Blow up of Solutions for Petrovsky Equation with Delay Term

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Abstract: *This work deals with a logarithmic Petrovsky equation with delay term. Under appropriate conditions, we prove the blow up of solutions in a finite time in a bounded domain. Our results are more general than the earlier results.*

Keywords: Blow up, Delay, Logarithmic source term, Petrovsky equation

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1 Introduction

In this work, we study the logarithmic Petrovsky equation as follows:

$$\begin{cases} u_{tt} + \Delta^2 u - \Delta u_t + \mu_1 u_t(x,t) + \mu_2 u_t(x,t-\tau) = u|u|^{p-2}\ln|u|^k, & \text{in } x \in \Omega,\ t \in (0,\infty), \\ u(x,t) = \frac{\partial u(x,t)}{\partial \upsilon} = 0, & \text{on } x \in \partial\Omega,\ t \in [0,\infty), \\ u_t(x,t-\tau) = f_0(x,t-\tau), & \text{in } x \in \Omega,\ t \in (0,\tau), \\ u(x,0) = u_0(x),\ u_t(x,0) = u_1(x), & \text{in } x \in \Omega, \end{cases} \tag{1}$$

with delay term where $\Omega \subset R^n$ is a bounded domain with sufficiently smooth boundary $\partial\Omega$. $p > 2$, k, μ_1 are positive constants, μ_2 is a real number, $\tau > 0$ denotes the time delay and υ is the unit outward normal vector on $\partial\Omega$. u_0, u_1, f_0 are the initial data functions to be specified later.

Logarithmic nonlinearity generally seems in super symmetric field theories and in cosmological inflation. From Quantum Field Theory, that such kind of $(u|u|^{p-2}\ln|u|^k)$ logaritmic source term seems in nuclear physics, inflation cosmology, geophysics and optics (see [2, 8]). Problems related to the mathematical behavior of solutions for PDEs with delay term have become interesting for many authors. Time delays often appear in many practical problems such as thermal, economic phenomena, biological, chemical, physical, mechanical applications, medicine and electrical engineering systems. Moreover, delay effects may destroy the stabilizing properties of a well-behaved system. There are several examples show that how time delays destabilize some boundary or internal control system [9, 10].

For the literature review, firstly, we begin with the studies of Birula and Mycielski [3, 4]. The authors investigated the equation with logarithmic term as follows

$$u_{tt} - u_{xx} + u - \varepsilon u \ln|u|^2 = 0. \tag{2}$$

This equation is a relativistic version of logarithmic quantum mechanics. They are the pioneer of these kind of problems. In 1980, Cazenave and Haraux [5] studied the equation as follows

$$u_{tt} - \Delta u = u \ln|u|^k, \tag{3}$$

and the authors proved the existence and uniqueness for the equation (3).

In 1986, Datko et al. [7] indicated that delay is a source of instability. In [19], Nicaise and Pignotti considered the wave equation with delay term as follows

$$u_{tt} - \Delta u + \mu_1 u_t(x,t) + \mu_2 u_t(x,t-\tau) = 0. \tag{4}$$

Under the condition $0 < \mu_1 < \mu_2$, they proved the stability. In the absence of delay, some other authors [15, 27] looked into exponentially stability for the equation (4).

Messaoudi [16], studied the equation as follows

$$u_{tt} + \Delta^2 u + |u_t|^{m-2} u_t = |u|^{p-2} u. \tag{5}$$

The author obtained the existence and showed that the solution is globally if $m \geq p$, and it blows up in finite time if $m < p$ and the initial energy is negative for the equation (5). Later, Chen and Zhou [6] improved this result. (see also [12, 13]).

Liu et al. [14], studied the Petrovsky equation as follows

$$u_{tt} + \Delta^2 u - \Delta u_t + |u_t|^{p-2} u_t = |u|^{q-2} u. \tag{6}$$

The authors established the existence, decay and blow up of solutions of (6). In 2013, Pişkin and Polat [24], obtained the global existence and decay of the equation (6).

Recently, Kafini and Messaoudi [11], looked into the following logarithmic equation with delay term

$$u_{tt} - \Delta u + \mu_1 u_t (x,t) + \mu_2 u_t (x, t-\tau) = u |u|^{p-2} \ln |u|^k . \tag{7}$$

They obtained the local existence result and they established the blow up result in a finite time for the equation (7). In recent years, some other authors investigate hyperbolic type equation with or without delay term (see [1, 17, 20, 21, 22, 23, 25, 26]).

