Function and Its Application in Real Life

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Abstract

The study presents ordered pairs, relations, and functions, examining their theoretical and practical applications. This study intends to implement fundamental mathematical concepts in practical scenarios. Examples and problem-solving methods were employed to examine functions and their real-world applications. Sequential instructional resources illustrate the observation of real-world functions. Secondary data was gathered from academic journals, research reports, and textbooks to support and explain these applications. The study advocates for problem-solving utilizing ordered pairs, relations, and functions following a literature and mathematical review. The study also addresses methods for instructing college students and researchers in these principles. The findings indicate that ordered pairs, relation, and functions facilitate the resolution of intricate global issues. The examination of water usage, academic schedules, and rice preparation illustrates the significance of essential functions. This study suggests that comprehending these concepts enhances everyday problem-solving and decision-making. Utilizing functions in everyday life improves mathematical understanding and application. The study indicated that acknowledging the significance of functions in real-world contexts enhances education and decision-making. The research seeks to promote comprehension and application of functions beyond basic mathematical principles. This study emphasizes that comprehending mathematical functions improves the capacity to model, analyze, and resolve real-life daily activities, particularly in educational contexts, water consumption, and rice preparation.

Key words: Ordered pair, Cartesian product, Relation, Function, real-life application

Introduction

A function is a fundamental idea in mathematics that delineates a relationship between two sets, where each input corresponds to a distinct output. Functions are essential in numerous practical applications, such as engineering, economics, medicine, and technology. Functions facilitate the modeling and resolution of real-world problems, ranging from forecasting population growth to enhancing corporate strategies. Comprehending functions and their applications improves problem-solving abilities and facilitates decision-making across various domains.

Literature Review

The term 'function' originates from a Latin word denoting 'activity' and was initially employed in the field of calculus. Rene Descartes (1596 - 1650) established a range of mathematical connections in his publication Geometry (1637) without explicitly employing the term "function." This was regarded as the initial comprehension of function. In the 1680s, the German mathematician G. W. Leibniz (1646 - 1716) introduced the term 'function' to represent the concept of a derivative.

Function is currently utilized in numerous domains of mathematics and its practical implementations. Various academic fields, such as physics, astronomy, engineering, economics, medical sciences, management, technology, and information and communication technology (ICT), utilize the idea of function to elucidate and utilize fundamental concepts and principles within their respective domains of study. Moreover, the function has been extensively employed in everyday applications to address a wide range of challenges in multiple domains, including diverse types of measurements, basic economics and business, information and communication technology, and more.

Various branches of mathematics are concerned with functions. Mathematical analysis examines functions with one, two, or multiple variables, investigating their properties and derivatives. The theories of differential and integral equations focus on solving equations where the unknowns are functions. Functional analysis deals with spaces composed of functions. Numerical analysis studies the methods for minimizing errors when evaluating various types of functions. Additional branches of mathematics explore notions that encompass generalizations or extensions of the concept of function. For instance, algebra examines operations and relations, whereas mathematical logic investigates recursive functions. The notion that functions should be a fundamental concept in secondary school mathematics has been a subject of debate for a long time (Klein, 1908/1945). The latest curriculum orientations strongly highlight the significance of functions (National Council of Teachers of Mathematics, 1989). The concept of function can be interpreted in various ways depending on the prevailing mathematical perspective, each of which has distinct implications for education. This paper examines key elements in the history of the idea of function, explores its connections with various scientific disciplines, and analyzes its application in the investigation of real-life scenarios. Lastly, the issue of a didactical approach is examined, with a particular focus on the fundamental nature of the conceptual framework guiding students' activities and the significance of various types of representation.

Research questions

I have formulated the following research questions to achieve my objectives:

i) How do ordered pairs, relations, and functions enhance the understanding of fundamental mathematical concepts?

ii) What are the most effective techniques for solving function-related problems systematically?

iii) How are functions applied in real-life situations to solve practical problems?

