Journal of Nepal Mathematical Society (JNMS)
ISSN: 2616-0153 (Print), 2616-0161 (Online)
Vol. 8, Issue 2, 2025 (Dec): 16-22
DOI: https://doi.org/10.3126/jnms.v8i2.87705

Research Article
Received Date: Oct. 21, 2025
Accepted Date: Dec. 07, 2025
Published Date: Dec. 29, 2025

Division in Vedic Mathematics

Krishna Kanta Parajuli

Department of Mathematics, Valmeeki Campus, Nepal Sanskrit University
Correspondence to: kknmparajuli@gmail.com

Abstract: *Vedic Mathematics, derived from ancient Hindu scriptures known as the Vedas, offers a unique and efficient approach to arithmetic, algebraic and trigonometric calculations. Among its 16 foundational sutras and 13 sub-sutras, division is addressed with remarkable simplicity and speed. Out of them "Nikhilam", "Urdhva Tiryagbhyam (Dhwajanka)" and "Paravartya" Sutras are particularly effective in performing division operations. These methods transform complex problems of division into manageable mental calculations, often eliminating the need for traditional long division techniques. The Vedic approach not only enhances computational speed but also promotes mental flexibility and problem-solving skills. Through some practical applications, this article demonstrates how Vedic Mathematics can simplify division, offering an alternative method that is valuable for complex computations. Its integration into modern education could revolutionize teaching methodologies, making mathematics more accessible and enjoyable for students.*

Keywords: Vedic Mathematics, Division, Nikhilam, Paravartya, Urdhva Tiryagbhyam

1 Introduction

The development of knowledge, civilization, and education from ancient times to the modern era is believed to have its primordial source in the Hindu's primary religious text, the Vedas. The renowned scholar (Rishi) Manu has said about the Vedas

"चातुर्वर्ण्यं त्रयो लोकाश्चत्वारश्च आश्रमाः पृथक् । भूतं भव्यं भविष्यं च सर्वं वेदात् प्रसिध्यति॥"

Meaning: The four varnas (social orders), the three worlds, the four stages of life-past, present, and future, everything is revealed and made known through the Vedas.

Mathematics, too, is not separate from this civilization, and its development appears to have occurred alongside the origin of the Vedas. If we look into the history of the word Mathematics, it is evident that its development originated in South Asia, a region stretching from sea level to the Himalayas. In Bishnupuran, the south Asian region is indicated in Sanskrit language as Bharatavarsha. The Vishnu Purana describes the extent of Bharatavarsha (ancient Bharat) with the following Sanskrit shloka as

"उत्तरं यत्समुद्रस्य हिमाद्रेश्चैव दक्षिणम् । वर्षं तद् भारतं नाम भारती यत्र सन्ततिः ॥"

Meaning: The land that lies to the north of the ocean and to the south of the Himalayas is known as Bharat, where the descendants of Bharat reside. In ancient times, the term Bharat referred to the region that is now known as South Asia. This area, rich in cultural and intellectual heritage, was shaped significantly by the Rishis and Sages who contributed to the evolution of civilization. The development of every civilization in this region appears to be guided by the Vedas, the ancient texts that form the foundation of knowledge and wisdom [13]. Ancient Egyptians including their use of repeated subtraction and Egyptian division methods [6]. Euclid introduces the Euclidean Algorithm for finding the GCD using a process of repeated division with remainders [8].The Bakhshali Manuscript, the book Aryabhatiya of Aryabhatya and commentaries of Bhaskara-I provided an early algorithmic approach to arithmetic, including long division, also demonstrates similar arithmetic techniques [7]. In Lilavati, Bhaskaracharya II articulated his concept of division in Sanskrit as follows [12] .

"भाज्याद्धरःशुद्धयवत यद्गुणःस्यादन्तत्यात्फलं तत्खलु भागहारे । समेन केनाप्यपनीय हारे भाज्ये भवेद्या सविता सम्भवे तु ॥"

Meaning: First, find the largest integer that you can multiply with the divisor and subtract from the leftmost digit or digits of the dividend. This integer becomes the first digit of the quotient. If the divisor and the dividend have a common factor, you can cancel that common factor. Then you perform the division using the remaining factors.

