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Learning Higher Mathematics with AI: A Phenomenological Study of Students' Experiences

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Abstract

This study is qualitative phenomenology research to explore the 'students' learning experience while using AI based learning resources in higher mathematics learning". The scope of the study is particularly limited to a content "osculating plane", which is a fundamental concept in Differential Geometry. This study employed interactive digital tools, specifically AI and JavaScript-based interactive simulations, to explore how it supports in enhancing student understanding in higher mathematics, as lived learning experience. The Technological Pedagogical Content Knowledge (TPACK) framework was used as a theoretical ground to analyze the data. This study is based among thirteen master's level students at Central Department of Education, Tribhuvan University, Nepal. The tools used in this study were classroom observation checklist and student interview guidelines. Being a phenomenological study, the data analysis focused on capturing and interpreting lived experiences and meanings, not on numerical comparison only. Based on the available data, interview transcripts and observation notes were coded, categorized into meaningful units, and synthesized into themes representing students lived experiences. Based on this analysis, this study found that integration of AI and JavaScript based interactive simulations in higher mathematics education learning resources, student enjoyed the learning with digital simulation and videos. These resources helped them to bridges the "conceptual learning gaps" and fosters "meaningful mathematical understandings".

Introduction

Higher mathematics is abstract and proof oriented. It is said that "Higher mathematics requires students to engage with mathematical formalization of mathematical symbols"(Yavich, 2025). It is also said that "this level of abstraction often creates a significant pedagogical disconnect, as students struggle to relate these symbolic structures to prior conceptual understanding"(Zainudin et al., 2025). Within this, the osculating plane in Differential Geometry is a fundamental concept to study both curves and surface in higher mathematics. The osculating plane represents a plane that "best fits" to a given curve at a given point. Mathematically, it is said that "osculating plane contains direction vector and acceleration vector (or, equivalently, the tangent and principal normal vectors) of a curve at a specific point" (Kock, 1998; Pasall Atmaca, 2010). The concept of osculating plane is also important to understand local behavior of a curve, for example curvature and torsion. However, the abstract nature of osculating plane often poses significant challenges to the students. Based on the empirical evidence of classroom experience, it can be said that traditional type of static diagrams and images being insufficient in conveying the dynamic relationship of osculating plane and its properties.

With the increasing integration of digital tools in education (Herring et al., 2014; Mishra et al., 2007; Mukhlis et al., 2024), understanding how individuals perceive and make sense of such learning experiences essential. Therefore, employing a phenomenological approach (Creswell et al., 2018; Denzin & Lincoln, 2018; Tyowua, 2023), this article discuss about how learner experience interactive visualization using modern web technologies, specifically with AI (Artificial Intelligence) and JavaScript based simulations and YouTube videos, which can bridge this "conceptual learning gap", as mentioned in the literature (Jirasatjanukul et al., 2023; Mukhlis et al., 2024; Nesvidomin et al., 2024). By allowing users to manipulate parameters and observe the real-time changes, for example, osculating plane, a deeper, more intuitive understanding can be fostered, as the author experienced (Dhakal, 2023). Therefore, using AI, the content and codes are generated, and osculating plane related learning content are prepared in a google blog using HTML and JavaScipt. Also, the AI and JavaScipt based interactive simulations and YouTube videos are prepared, and students learning experience are explored.

Given "C: $\vec{r} = \vec{r}(s)$ be a space curve of class >2 and P, Q, R be three neighboring points on it. Then osculating plane at P is defined as the limiting position of a plane determined by PQR as Q, R→P"(Koirala & Dhakal, 2024; Pundir et al., 2021).

It is written as

Osculating plane at P= $\lim_{Q,R\to P} Plane\ PQR$

The osculating plane at P contains the tangent line, and it has three-point contact with curve at P

Now, tangent vector at P is defined as

$$\vec{r}' = \vec{t}$$

This vector indicates the direction of motion along the curve.

The second derivative of gives the acceleration vector

$$\vec{r}'' = \kappa\vec{n}$$

This vector describes how the velocity vector is changing, both in magnitude and direction (Koirala & Dhakal, 2024; Pundir et al., 2021).

The osculating plane at a point P on the curve (corresponding to parameter t) is defined by the point P itself and the two vectors $\vec{r}'$ and $\vec{r}''$. For the osculating plane to be uniquely defined, these two vectors must be linearly independent, which is given by

$[R-\vec{r}, \vec{r}', \vec{r}'']=0$

The normal vector to the osculating plane at a point t is given by the cross product of the tangent and acceleration vectors, which is

$$\vec{b} = \vec{t} \times \vec{n}$$

This vector is perpendicular to both $\vec{t}$ and $\vec{n}$ and thus perpendicular to the plane they define.