Research objectives

The main goal of this research is to solve the problems stated in the problem statement. To achieve this, I have established the following objectives.

i. To analyze the role of ordered pairs, relations, and functions in enhancing the understanding of mathematical concepts.

ii. To identify and demonstrate effective problem-solving techniques related to functions.

iii. To explore and illustrate real-life applications of functions in various practical scenarios.

Methodology

This study adhered to the problem-solving methodology. Through the utilization of problem-solving methodologies, individuals can examine a given scenario or difficulty and execute efficient resolutions. It is possible to improve various approaches for various applications. I utilized concrete examples based on real-life experiences to tackle these challenges and achieve easily understandable results.

Results and Discussion

Definition 3.1: An ordered pair refers to a set of two components that are separated by commas and enclosed between tiny brackets. In the ordered pair (a, b), the first element, a, is commonly known as the x-component or x-coordinate, whereas the second element, b, is commonly known as the y-component or y-coordinate.

The ordered pairs (a, b) and (c, d) are considered to be equal, expressed as (a, b) = (c, d), if and only if a is equal to c and b is equal to d.

Definition 3.2: The cartesian product of two sets A and B, denoted as A×B (pronounced A cross B), is the set that contains all possible ordered pairings (a, b) where an is an element of A and b is an element of B. The symbol A × B is defined as the set of ordered pairs (a, b) where a belongs to A and b belongs to B.

In symbol, we define

$A \times B = \{(a, b): a \in A \text{ and } b \in B\}$

Example 1: Let A = {3, 4} and B = {2, 3, 4}. find A × B

Solution: A × B = {(3, 2), (3, 3), (3, 4), (4, 2), (4, 3), (4, 4)}

Definition 3.3: A relation from set A to set B is a subset of the cartesian product A × B. The relation R can be denoted as xRy, where (x, y) is an element of R, or simply as R. A relation on set A is a connection that maps elements from A to itself.

Examples 2: If A = {1, 2, 3} and B = {1, 2}. find the relation x > y from set A to set B.

Solution: A×B = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 1), (3, 2)}

Then, $R = \{(2, 1), (3, 1), (3,2)\} \in A\times B$.

Definition3.4: Consider two non-empty sets, A and B, and let R be a relation from A to B. The domain of a relation R is the set of all first elements of the ordered pairs in R, whereas the range of R is the set of all second elements of the ordered pairs in R. The domain of a function R is represented by the notation Dom (R), whereas the range is represented by the notation Ran (R).

Symbolically, $\text{Dom}(R) = \{x: (x, y) \in R\}$ and $\text{Ran}(R) = \{y: (x, y) \in R\}$.

Example 3: If R = {(5, 5), (7, 4), (9, 2), (9, 2)} be a relation from a set A to a set B. Then find the domain and range of the relation.

Solution: Domain of R = {5,7,9}

Range of R = {5,4,2}

Definition3.5: Consider a relation R that maps elements from set A to set B. The inverse relation of

R, indicated by R–1, is formed by swapping the first and second items in the ordered pairs of relation R. So, if

$R = \{(x, y): x \in A, y \in B\} \in A \times B$

then, $R–1 = \{(y, x): y \in B, x \in A\} \in B \times A$

Example 4: If $R = \{(5, 1), (6, 2), (7, 3)\}$, find R–1.

Solution: $R–1 = \{(1, 5), (2,6), (3,7)\}$

Some Special Types of Relations

Type1: Reflexive Relation

A relation R on a set A, denoted as $R \in A\times A$, is considered reflexive if $(x, x) \in R$ for every $x \in A$. To clarify, if each initial member of R is connected to itself, then R is considered a reflexive relation.

A relation R in a set A is not reflexive if there exists at least one element x in A such that the ordered pair (x, x) is an element of R.

Example 5: If $A = \{1, 2\}$. Then show that, A relation R in a set A is reflexive.

Solution: $A \times A = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

If we define $R = \{(x, y) : x = y\} \in A \times A$,

then $R = \{(1, 1), (2, 2)\}$

Since $1, 2 \in A$. Therefore,

$\in (1, 1), (2, 2) \in R$.