The technique of Lilavati is remarkably similar to modern long division with demonstrating the systematic approach of identifying the largest multiple, subtracting, and bringing down the next digit. Overall, a variety of cultures and historical periods contributed to the development of the division technique, rather than a single individual. Hindu mathematician like Bhaskaracharya-II is most closely linked to the modern long division method as we know it today [9].

Since the time of the Indus Valley Civilization, society has gone through many changes, and along with this, various branches of mathematics have also developed. Many mathematicians emerged during this time and made significant contributions to mathematics, such as Aryabhatta, Bhaskara-I, Varahamihira, and Bhaskaracharya-II in the 12th century [5]. Although mathematical techniques evolved over time, the method of division remained mostly unchanged. However, in the 19th century, the great mathematician Bharati Krishna Tirthaji introduced a very simple and easy method for mathematical calculations using Vedic Sutras across many mathematical fields. This article aims to present his division techniques.

Since the conventional method for basic level mathematical calculations of arithmetical division is already familiar to both teachers and students. Being commonly practiced, there is no need to present it in detail in this paper. However, among basic arithmetical operations, the division of large numbers is often found to be difficult, time-consuming, and mentally stressful for both students and teachers when using the conventional method. Nevertheless, due to the lack of alternatives, students are compelled to use it. In this context, this paper aims to present Bharati Krishna Tirthaji s division method as a possible alternative that can make long division simpler and more efficient.

Vedic Mathematics, propounded by Bharati Krishna Tirthaji, presents a simplified and systematic approach to arithmetic operations, including division, through Vedic Sutras (mathematical formulas) out of his 16 sutras. The division method in Vedic Mathematics by Tirthaji primarily utilizes the "Nikhilam Sutra" (All from 9 and the last from 10) and the "Paravartya Yojayet" (Transpose and Apply) for short division and long division techniques. Paravartya Yojayet is related as + to - , × to ÷ , left to right, numerator to denominator and vice-versa etc.

2 Division by Tirthaji's Sutra

Mainly, Tirthaji has used three methods for division, namely, Nikhilam, Paravartya Yojayata, and Dwajanka. Nikhilam and Paravartya Yojayata can only be used for specific cases, while Dwajanka is general which can be used for dividing even large numbers of complex computations. This article focuses on illustrating these methods.

2.1 Division By Nikhilam Sutra

The Nikhilam Sutra is useful when the divisor is close to power of 10 but less than that of it. Using Nikhilam sutra we can easily divide when the divisors are like 8, 9, 94, 97, 993, 996, 9987, 98995 etc. It should also be noted that, in the process of division by Nikhilam methods, there is very important of zeros in the complement of divisor in the divisor column. The complement of 9 is 1, the complement of 99 is 01, the complement of 999 is 001 being the bases 10, 100, 1000 respectively. Therefore, it should not forget to write the complement of divisor from the base in the divisor column.

Methods: To apply this, we draw a slash in the dividend starting from the right, separating it into two parts: the right part is called the remainder block, and the left part is the quotient block. We then carry down the first digit of the quotient block to start the quotient. This digit is multiplied by the new divisor, and the product is written under the next digit of the dividend. At each step, we add the digits in the column to find the next digit of the quotient. This process is repeated until all digits have been processed, leaving a number in the remainder section. If this remainder exceeds the original divisor, we divide it once more by the original divisor, and add the new quotient to the existing quotient. The remainder from this final step becomes the final remainder, completing the division [4, 14, 15].
When 1221340 is divided by 8987.

Divisor Column	Quotient Column			Remainder Column			
8987	1	2	2	1	3	4	0
Complement = 1013		1	0	1	3		
(1000 - 8987 = 1013)			3	0	3	9	
				5	0	6	5
	1	3	5	8	0	9	5

Here, quotient = 135 and remainder = 8095.

2.2 Division by Parāvartya Yojayet Sutra

Paravartya means by transposing or transforming, Yojayet means connect or join or apply. Therefore, the literal meaning of the formula Paravartya Yojayet is "Transpose and Apply" [15]. Here the word transpose indicates the operation of change of the signs or operations like from plus to minus or multiplication to division and conversely. It also indicates the interchange of functions and their inverse functions [11].

