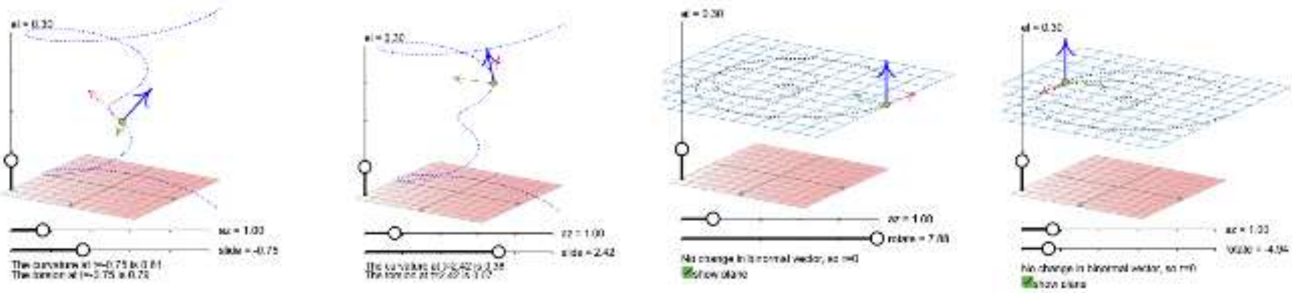
In the case of a space curve, the curve exhibits both bending and twisting as it extends out of a single plane. Therefore, analyzing curvature alone is not sufficient; it is also necessary to quantify the extent of the curve's twisting, which is described by its torsion.

Regarding the understanding of torsion, student was in confusion for its geometrical interpretation just reading the textbook definition. For example, in the textbook it is defined as

> "the torsion of a space curve C, represented as C: $\vec{r} = \vec{r}$ (s), at a point P quantifies the curve's deviation from planarity. Specifically, it is defined as the rate of change in the binormal vector's direction at P, with its magnitude commonly denoted by τ
>
> $$\tau = \lim_{\delta s \to 0} \frac{\delta\theta}{\delta s} = \frac{d\theta}{ds}$$
>
> where δθ is angle between binormals at two neighboring points P and Q on the curve along arc length δs as δs→0" (Koirala & Dhakal, 2024; Pundir et al., 2021; Richardson & King, 2002).

Figure 4:

Local properties of torsion



Students using the applet in Figure 4 learned that the vector $\vec{b}'$ is known as the torsion vector, and its magnitude, denoted as $|\vec{b}'|$, represents the torsion itself. As mentioned in literature (Richardson & King, 2002), it is defined as

$|\vec{b}'|$=- τ.

They also understood that the radius of torsion, σ, is the reciprocal of torsion, or $\sigma = \frac{1}{\tau}$. Furthermore, they recognized that torsion varies from point to point along a curve and indicates how much the curve deviates from being planar.

This understanding is supported by a specific quote as below.

> the value of torsion changes from point to point along the curve, and the torsion measure of deviation of curve from lying in a plane, as I see the torsion of circle like curve is zero, in the
>
> applet.

While student used the applet given in Figure 4, students understand that the vector $\vec{b}'$ is called torsion vector, as the magnitude of $\vec{b}'$ is called torsion, which is denoted by $|\vec{b}'|$ and defined by $|\vec{b}'|$=- τ (Richardson & King, 2002). The reciprocal of torsion is denoted by $\sigma = \frac{1}{\tau}$, and it is called radius of torsion. They also perceived that torsion varies point to point is measure of deviation of curve from lying in a plane. It is justified by a quote

The visualization given in Figure 4 helped students to understand number of textbook theorems (Koirala & Dhakal, 2024; Pundir et al., 2021), which is

1. iff $\tau = 0$ at all points on the curve, then the curve is plane curve
2. iff $[\vec{r}', \vec{r}'', \vec{r}'''] = 0$ at all points, then the curve is plane curve
3. iff $[\dot{\vec{r}}, \ddot{\vec{r}}, \dddot{\vec{r}}] = 0$ at all points, then the curve is plane curve

the logic of the theorems was based on the mathematical deductions

$[\vec{r}', \vec{r}'', \vec{r}'''] = \kappa^2 \tau$, and $[\dot{\vec{r}}, \ddot{\vec{r}}, \dddot{\vec{r}}] = \kappa^2 \tau \dot{s}^6$

together, the visualization of curvature and torsion, helped students for a deeper discussion around the Fundamental Theorem of space curves, which states that a curve in space is uniquely determined by its curvature and torsion functions, which is also mentioned in a literature (Richardson and King, 2002).

Discussion and Findings

Learning differential geometry demands that students draw upon a wide range of foundational mathematical disciplines (Huang, 2018). Mastery of these areas not only supports their understanding of core concepts in local differential geometry-such as curvature and torsion-but also highlights the interdisciplinary nature and value of prerequisite courses.

The findings of this study align with existing literature suggesting that interactive technology tools enhance conceptual understanding (Dhakal, 2019, 2023). JavaScript simulations allowed for real-time manipulation, which is critical for grasping dynamic properties like curvature and torsion, which is an essence of TPACK (Herring et al., 2014; Koehler et al., 2014; Mishra et al., 2007).

In a literature it is noted that students gain better spatial understanding when interactive elements are introduced (Mukhlis et al., 2024; Onwu Iji & Abah, 2018). Also, a literature emphasized the role of dynamic geometry software in reshaping mathematical thinking (Dhakal, 2023; Tamam & Dasari, 2021). A study highlighted connetivist approaches help

student to learn mathematical concepts, where students build knowledge through interaction with digital tools (Mukhlis et al., 2024). Based on this literature, in can be said that JavaScript simulation can unpack the dynamics of curvature and torsion as discussed in TPACK (Herring et al., 2014; Koehler et al., 2014; Mishra et al., 2007).