Hence, It is reflexive relation.

Type 2: Symmetric Relation

A relation on a set A is called a symmetric relation if $(x, y) \in R \Rightarrow (y, x) \in R$.

Example 6: Is $A = \{1, 2, 3\}$. Then the relation A defined on $x > y$ is symmetric or not?

Solution: $A \times A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

If we define $R = \{(x, y) : x > y\} \in A \times A$, then

$R = \{(2, 1), (3, 1), (3, 2)\}$

is not symmetric because

$(2, 1) \in R \Rightarrow (1, 2) \in R$,

$(3, 1) \in R \Rightarrow (1, 3) \in R$

And, $(3, 2) \in R \Rightarrow (2, 3) \in R$.

Type 3: Transitive Relation

A relation R on a set A is called a transitive relation if $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$.

Example 7: Prove that the relation $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (1, 3)\} \in A \times A$ defined on the set A = {1, 2, 3} is a transitive relation.

Solution: Since, $(1, 2) \in R$ and $(2, 3) \in R$ implies $(1, 3) \in R$

$(1, 2) \in R$ and $(2, 1) \in R$ implies $(1, 1) \in R$ and so on

Hence, the relation R is transitive relation.

Type 4: Equivalence Relation

An equivalence relation R on a set A is defined as a relation that satisfies the properties of reflexivity, symmetry, and transitivity.

Examples 8: Prove that, If R be the set of real numbers. A relation R defined by "p is equal to q" is equivalence relation.

Solution: Since, p is equal to p, so relation is reflexive.

p is equal to q $\Rightarrow$ q is equal to p so relation is symmetric and

p is equal to q and q is equal to r $\Rightarrow$ p is equal to r so relation is transitive hence relation p is equal to q equivalence relation

Definition3.6: Consider two sets, A and B, which are both non-empty. A function f, defined as a mapping from set A to set B, is a rule or correspondence where each element x in A is uniquely associated with an element y in B. A function is synonymous with a mapping.

A function f mapping the set A to the set B is represented as $f : A \longrightarrow B$. Typically, functions are represented by symbols such as f, g, h, ∅, and so on.

Definition3.7: A function $f : A \longrightarrow B$. The set A on which the function f is defined is called the domain of the function f and the set B is called its co-domain. If the function f associates an element $x \in A$ to a unique element $y \in B$, then we write $y = f(x)$ and call y or f (x) as the f-image of x or the value of the function f for x. The element x is called the pre-image of y or f (x) under the function f. The range of the function f is defined as the set of all images of its domain (i.e. the elements of A) and is denoted by R (f) or f (A). It is clear that R (f) is a subset of B i.e. R (f) ❑ B.

$f(A) = R(f) = \{f(x) : \forall\ x \in A\}$

Type 1: One-one (or Injective) function

A function f: $A \rightarrow B$ is said to be a one-one or an injective function if different elements in A have different images in B. This means if $x_1, x_2 \in A$ and $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$ or equivalently $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

Examples 9: Prove that if Z be the set of integers and let f: $Z \rightarrow Z$ be defined by $f(x) = 3x+5$. Then f is a one-one function.

Solution: $f(x_1) = f(x_2) \Rightarrow 3x_1+5 = 3x_2+5$

$\Rightarrow 3x_1 = 3x_2$

$\Rightarrow x_1 = x_2$

Hence, the given function is one to one.

Type 2: Onto (or Surjective) function

A function f: A→B is said to be an onto or a surjective function if every element in B has at least one pre-image in A. This means, there is no element left over in B which has no pre-image in A. In this case, the range of f is equal to the co-domain of f i.e. f (A) = B. Thus, in onto mappings, the co-domain B is completely covered by the f -images of the domain A.

Examples 10: Prove that, A function f: A→B is defined by f (x) = x^2 where A = {–1, 1, 2, –2} and B = {1, 4} is onto function.

Solution: Here, f (x) = x2

then f (A) = {f(– 1), f(1), f(2), f(– 2)} = {$(-1)^2$, 12, 2^2, $(-2)^2$} = {1, 4} = B so that f is an onto function.