There is no research, to our best knowledge, about Petrovsky equation with delay term and logarithmic source term, therefore, our paper is generalization of the previous ones. Our goal is to get the blow up results of the Petrovsky equation with delay term ($\mu_2 u_t (x, t-\tau)$) and logarithmic source term ($u |u|^{p-2} \ln |u|^k$).

The plan of this paper is as follows: In Section 2, we give some materials which will be used later. In Section 3, we prove the blow up results in a finite time for negative initial energy in a bounded domain.

2 Preliminaries

In this part, we show the material needed for our results. We use the standard Lebesgue space $L^2(\Omega)$ and Sobolev space $H_0^2(\Omega)$ with their usual scalar products and norms. C is a positive constant throught this paper. Firstly, we introduce the new variable z similar to [18],

$$z(x,\rho,t) = u_t (x, t-\tau\rho), \ x \in \Omega, \ \rho \in (0,1), \ t > 0,$$

which implies that

$$\tau z_t (x,\rho,t) + z_\rho (x,\rho,t) = 0, \ x \in \Omega, \ \rho \in (0,1), \ t > 0.$$

Thus, problem (1) can be transformed as follows:

$$\begin{cases} u_{tt} + \Delta^2 u - \Delta u_t + \mu_1 u_t (x,t) + \mu_2 z(x,1,t) = u |u|^{p-2} \ln |u|^k , & \text{in } \Omega \times (0,\infty), \\ \tau z_t (x,\rho,t) + z_\rho (x,\rho,t) = 0, & \text{in } \Omega \times (0,1) \times (0,\infty), \\ z(x,\rho,0) = f_0 (x, -\rho\tau), & \text{in } \Omega \times (0,1), \\ u(x,t) = \frac{\partial u(x,t)}{\partial v} = 0, & \text{on } \partial\Omega \times [0,1), \\ u(x,0) = u_0(x), \ u_t(x,0) = u_1(x), & \text{in } \Omega. \end{cases} \tag{8}$$

We define the energy functional related to the problem (8):

$$\begin{aligned} E(t) &= \frac{1}{2} \|u_t\|^2 + \frac{1}{2} \|\Delta u\|^2 + \frac{k}{p^2} \|u\|_p^p - \frac{1}{p} \int_\Omega |u|^p \ln |u|^k \, dx \\ &\quad + \frac{\xi}{2} \int_\Omega \int_0^1 |z(x,\rho,t)|^2 \, d\rho dx, \end{aligned} \tag{9}$$

where

$$\tau\left|\mu_2\right|<\xi<\tau\left(2\mu_1-\left|\mu_2\right|\right) \text{ and } \mu_1>\left|\mu_2\right|. \tag{10}$$

Now, to prove our main result, we define

$$\begin{aligned} H(t) &= -E(t)=-\frac{1}{2}\left\|u_t\right\|^2-\frac{1}{2}\|\Delta u\|^2-\frac{k}{p^2}\|u\|_p^p \\ &\quad +\frac{1}{p}\int_{\Omega}|u|^p \ln|u|^k dx-\frac{\xi}{2}\int_{\Omega}\int_0^1|z(x,\rho,t)|^2 d\rho dx. \end{aligned} \tag{11}$$

We have the following lemmas to get our main result:

Lemma 1. *For $C>0$,*

$$\left(\int_{\Omega}|u|^p \ln|u|^k dx\right)^{s/p} \leq C\left[\int_{\Omega}|u|^p \ln|u|^k dx+\|\Delta u\|_2^2\right]$$

satisfies, for any $u \in H_0^2(\Omega)$ and $2 \leq s \leq p$, provided that $\int_{\Omega}|u|^p \ln|u|^k dx \geq 0$.

Proof. In [11], by Lemma 3.2 we know that

$$\left(\int_{\Omega}|u|^p \ln|u|^k dx\right)^{s/p} \leq C\left[\int_{\Omega}|u|^p \ln|u|^k dx+\|\nabla u\|_2^2\right]$$

is satisfied, by using Sobolev Embedding Theorem we get this result. □

From [11], we have the lemmas as follows:

Lemma 2. *Depending on Ω only, assume that $C>0$, such that*

$$\|u\|_2^2 \leq C\left[\left(\int_{\Omega}|u|^p \ln|u|^k dx\right)^{2/p}+\|\Delta u\|_2^{4/p}\right], \tag{12}$$

provided that $\int_{\Omega}|u|^p \ln|u|^k dx \geq 0$.