The format and working rule of Nikhilam and Paravartya methods are same. However, there is a slight difference between Nikhilam and Paravartya sutra for division. Paravartya sutra works effectively when the first digit of the divisor is 1. We will follow almost the same methods in Paravartya as in the Nikhilam method. In the Nikhilam method, we study division by numbers that are less than the base. In the Paravartya method, we learn division by numbers that are greater than the base. In the Nikhilam method, when the divisor is less than the base, we take its complement. This new divisor becomes a positive number. In the Paravartya method, the divisor is greater than the base and still find its complement, but this complement becomes negative. Therefore, our new divisor is negative. Because of this, we do not add the digits to get the next quotient digit; instead, we subtract them. This is because the multiplication of the quotient digit with the negative complement gives a negative value. In Paravartya Sutra, first we find the complement from the base as done in Nikhilam method. This complement will be revised by Paravartya sutra by writing each digit of the complement with a changed sign separately. For example, if the divisor is 112, the nearest base is 100, excess 12 and revised complement = -1 -2. This revised complement will now be the basis of division, (i.e. the excess of the divisor is made the new divisor with a negative sign) [1,3,18].

It should be noted that if any digit in the remainder is negative, we must calculate the actual remainder again. If the remainder becomes positive, it is our final remainder. But if the remainder is still negative, we use another step. We subtract 1 from the quotient to get the final quotient. Then we subtract the negative remainder from the divisor to get the final remainder.
To clarify more, when 25985 is divided by 123.

Divisor Column	Quotient Column			Remainder Column	
1 2 3	2	5	9	8	5
Complement = 23		-4	-6		
Revised complement			-2	-3	
= -2 −3				-2	-3
	2	1	1	3	2

Here, Quotient = 211 and Remainder = 32.
When 1221340 is divided by 12132.

Divisor Column	Quotient Column			Remainder Column			
1 2 1 3 2	1	2	2	1	3	4	0
Complement = 2131		-2	-1	-3	- 2		
Revised complement			-0	-0	-0	-0	
= -2 -1 -3 -2				-2	-1	-3	-2
	1	0	1	-4	0	1	-2

Remainder would be = -4000 + 000 + 10 − 2 = -3992 (is negative).
So, actual remainder = 12132 − 3992 = 8140 then quotient = 101 − 1 = 100.
Hence, Q = 100 and R = 8140.
When 104135 is divided by 1112.

Divisor Column	Quotient Column			Remainder Column		
1112	1	0	4	1	3	5
Complement = 112		-1	-1	-2		
Revised complement			1	1	2	
= -1 -1 -2				-4	-4	-8
	1	-1	4	-4	1	-3

Remainder would be -400 + 10 − 3 = -393 (is negative).
So, actual remainder = 1112 − 393 = 719 and quotient =100 − 10 +4 =94.
Hence, actual quotient = 94 − 1 = 93 and Remainder = 719.

2.3 Division by Urdhva Tiryagbhyam

The meaning of Urdhva Tiryagbhyam is vertically and crosswise. Some Vedic mathematician interduced the Dhwajank as the subsutra of Urdhva Tiryagbhyam. Any way they are co-related to each other. So, the division method here we are going to study now is a mix of Vedic sutra Urdhva Tiryagbhyam and subsutra Dhwajank. This mix-up Vedic sutra is also known as straight division because we get the answer as a quotient and remainder in one line. Division is basically the reverse of multiplication. In the normal method of division, we first guess a quotient digit. Then we multiply this digit by the divisor and find the remainder. If the remainder is negative or if it shows that more multiples of the divisor are still included, we must change the quotient digit. When the divisor has more digits, this whole process becomes slow and takes more time [10, 15].

2.3.1 Structural format of Division in Dhwajanka Method

In the process of division by Dhwajank method, it should be formed a structural format, where there are three columns. In the first column, we put the divisor (base and its Dhwajank), in the second; we put quotient below the dividend and in the third column we put remainder. If we have to divide the number abcdef (6 digits number) by pqr (3 digits number) then we should form the structural format as [13, 15, 16]

1^{st} Column	2^{nd} Column				3^{rd} Column	
p^{qr}	a	b	c	d	e	f
	Quotient part				Remainder	

Here, our new divisor is p and q r is called the Dhwajank (i.e., flag digit).
When 523674 is divided by 63 with a flag of one digit.