Type 3: One-one Onto (or Bijective) function

A function f : A→B is said to be a one-one onto or a bijective function if it is both one-one and onto. A bijective function is also known as a one-to-one correspondence, whose arrow diagram shown as below.

Example 11: Let $\mathbb{R}$ be a set of real numbers. The show that f : $\mathbb{R} \to \mathbb{R}$ such that f(x) = 4x + 7 for all $x \in \mathbb{R}$, is bijective.

Solution: Let f : $\mathbb{R} \to \mathbb{R}$ is a function and is defined by f(x) = 4x + 7 for all $x \in \mathbb{R}$.

For one-to-one

Let $x_1, x_2 \in \mathbb{R}$. Then $f(x_1) = 4x_1 + 7$, $f(x_2) = 4x_2+7$

Now,

$f(x_1) = f(x_2) \Rightarrow 4x_1 + 7 = 4x_2+7$

$\Rightarrow 4x_2 = 4x_2$

⇒ x1 = x2.

This means f is one to one.

For onto

Let y = f(x) = 4x + 7 for $x \in \mathbb{R}$.

That is

y = 4x + 7 ⇒ x = (y-7)/4.

Clearly y ∈ R. So, (y-7)/4 is defined for all y y ∈ R. And (y-7)/4 ∈ R.

This means f is onto. Hence, f is one-to-one and onto. So, f is bijective.

Definition3.8: Let f : A→B be a one-one onto function then the new function defined from B to A is said to be inverse the function f if every elements of set B associates with unique elements of set A. The inverse of the function f is denoted by f –1.

Examples 12: If a function f: R→R defined by f(x) = 9x +5 then find f –1 (x).

Solution: Here, f(x) = 9x +5 or, y = 9x +5

Interchanging the role of x and y, we get.

or, $x = 9y + 5$

or, $x - 5 = 9y$

or $y = (x-5)/9 \quad \therefore f^{-1}(x) = (x-5)/9$

Definition 3.9: Let A, B and C be any three non-empty sets. Let f: A → B and g: B → C the two functions then the new function defined from A to C called the composite function of f and g if every element of set A associate with unique elements of set C. The composite function of set f and g is denoted by g o f and is defined by g o f: A → C such that (g o f) (x) = g(f (x)). g o f is read as 'g circle f' or 'g composite f'.

Example 13: Let f : R → R and g: R → R be defined by f (x) = x + 7 and g(x) = x2. Then find gof and fog.

Solution: $(g \circ f)(x) = g(f(x)) = g(x + 7) = (x + 7)^2$

and $(f \circ g)(x) = f(g(x)) = f(x^2) = x^2 + 7$

4. Applications of Function

In real life, function is applied in many different fields. I've covered fundamental applications for Order Pair, Cartesian Product, Relation and Function.

4.1 Real Life Example related to Order Pair

Suppose in a school library, books of different subjects of different class are arranged in row and column. If we don't know about the concept of ordered pair we cannot find the book. So by using ordered pair we can find the exact location of a specific book. Let the books is in (4, 5) ordered pair it means that the book is in fourth row and fifth column.

4.2 Real Life Example related to Cartesian Product

The Cartesian Product can be utilized to ascertain the potential outcomes that may arise during an experiment. Let A represent the possible outcomes of a die, which are {1,2,3,4,5,6}. Let B represent the possible outcomes of a coin, which are {H,T}. The possible outcomes when both the die and the coin are thrown simultaneously can be obtained by taking the Cartesian product of A and B, denoted as A x B. This results in the set {(1,H), (2,H), (3,H), (4,H), (5,H), (6,H), (1,T), (2,T), (3,T), (4,T), (5,T), (6,T)}.

4.3 Real Life Example related to Relation

Consider Ram as the father of Hari and Hari as the grandchild of Shyam. By employing the concept of relation, we may determine that the relationship between Ram and Shyam is that Shyam is Ram's father.