Fo examplify this mathematical intuition, we can consider an example of circular helix given in a text book written by Dhakal & Koirala and also by Pundir et al (Koirala & Dhakal, 2024; Pundir et al., 2021), in which circular helix is parameterized by an equation

"$\vec{r}$=(a cos t, a sin t, b t)

where a and b are positive constants.

With a bit of calculation, loike taking the first derivative, tangent vector is given by

$\vec{r}'$=(-a sin t,a cos t,b)" (Koirala & Dhakal, 2024; Pundir et al., 2021

Similarly, the acceleration vector is given by

"$\vec{r}''$=(-a cos t, -a sin t, 0)" (Koirala & Dhakal, 2024; Pundir et al., 2021

Finally, the equation of osculating plane is obtained by

"[(x,y,z)- $\vec{r}$, $\vec{r}'$, $\vec{r}''$]= 0" (Koirala & Dhakal, 2024; Pundir et al., 2021

bx sin t- by cos t + az - abt = 0

The aim of this research is to visualize the mathematical version of osculating plane using AI related tools to enhance the dynamics and support students to formulate "concept image" as learning schema.

Research Question

This study is based on students lived experience on digital and interactive learning resources while learning higher mathematics, particularly AI and JavaScript based interactive simulation and related videos. Therefore, this research seeks to answer the underlying learning experiences of students, their engagement and interaction with digital resources. Thus, the research question of this study is

"What are the students' learning experience while using AI based interactive resources in higher mathematics?

Theoretical Framework

"The Technological Pedagogical Content Knowledge (TPACK) framework is a concept about technology use in pedagogy to leverages technology in pedagogically effective way and to facilitate student learning" (Herring et al., 2014; Koehler et al., 2014; Mukhlis et al., 2024).

This TPACK framework is considered as a guiding lens in this study. This framework emphasizes the interconnectedness and interplay amonth three core components: (a) the technological knowledge (how to use JavaScript for graphics and simulation), (b) pedagogical knowledge (how to teach mathematical concepts), and (c) content knowledge (the dynamics of osculating plane). The goal is to develop digital simulation as a learning tool that not only accurately represents the mathematical content but also emphasize engaged and interactive learning, which is the researcher learned and deployed using AI related tools.

Methods

This research is a phenomenology based qualitative research. This study intends to explore how students learning experience, as a phenomenon, shape the mathematical understanding while using interactive simulations. Particularly, the study analyzes how learners understand the concept of osculating plane, and how they interact with the interactive visualization, which is based on AI and JavaScript.

The participants of this study were master level students studying "Differential Geometry" at Tribhuvan University Nepal, which is thirteen students (five girls, eight boys) during 2024 academic year. The small sample size is characteristic of qualitative phenomenological studies, which prioritize in-depth study of students learning experience.

In this study, research data were collected through two primary tools (a) observation check list and (b) interview guidelines. The observation was used to capture student' learning experiences related information, which can be helpful to materialize the conceptual confusion while learning mathematics. Next, the interview guidelines were used to get insight into students' perceptions and performance while using interactive learning tools both simulation and videos, and to explore how these resources helped student to gain the conceptual benefits that they have experienced.

The prompt question for the interview was to analyze the performance, which was based on prescribed textbook, which consists of five items, which was based on the concept of osculating plane, across all levels of learning taxonomy. The test was conducted to understand who experienced what while using JavaScript simulations and videos while learning the content “osculating plane”.

Based on the available data, interview transcripts and observation notes were coded, categorized into meaningful units, and synthesized into themes representing students lived experiences. In this way, data analysis was carried out using the basic analysis stages such as catagorizing data, reading quotations, describing and classifying themes, then presented descriptively for the results to capture students lived experiences.

Results

Differential Geometry is a core component in higher mathematics, often requiring visual and conceptual clarity. In traditional settings, classroom interaction plays a central role. However, in increasingly flexible learning environments, students may access alternative paths to understanding.

This article investigates students learning experience on "osculating planes".