Division Column(I)	Quotient Column(II)					Remainder Column(III)
6^3	5	2	${}_4 3$	${}_1 6$	${}_1 7$	${}_2 4$
	8		3	1	2	18

Process: To divide 523674 by 63 using the flag method, first take 6 as the main divisor and 3 as the flag digit. Since there is one flag digit in the divisor, only the last digit will belong to the remainder column. Begin by dividing 52 by 6, which gives the first quotient ($Q_1 = 8$) and remainder ($R_1 = 4$). This remainder 4 is placed before the flag digit 3, forming a new gross dividend (GD) of 43. The net dividend (ND) is then calculated as 43 - (8 times 3) = 19. Next, divide 19 by 6 to obtain ($Q_2 = 3$) and ($R_2 = 1$). Place this remainder 1 before 6 to form a new GD of 16, and compute the ND as 16 − (3 times 3) = 7. Dividing 7 by 6 gives ($Q_3 = 1$) and ($R_3 = 1$). This remainder is placed before 7 to obtain a new GD of 17, and the ND becomes 17 - (1 times 3) = 14. Dividing 14 by 6 yields ($Q_4 = 2$) and ($R_4 = 2$). The process then moves into the remainder section. Place the remainder 2 before 4 to get a GD of 24, and compute the final ND as 24 - (2 times 3) = 18. With this, the division process is complete. Therefore, when 523674 is divided by 63, the quotient is 8312 and the remainder is 18.

When there are two flagged digits [10, 16, 18]: In the first stage, the product of the first flagged digit and the first quotient digit is subtracted from the gross dividend (GD) to obtain the net dividend (ND). In the second stage, the cross product of the two flagged digits and the last two quotient digits is subtracted from the GD to determine the ND. In the third stage, this process continues, where after completing stage II, the subtraction of the cross product of the two flag digits and the last two quotient digits from the GD is repeatedly carried out to get the subsequent net dividends. For more clarity, When 2329989 is divided by 514 with a flag of 2 digits.

Column(I)	Column(II)					Column(III)	
5^{14}	2	3	$_{3}2$	$_{3}9$	$_{3}9$	$_{1}8$	9
		4	5	3	3	27	

Process: First taking 5 as the main or base divisor and 14 as the flag digit, which means the remainder column will contain two digits. We consider the first two digits 23 and dividing 23 by 5 gives the first quotient ($Q_1 = 4$) and remainder ($R_1 = 3$). This remainder 3 is placed before the third digit of the dividend, forming a new gross dividend (GD) of 32. The net dividend (ND) is then calculated as 32 - (4 times 1) = 28. Dividing 28 by 5 yields ($Q_2 = 5$) and ($R_2 = 3$). This remainder 3 is placed before the fourth digit of the dividend, making the GD equal to 39. The ND is obtained by subtracting the sum of the cross products of quotient digits 45 and flag digits 14, giving 39 - (1 times 5 + 4 times 4) = 18. Next, dividing 18 by 5 gives ($Q_3 = 3$) and ($R_3 = 3$). The new GD becomes 39 and the ND is computed by subtracting the sum of the cross products of quotient digits 53 and flag digits 14, resulting in 39 - (5 times 4 + 1 times 3) = 16. Dividing 16 by 5 produces ($Q_4 = 3$) and ($R_4 = 1$). Now our calculation has moved in remainder part. Here, GD = 189 and ND = GD − (Sum of cross product of the quotient 33 and flag digit 14) $\times$ 10 − (last flag digit $\times$ last quotient) = 189 − (4 $\times$ 3 + 3$\times$1) $\times$ 10 - (4 $\times$ 3) = 189 -162 = 27. We stop the process. Hence, Quotient (Q) = 4533 and remainder (R) = 27.

When there are three flagged digits [10, 16, 18]: First, subtract the product of the first flagged digit and the first quotient digit from the Gross Dividend (GD) to obtain the Net Dividend (ND). Then, subtract the cross-product of the first two flagged digits and the first two quotient digits from the GD to get the ND. After that, subtract the cross-product of the three flagged digits and the three quotient digits from the GD to obtain the ND. Finally, continue the subtraction of the cross-product of the three flagged digits and the three quotient digits in the same manner to get the updated ND. It should be noted that, if a decimal representation of the quotient (answer) is desired, the division process continues in the same manner without any fundamental change in the procedure.