Lemma 3. *Depending on Ω only, suppose that $C>0$, such that*

$$\|u\|_p^s \leq C\left[\|u\|_p^p+\|\Delta u\|_2^2\right], \tag{13}$$

for any $u \in H_0^2(\Omega)$ and $2 \leq s \leq p$.

Lemma 4. *Suppose that (10) satisfies. Then, for $C_0 \geq 0$, we get*

$$E'(t) \leq -C_0\int_{\Omega}\left(\left|u_t\right|^2+|z(x,1,t)|^2\right) dx \leq 0. \tag{14}$$

Proof. Multiplying the first equation in (8) by u_t and integrating over Ω and using integration by parts, we have

$$\begin{aligned} & \frac{d}{dt}\left(\frac{1}{2}\left\|u_t\right\|^2+\frac{1}{2}\|\Delta u\|^2+\frac{k}{p^2}\|u\|_p^p-\frac{1}{p}\int_{\Omega}|u|^p \ln|u|^k dx\right) \\ & +\left\|\nabla u_t\right\|^2+\mu_1\left\|u_t\right\|^2+\int_{\Omega}\mu_2 z(x,1,t) u_t dx \\ = \;& 0. \end{aligned} \tag{15}$$

Multiplying the second equation in (8) by $(\xi/\tau)z$ and integrating over $\Omega\times(0,1)$, $\xi>0$, we have

$$\frac{\xi}{2}\frac{d}{dt}\int_{\Omega}\int_0^1 z^2(x,\rho,t) d\rho dx+\frac{\xi}{\tau}\int_{\Omega}\int_0^1 z(x,\rho,t) z_\rho(x,\rho,t) d\rho dx=0. \tag{16}$$

Noting that

$$\begin{aligned}
& -\frac{\xi}{\tau}\int_{\Omega}\int_{0}^{1} z\left(x,\rho,t\right) z_{\rho}\left(x,\rho,t\right) d\rho dx \\
= & -\frac{\xi}{2\tau}\int_{\Omega}\int_{0}^{1} \frac{\partial}{\partial\rho} z^{2}\left(x,\rho,t\right) d\rho dx \\
= & \frac{\xi}{2\tau}\int_{\Omega}\left(z^{2}\left(x,0,t\right)-z^{2}\left(x,1,t\right)\right) dx \\
= & \frac{\xi}{2\tau}\left(\int_{\Omega} u_{t}^{2} dx-\int_{\Omega} z^{2}\left(x,1,t\right) dx\right).
\end{aligned} \tag{17}$$

Combining (15) and (16) and taking into consideration (17), we have

$$\begin{aligned}
E'\left(t\right) = & -\left(\mu_{1}-\frac{\xi}{2\tau}\right)\int_{\Omega}\left|u_{t}\left(x,t\right)\right|^{2} dx-\frac{\xi}{2\tau}\int_{\Omega}\left|z\left(x,1,t\right)\right|^{2} dx \\
& -\left\|\nabla u_{t}\right\|^{2}-\mu_{2}\int_{\Omega} z\left(x,1,t\right) u_{t}\left(x,t\right) dx.
\end{aligned} \tag{18}$$

Utilizing Young's inequality, we obtain

$$-\mu_{2}\int_{\Omega} z\left(x,1,t\right) u_{t}\left(x,t\right) dx \leq \frac{\left|\mu_{2}\right|}{2}\int_{\Omega}\left(\left|u_{t}\left(x,t\right)\right|^{2}+\left|z\left(x,1,t\right)\right|^{2}\right) dx.$$

Thus, by (18), we get

$$\begin{aligned}
E'\left(t\right) \leq & -\left(\mu_{1}-\frac{\xi}{2\tau}-\frac{\left|\mu_{2}\right|}{2}\right)\int_{\Omega}\left|u_{t}\left(x,t\right)\right|^{2} dx \\
& -\left(\frac{\xi}{2\tau}-\frac{\left|\mu_{2}\right|}{2}\right)\int_{\Omega} z^{2}\left(x,1,t\right) dx.
\end{aligned} \tag{19}$$

From (10), for $C_0 > 0$, we obtain

$$E'\left(t\right) \leq -C_{0}\int_{\Omega}\left(u_{t}^{2}+z^{2}\left(x,1,t\right)\right) dx \leq 0.$$

□

3 Blow up of solutions

In this part, we obtain the blow up results in a finite time for the problem (8) in a bounded domain.