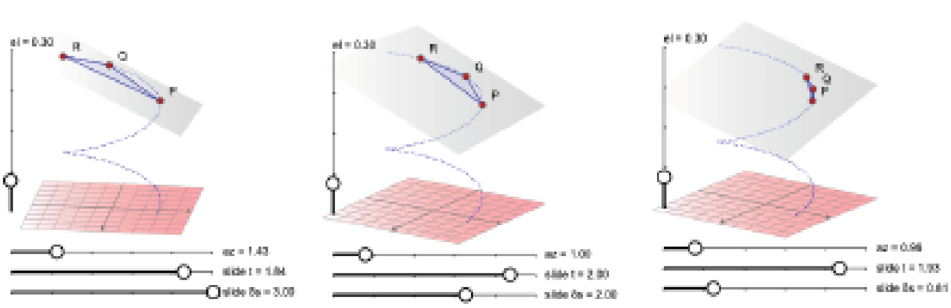

Figure 2: osculating plane is limiting position of a plane

For example, the textbook definition explicitly states that the osculating plane is the limiting position of a plane determined by PQR as $Q, R \to P$ (Koirala & Dhakal, 2024; Pundir et al., 2021). The figure directly illustrates this. Students used it and traced the points P, Q, and R on the simulation (represented by the blue line). The dotted lines from Q and R to the plane suggest their proximity to P, which aligns with the limiting process. So, that students became able to understand the concept. This is exemplified bya quotation of a student, who said that

"*The concept of a limit was abstract for me, now the visual thing, especially through the interactive-ness, it helped me to understand how Q, R tends to P, and the osculating plane is formed*".

The arrows and dashed lines indicating the "movement" or "approach" of the plane from below (or through the curve) are vital. Students observed that how as Q and R get closer to P, the plane containing P, Q, and R "settles" into a specific orientation, becoming the osculating plane. This dynamic visualization became more effective than static images for grasping the limiting concept.

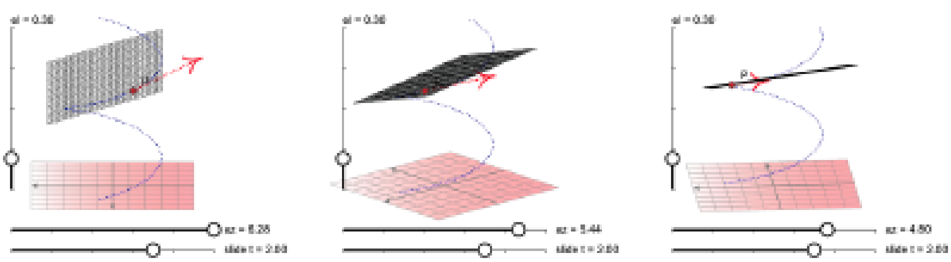

Figure 3: osculating plane contains the tangent line

In the textbook, it is mentioned that the osculating plane at P contains the tangent line at P. Using this simulation, students experienced that by tryin to identify the tangent line at P. The plane, in its limiting position, has naturally enclose this tangent, which they experienced through the simulation. This reinforces the idea that the osculating plane is the "best fit" plane to the curve at that point.

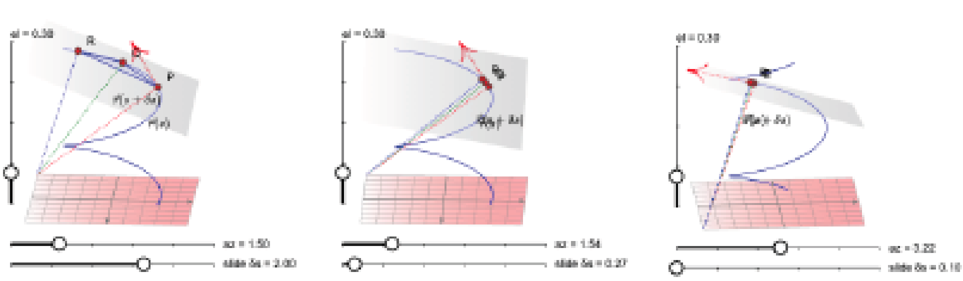

Figure 4: osculating plane has three point contact with the curve

Also, in the textbook, it is mentioned that, "osculating plane at P has three point contact with the curve at P" (Koirala & Dhakal, 2024; Pundir et al., 2021), this is they directly experienced through the visual representation of P, Q, and R. Although Q and R are distinct points in the initial setup, the limiting process implies that the osculating plane touches the curve "at least" at three infinitesimally close points (P, and the "merged" Q and R). This visual representation reinforced students to understand the three point contact of the osculating plane with the curve. This justified by the statement

"*With this 3D visualization, we get chance to rotate and view the setup of osculating plane from different angles and different orientations, which became very valuable in our understanding*".

The animation parameter helped student to experience the dynamics of osculating plane from different angles, which is crucial for spatial reasoning in 3D geometry. They observed explained that position of osculating plane "cuts" or "lies" relative to the curve in various points, as the points moves along the curve, the orientation of osculating plane may vary accordingly. This understanding helped in enhancing their understanding of the orientation of osculating plane in space. This is verified by the student's experience, as it is mentioned that

"*We had a confusion like, osculating plane is similar to the xy-plane of 3D geometry, but this specific visualization helped us to understand that osculating plane is the byproduct of a curve which is drawn on rectangular plane, the representation of P, Q, R and their limiting behavior helped us to differentiate it*".