4.4 Real Life Example related to Function

Type 1: Real life application of function in cooking rice

Sunita informed me that one cup of rice necessitates two cups of water for preparation. I depicted this relationship using the function $f(x) = 2x$. Consequently, for one cup of rice, $f(1) = 2 \times 1 =$ 2 cups of water; for two cups of rice, $f(2) = 2 \times 2 = 4$ cups of water; for three cups of rice, $f(3) = 2 \times 3 =$

6 cups of water, and so on.

Type 2: Real life application of function for water usage

Let (t) denote the volume of water utilized (in liters) as a function of time t (in minutes) throughout the process of filling the tub or operating the shower. If water flows at a uniform rate of k liters per minute:

$f(t) = k.t$

Water flows at a rate of 8 liters per minute.

For $t = 4$ minutes, the function computes: $f(4) = 8\times4 = 32$ liters

For $t = 7$ minutes, the function computes: $f(7) = 8\times7 = 56$ liters

For $t = 10$ minutes, the function computes: $f(10) = 8\times10 = 80$ liters

Type 3: Real life application of function for teaching period

A college schedule commences at 10:45 AM. The first two periods each 45 minutes. The third period is shorter than the first two period than 5 minutes. The fourth period matches the duration of the first two periods, followed by a break of the same length. The fifth period also lasts 45 minutes. The sixth and seventh periods are each 5 minutes shorter than the first two periods. The final period is equivalent in duration to the first two periods.

To mathematically represent the college schedule, let's define variables and functions to represent the durations and timing of the periods and break:

Variables

- P1=P2=P4=P5=P8=45 minutes (first, second, fourth, fifth, and eighth periods)
- P3=P6=P7=45−5=40 (third, sixth and seventh periods are 5 minutes shorter)
- B=45 minutes (break duration, matching the first period)

Start Time

Let the schedule start at T_0=10:45 AM.

Schedule Representation

Define the start time of each period Ti (where i=1, 2,-------,8) as:

$$Ti=Ti-1+Pi-1, \quad i\geq 2$$

Here, Pi−1 is the duration of the period (or break).

Calculating Start Times

Period 1: Starts at T_1= 10:45.

Period 2: $T_2 = T_1 + P_1$ = 10:45 +0:45= 11:30.

Period 3: $T_3 = T_2 + P_2$ = 11:30 +0:45= 12:15.

Period 4: $T_4 = T_3 + P_3$ = 12:15+0:40 =12:55.

Period 5: $T_5 = T_4 + P_4$= 12:55+0:45 =1:40.

Break: $T_B = T_5 + B$ = 1:40+0:45 =2:25.

Period 6: $T_6 = T_B + P_5$ = 2:25+0:45 =3:10.

Period 7: $T_7 = T_6 + P_6$ = 3:10+0:40 =3:50.

Period 8: $T_8 = T_7 + P_7$ = 3:50+0:40 =4:30.

Period 9: $T_9 = T_8 + P_8$ = 4:30+0:45 =5:15.

Mathematical Representation

$$Ti=Ti-1+Pi-1, \quad i\geq 2 \text{ where } Pi-1 \in \{45, 40\}$$

Where Pi−1 depends on the period.

- P1=P2=P4=P5=P8=45
- P3=P6=P7=40
- B = 45

Conclusion

This article explores the fundamental ideas of ordered pairs, relations, and functions, providing detailed explanations and methodical techniques for solving these problems through several examples. I focused on assisting both students and researchers in understanding these mathematical concepts effectively. I emphasized practical applications of functions to illustrate their importance in everyday life. Understanding the real-life application of functions in the context of water usage and the teaching period. The real-life use of functions in cooking rice connects to more sophisticated applications in science and technology; this article functions as a bridge between theoretical mathematics and practical implementation. My goal was to ensure a thorough understanding of these concepts, not just in theory but also in their practical application. By illustrating these connections, I hope to inspire learners to see mathematics as a vital tool in various aspects of their daily lives. Whether measuring ingredients or predicting outcomes, recognizing the relevance of functions can enhance both academic performance and everyday decision-making.

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