Theorem 5. *Suppose that (10) holds. Assume further that*

$$\begin{cases} p \geq 2, & \text{if } n \leq 4, \\ 2 \leq p \leq \frac{2(n-2)}{n-4}, & \text{if } n > 4, \end{cases}$$

and

$$E\left(0\right) < 0. \tag{20}$$

Hence, the solution of (8) blows up in finite time.

Proof. From (14), we see that

$$E\left(t\right) \leq E\left(0\right) < 0.$$

Therefore,

$$H'\left(t\right) = -E'\left(t\right) = C_{0}\int_{0}^{1}\left(u_{t}^{2}+z^{2}\left(x,1,t\right)\right) dx \geq C_{0}\int_{0}^{1} z^{2}\left(x,1,t\right) dx \geq 0 \tag{21}$$

and

$$0 < H(0) \leq H(t) \leq \frac{1}{p}\int_{\Omega} |u|^{p} \ln |u|^{k} dx. \tag{22}$$

We introduce

$$L(t) = H^{1-\alpha}(t) + \varepsilon \int_{\Omega} u u_t dx + \frac{\varepsilon}{2}\int_{\Omega} |\nabla u|^{2} dx + \frac{\mu_1 \varepsilon}{2}\int_{\Omega} u^{2} dx,\ t \geq 0,$$

where $\varepsilon > 0$ to be specified later and

$$\frac{2(p-2)}{p^{2}} < \alpha < \frac{p-2}{2p} < 1. \tag{23}$$

Using the first equation in (8), we have

$$\begin{aligned} L'(t) &= (1-\alpha) H^{-\alpha}(t) H'(t) + \varepsilon \|u_t\|^{2} + \varepsilon \int_{\Omega} u u_{tt} dx \\ &\quad + \varepsilon \int_{\Omega} \nabla u \nabla u_t dx + \varepsilon \mu_1 \int_{\Omega} u u_t dx \\ &= (1-\alpha) H^{-\alpha}(t) H'(t) + \varepsilon \|u_t\|^{2} - \varepsilon \|\Delta u\|^{2} \\ &\quad - \varepsilon \mu_2 \int_{\Omega} u z(x,1,t) dx + \varepsilon \int_{\Omega} |u|^{p} \ln |u|^{k} dx. \end{aligned} \tag{24}$$

Thanks to Young's inequality, we obtain

$$-\varepsilon \mu_2 \int_{\Omega} u z(x,1,t) dx \leq \varepsilon |\mu_2| \left(\delta \int_{\Omega} u^{2} dx + \frac{1}{4\delta}\int_{\Omega} z^{2}(x,1,t) dx \right),\ \forall \delta > 0. \tag{25}$$

Hence, by (24), we get

$$\begin{aligned} L'(t) &\geq \left[(1-\alpha) H^{-\alpha}(t) - \frac{\varepsilon |\mu_2|}{4\delta C_0} \right] H'(t) + \varepsilon \|u_t\|^{2} - \varepsilon \|\Delta u\|^{2} \\ &\quad + \varepsilon \int_{\Omega} |u|^{p} \ln |u|^{k} dx - \varepsilon \delta |\mu_2| \|u\|^{2}. \end{aligned} \tag{26}$$

By taking δ so that $|\mu_2| / 4\delta C_0 = \kappa H^{-\alpha}(t)$, for large κ to be specified later and substituting in (26), we obtain

$$\begin{aligned} L'(t) &\geq [(1-\alpha) - \varepsilon \kappa] H^{-\alpha}(t) H'(t) + \varepsilon \|u_t\|^{2} - \varepsilon \|\Delta u\|^{2} \\ &\quad - \frac{\varepsilon |\mu_2|^{2}}{4\kappa C_0} H^{\alpha}(t) \|u\|^{2} + \varepsilon \int_{\Omega} |u|^{p} \ln |u|^{k} dx. \end{aligned}$$