From the student's experience, it resulted that the interactive simulation, serves as a dynamic and intuitive representation of mathematical concept. It allows them to visually grasp the concept, connect the definition to the properties of the osculating plane (for example, tangent line and three-point contact), and explore its spatial orientation, thereby enriching their learning experience.

This result also indicates that using the interactive visualization fostered a deeper, integrated understanding of the osculating plane, not just as a mathematical concept (Content Knowledge), but also in terms of how it can be effectively taught and learned using technology (Pedagogical and Technological Knowledge). Specifically, the visualization tool facilitated (a) to enhance conceptual understanding in mathematics, (b) to demonstrate learninhg schema of interplay among curve, tangent, acceleration, and the osculating plane, and (c) to understand the orientation of osculating plene relative to the curve.

Discussion

Based on the result of the data analysis, this study found that TPCK of osculating plane highlighted the transformative potential of educational technology. The interactive nature of the JavaScript tool allowed learners and tutors to transform static representations to interactive representation of mathematical contents. This enabled learners for active and interactive learning participation. This is also aligned with connectivist learning theories (Mukhlis et al., 2024), where learners build knowledge through live experience with digital tools.

The findings of this study visualized the effectiveness of AI tools to help learner to understand complex mathematical concepts. The JavaScript based dynamic visualizations were found not only as a supplementary tool, but as a central learning resource. This kinds of usefulness of digital resources are also explained in number of literatures, for example Jirasatjanukul et al, Mukhlis et al and Nesvidomin et al (Jirasatjanukul et al., 2023; Mukhlis et al., 2024; Nesvidomin et al., 2024). The integration of TPACK provided valuable insights into the learning experience, this justifies the pedagogical effectiveness of TPACK and AI use in higher mathematics. As mentioned in literature, such digital tools and AI use cam "offer a robust digital tool, like JavaScript framework in this reseach, to design pedagogical thought integration of learning activities" (Herring et al., 2014; Koehler et al., 2014).

Form the study, it is seen that, mathematical concepts of osculating plane are translated into a dynamic visual representation using JavaScript. Such "text and content visualization types of research works are exemplified by number of literatures; for example Onwu Iji and Slavickova" (Onwu Iji & Abah, 2018; Slavickova, 2021). Therefore, this research work has provided an opportunity for learners, allowing users navigate the content. For example, in this research work

'*Learner can move a point P along the curve, change the curve's parameters, and rotate the 3D view, which can provide an immersive experience to the learner on how the osculating plane changes its orientation*'.

Similar finding are also mentioned in number of research works (Becerra-Romero et al., 2019; Jirasatjanukul et al., 2023; Zykova et al., 2018). Therefore,form the study, it is seen that, abstract concept like osculating plane can be materialized, which are also the essence of number of research works. (Liu et al., 2023; Nesvidomin et al., 2024; Saglam et al., 2004). The interactive JaaScript simulation has provided an opportunity for learners, to experience the visual dynamics, as suggested by TPCK (Herring et al., 2014; Koehler et al., 2014) and connectivism (Mukhlis et al., 2024).

Being a phenomenological study, this study has explained the students experience, showing a direction for intellectual growth through digital simulations, especially when digital materials are designed with pedagogical intent, and using AT tools, as discussed by erring et al and others (Herring et al., 2014; Koehler et al., 2014). So the student were benefited from this digital materials as learning resources, such outcomes are also mentioned in research works by Dhakal & Sharma and Onwu Iji & Abah (Dhakal & Sharma, 2016; Onwu Iji & Abah, 2018). Therefore, the effectiveness of the AI tools has been exemplified in this research work.

Conclusion

This phenomenological study illustrated that thoughtfull designed digital resources does not only supplement the learning resources but helps to exceed the impact of traditional one-way traffic type of instruction. For students, use of interactive digital resources provides inclusive and equitable learning pathways to mastery intented mathematical concepts. By employing TPACK as a grounded theoritical framework, this study demonstrated that AI and JavaScript based interactive learning resources can foster a deep and meaningful understanding while mathematical concept. Therefore, future work could involve more formal experimental type studies to justify the effectiveness of AI and digital tools in higher mathematics education.

Data availability

The content of this research, for example JavaScript simulations, interactive visualizations, videos and text can be accessed at https://www.bedprasaddhakal.com.np/2024/06/osculating-plane.html. This research conducted independently by the reseaercher during the regular class at university. No funding or financial support were received. Also, researcher declares no conflicts of interest to the content of this article.

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