However, not all research supports this view unconditionally, for example participants raised concerns that excessive reliance on technology may weaken students' abilities to symbolic computation. So, this study clearly appreciates the views that digital tools can sometimes lead to superficial learning if not coupled with mathematical computations. So, as suggested by TPACK (Herring et al., 2014; Koehler et al., 2014; Mishra et al., 2007), the technology integration must be with pedagogically thoughtfulness.

The findings of the study, therefore, suggest that while JavaScript tools offer significant pedagogical affordances, they must be embedded in thoughtful pedagogical strategies, such as guided inquiry, scaffolded learning, and reflection sessions to uncover algebraic or symbolic computations, which a beauty of mathematical content, as explained in TPACK (Herring et al., 2014; Koehler et al., 2014; Mishra et al., 2007).

In this context, this study revealed that engaging with differential geometry through interactive, simulation-based environments, it helps students understand and apply foundational concepts in meaningful ways for further learning. Moreover, it promoted the ability to express abstract ideas visually and verbally, such as ”Curvature is the rate of change of tangent direction that becomes a straight line, torsion is the rate of change of osculating plane or distortion along the plane” which is crucial for interpreting and applying geometric concepts in both theoretical and applied contexts, which can be articulated with the theoretical foundation of TPACK (Herring et al., 2014; Koehler et al., 2014; Mishra et al., 2007).

Recommendation

Based on the findings of this study, it is recommended for teachers to use of digital tools like Javascript tools and TPACK-aligned pedagogy to effectively guide students through interactive learning experiences. So, future studies should explore the impact of such tools on diverse learner groups and investigate long-term retention of concepts learned through simulation-based pedagogy cum computational tool like Mathematica, Maple, MatLab.

Data availability

The datasets and illustrative materials that support the findings of this study are publicly available on the author’s educational websites, which can be accessed at

https://www.bedprasaddhakal.com.np/2024/07/curvature-and-torsion.html. These sources contain the original data, code snippets, and interactive content used to support the analytical and visual results presented in the article.

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References

Creswell, J. W., Creswell, J. D., Creswell, J. W., & Creswell, J. D. (2018). *Research design: Qualitative, quantitative, and mixed methods approaches,* (Fifth edition). SAGE.

Denzin, N. K., & Lincoln, Y. S. (Eds.). (2018). *The SAGE handbook of qualitative research* (Fifth edition). SAGE.

Dhakal, B. P. (2019). *VLE for Engaged and Interactive learning in Higher Mathematics Education*. Tribhuvan University.

Dhakal, B. P. (2023). Digital Pedagogy: An effective Model for 21st Century Education. *Academic Journal of Mathematics Education, Bpd_dp*, *5*(1), 1–9. https://doi.org/10.3126/ajme.v5i1.54534

Dhakal, B. P. (2025). Learning Higher Mathematics with AI: A Phenomenological Study of Students' Experiences. *Interdisciplinary Research in Education*, *10*(2), 1–9. https://doi.org/10.3126/ire.v10i2.88090

Herring, M. C., Koehler, M. J., Mishra, P., & Published By The Aacte Committee O (Eds.). (2014). *Handbook of Technological Pedagogical Content Knowledge (TPCK) for Educators* (0 ed.). Routledge. https://doi.org/10.4324/9781315759630

Huang, Y. (2018). Research on Relationship between Curvature and Torsion of Space Curve. *Proceedings of the 2017 4th International Conference on Machinery, Materials and Computer (MACMC 2017)*. 2017 4th International Conference on Machinery, Materials and Computer (MACMC 2017). https://doi.org/10.2991/macmc-17.2018.142

Koehler, M. J., Mishra, P., Kereluik, K., Shin, T. S., & Graham, C. R. (2014). The Technological Pedagogical Content Knowledge Framework. In *Handbook of Research on Educational Communications and Technology* (pp. 101–111). Springer New York. https://doi.org/10.1007/978-1-4614-3185-5_9

Koirala, S. P., & Dhakal, B. P. (2024). *Differential Geometry*. Sunlight Publication.

Mishra, P., Peruski, L., & Koehler, M. (2007). *Developing Technological Pedagogical Content Knowledge (TPCK) Through Teaching Online*.

Mukhlis, H., Haenilah, E., Sunyono, Maulina, D., Nursafitri, L., Nurfaizal, & Noerhasmalina. (2024). Connectivism and digital age education: Insights, challenges, and future directions. *Kasetsart Journal of Social Sciences*, *45*, 803–814. https://doi.org/10.34044/j.kjss.2024.45.3.11

Onwu Iji, C., & Abah, J. (2018). *Mathematics education for all through information technology innovations*.

Pundir, S. K., Gupta, P. P., & Malik, G. (2021). *Differential Geometry*. Anu Books.

Richardson, G., & King, J. R. (2002). The evolution of space curves by curvature and torsion. *Journal of Physics A: Mathematical and General*, *35*(46), 9857–9879. https://doi.org/10.1088/0305-4470/35/46/310

Steffe, L. P., & Nesher, P. (Eds.). (2009). *Theories of mathematical learning*. Routledge, Taylor & Francis Group. ICME.

Tamam, B., & Dasari, D. (2021). The use of Geogebra software in teaching mathematics. *Journal of Physics: Conference Series*, *1882*(1), 012042. https://doi.org/10.1088/1742-6596/1882/1/012042