Thus, we get

$$\begin{aligned} L'(t) &\geq [(1-\alpha) - \varepsilon \kappa] H^{-\alpha}(t) H'(t) + \varepsilon a \int_{\Omega} |u|^{p} \ln |u|^{k} dx + \varepsilon \frac{p(1-a)+2}{2} \|u_t\|^{2} \\ &\quad + \varepsilon \left(\frac{p(1-a)}{2} - 1 \right) \|\Delta u\|^{2} + \frac{\varepsilon (1-a) k}{p} \|u\|_{p}^{p} - \frac{\varepsilon |\mu_2|^{2}}{4\kappa C_0} H^{\alpha}(t) \|u\|^{2} \\ &\quad + \varepsilon p (1-a) H(t) + \frac{\varepsilon (1-a) p \xi}{2} \int_{\Omega} \int_{0}^{1} z^{2}(x,\rho,t) d\rho dx, \end{aligned} \tag{27}$$

for $0 < a < 1$.

Thanks to (12) and (22), we obtain

$$\begin{aligned} H^{\alpha}(t) \|u\|_{2}^{2} &\leq \left(\int_{\Omega} |u|^{p} \ln |u|^{k} dx \right)^{\alpha} \|u\|_{2}^{2} \\ &\leq \left[\left(\int_{\Omega} |u|^{p} \ln |u|^{k} dx \right)^{\alpha + 2/p} + \left(\int_{\Omega} |u|^{p} \ln |u|^{k} dx \right)^{\alpha} \|\Delta u\|_{2}^{4/p} \right]. \end{aligned}$$

Utilizing Young's inequality, we obtain

$$\begin{aligned} H^{\alpha}(t)\|u\|_2^2 &\leq \left(\int_\Omega |u|^p \ln|u|^k\,dx\right)^{\alpha}\|u\|_2^2 \\ &\leq \left[\begin{array}{c} \left(\int_\Omega |u|^p \ln|u|^k\,dx\right)^{(p\alpha+2)/p} \\ +\frac{2}{p}\|\Delta u\|^2+\frac{p-2}{p}\left(\int_\Omega |u|^p \ln|u|^k\,dx\right)^{\alpha p/(p-2)} \end{array}\right]. \end{aligned}$$

Therefore, we have

$$\begin{aligned} H^{\alpha}(t)\|u\|_2^2 &\leq \left(\int_\Omega |u|^p \ln|u|^k\,dx\right)^{\alpha}\|u\|_2^2 \\ &\leq C\left[\begin{array}{c} \left(\int_\Omega |u|^p \ln|u|^k\,dx\right)^{(p\alpha+2)/p}+\|\Delta u\|^2 \\ +\left(\int_\Omega |u|^p \ln|u|^k\,dx\right)^{\alpha p/(p-2)} \end{array}\right], \end{aligned}$$

where $C=\max\left\{\frac{2}{p},\ \frac{p-2}{p}\right\}$.

Exploiting (23), we have

$$2<\alpha p+2\leq p \text{ and } 2<\frac{\alpha p^2}{p-2}\leq p.$$

Therefore, lemma 1 satisfies

$$H^{\alpha}(t)\|u\|_2^2\leq C\left(\int_\Omega |u|^p\ln|u|^k\,dx+\|\Delta u\|_2^2\right). \tag{28}$$

Combining (27) and (28), we get

$$\begin{aligned} L'(t) \geq\ & [(1-\alpha)-\varepsilon\kappa]\,H^{-\alpha}(t)\,H'(t)+\varepsilon\left(a-\frac{C|\mu_2|^2}{4\kappa C_0}\right)\int_\Omega |u|^p\ln|u|^k\,dx \\ &+\varepsilon\left(\frac{p(1-a)-2}{2}-\frac{C|\mu_2|^2}{4\kappa C_0}\right)\|\Delta u\|^2+\frac{\varepsilon(1-a)k}{p}\|u\|_p^p \\ &+\varepsilon\frac{p(1-a)+2}{2}\|u_t\|^2+\varepsilon p(1-a)H(t) \\ &+\frac{\varepsilon(1-a)p\xi}{2}\int_\Omega\int_0^1 z^2(x,\rho,t)\,d\rho dx. \end{aligned} \tag{29}$$

Since, choosing $a>0$ so small, such that

$$\frac{p(1-a)-2}{2}>0,$$

and choosing κ large enough, we have

$$\begin{cases} \frac{p(1-a)-2}{2}-\frac{C|\mu_2|^2}{4\kappa C_0}>0\ , \\ a-\frac{C|\mu_2|^2}{4\kappa C_0}>0. \end{cases}$$

Once κ and a are fixed, picking ε so small, such that

$$(1-\alpha)-\varepsilon\kappa>0,$$

$$H(0)+\varepsilon\int_\Omega u_0u_1dx>0.$$

Therefore, for some $\lambda > 0$, estimate (29) becomes

$$\begin{aligned} L'(t) &\geq \lambda \left[H(t) + \|u_t\|^2 + \|\Delta u\|^2 + \|u\|_p^p \right] \\ &\quad + \lambda \left[\int_\Omega \int_0^1 z^2(x,\rho,t)\, d\rho dx + \int_\Omega |u|^p \ln |u|^k\, dx \right], \end{aligned} \tag{30}$$

and

$$L(t) \geq L(0) > 0, \ t \geq 0. \tag{31}$$

Thanks to the embedding $\|u\|_2 \leq C\|u\|_p$ and Hölder's inequality, we obtain

$$\int_\Omega u u_t dx \leq \|u\|_2 \|u_t\|_2 \leq C\|u\|_p \|u_t\|_2,$$

then exploiting Young's inequality, we get

$$\left| \int_\Omega u u_t dx \right|^{1/(1-\alpha)} \leq C\left(\|u\|_p^{\mu/(1-\alpha)} + \|u_t\|_2^{\theta/(1-\alpha)} \right), \text{ for } 1/\mu + 1/\theta = 1. \tag{32}$$

By Lemma 3, we take $\theta = 2(1-\alpha)$ which gives $\mu/(1-\alpha) = 2/(1-2\alpha) \leq p$. Hence, for $s = 2/(1-2\alpha)$, estimate (32) satisfies

$$\left| \int_\Omega u u_t dx \right|^{1/(1-\alpha)} \leq C\left(\|u\|_p^s + \|u_t\|_2^2 \right).$$

Hence, Lemma 3 provides

$$\left| \int_\Omega u u_t dx \right|^{1/(1-\alpha)} \leq C\left[\|\Delta u\|^2 + \|u_t\|^2 + \|u\|_p^p \right]. \tag{33}$$

Therefore,

$$\begin{aligned} L^{1/(1-\alpha)}(t) &= \left(H^{1-\alpha}(t) + \varepsilon \int_\Omega u u_t dx + \frac{\varepsilon}{2} \int_\Omega |\nabla u|^2 + \frac{\mu_1 \varepsilon}{2} \int_\Omega u^2 dx \right)^{1/(1-\alpha)} \\ &\leq C\left[H(t) + \left| \int_\Omega u u_t dx \right|^{1/(1-\alpha)} + \left| \int_\Omega |\nabla u|^2 dx \right|^{1/(1-\alpha)} + \|u\|_2^{2/(1-\alpha)} \right] \\ &\leq C\left[H(t) + \left| \int_\Omega u u_t dx \right|^{1/(1-\alpha)} + \left| \int_\Omega |\nabla u|^2 dx \right|^{1/(1-\alpha)} + \|u\|_p^{2/(1-\alpha)} \right] \\ &\leq C\left[H(t) + \|\Delta u\|^2 + \|u_t\|^2 + \|u\|_p^p \right], \ t \geq 0. \end{aligned} \tag{34}$$

Combining (30) and (34), we obtain

$$L'(t) \geq \Lambda L^{1/(1-\alpha)}(t), \ t \geq 0, \tag{35}$$

where Λ is a positive constant depending only on λ and C. An integration of (35) over $(0,t)$ yields

$$L^{\alpha/(1-\alpha)}(t) \geq \frac{1}{L^{-\alpha/(1-\alpha)}(0) - \Lambda \alpha t/(1-\alpha)}.$$

Hence, $L(t)$ blows up in time T^*

$$T \leq T^* = \frac{1-\alpha}{\Lambda \alpha L^{\alpha/(1-\alpha)}(0)}.$$

Consequently, the proof is completed. $\square$

4 Conclusions

In recent years, there has been published much work concerning the wave equation with constant delay or time-varying delay. However, to the best of our knowledge, there was no blow up of solutions for the logarithmic Petrovsky equation with delay term. Under suitable conditions, we have been obtained the blow up results in a finite time in a bounded domain.

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