3 A Qualitative Inquiry of the Formulae

Out of the mentioned explanation, this qualitative inquiry wants to explore the mathematical dimensions of Vedic formulas Nikhilam, Paravartya Yojayeta, and Urdhva Triyagbhyam. This exploration aims to understand these ancient techniques by analyzing their conceptual foundations and practical applications in Vedic thought together with how they may inform contemporary mathematical strategies.

3.1 The Special Case of Complement in Nikhilam Method

Nikhilam method itself a special case of division in Vedic mathematics propounded by Bharati Krishna Tirthaji. Among them, complement in the division process by Nikhilam method is again a special case under it, i.e., division of any number by 9. Using the process of this special method, the result will be obtained as [10]: the first digit of the quotient can be obtained by taking the first digit of the dividend. The second digit of the quotient can be obtained by adding the first two digits of the dividend. The third digit of the quotient can be obtained by adding the first, second and third digit of the dividend and so on. The remainder can be obtained by adding all the digits of the dividend. If the remainder is 9 add 1 to the quotient and change the remainder to 0. If it is greater than 9 then repeat the process.
For more clarity, when 51342 is divided by 9. Here, first digit of quotient = 5; Second digit of quotient = 5 + 1 = 6; Third digit of quotient = 5 + 1 + 3 = 9; Fourth digit of the quotient = 5 + 1 + 3 + 4 = 13 (In the quotient part, we put down the unit digit and carry over the other digits to the previous digit of the quotient). Thus, the quotient = 5693, i.e., 5703 and remainder = 5 + 1 + 3 + 4 + 2 = 15 (The number is greater than 9. When we divide 15 by 9, the quotient is 1 and the remainder is 6). Hence, quotient becomes 5703 + 1 = 5704 and remainder becomes 6.

3.2 Use of Paravartya Yojayeta

Vedic formula Paravartya Yojayeta is used for both arithmetic as well as algebraic division. It is also used to solve simple linear equations, quadratic equations, cubic equations, partial fractions etc. Paravartya-Yojayet sutra has very close relation with the Remainder Theorem and Horner process of synthetic division in the aspect of algebraic division [11]. Nikhilam method is useful for particular cases where the divisor-digits are big ones but it is difficult to use when the divisor consists of small digits. To cover these cases, Paravartya Yojayet is useful. In this formula, the rule, transposition related as + to - , × to ÷ , left to right, numerator to denominator and vice-versa etc. in the related concerned problems. Therefore, Paravartya Yojayeta indicates the interchange of functions and its inverse functions [2,10,17]. Nikhilam and Paravartya sutra both are used to special cases of numbers for division. In Nikhilam, we studied the division by the numbers less than the base but in Paravartya yojayeta more than the base. There is no subtraction process to be carried out at all in Nikhilam whereas in Paravartya Yojayeta, if any digit in the remainder and quotient in the division process is occurred negative, the actual results would be readjusted. It is difficult to use for every type of number (i.e., not use for general cases).

3.3 Cross Checking Tool in Vedic Mathematics

Most of the errors in mathematical calculations arise when fundamental operations are operated incorrectly. Among them, division and multiplication are most error-prone areas. To check the error in such fundamental operations there is not any cross-checking system (mentally fast) in conventional method of mathematics besides Vedic. Vedic mathematics has developed a tool of basic rule for cross checking the faults of fundamental operations, popularly known as Navashesh. The meaning of Navashesh is casting out nine [8]. Therefore, the Navashesh method will undoubtedly boost the level of confidence. In the process, we take sum of the digits of numbers which is also known as Bijanka in Sanskrit. Bijanka is found by adding all the digits of a number. If the result is a two-digit numbers then add the digits again to find the Bijanka. A digit sum of 9 is the same as a digit sum of zero. For example: Bijanka of 487935 is $4 + 8 + 7 + 9 + 3 + 5 = 36$. Since, 36 is a double-digit number, to get single digit, we have to add it again as $3 + 6 = 9$, which is taken as zero. Bijanka of 1927318697 is $1 + 9 + 2 + 7 + 3 + 1 + 8 + 6 + 9 + 7 = 44$. Again (being double digit), $4 + 4 = 8$.
It is universal that, Dividend = Divisor × Quotient + Remainder.
When $2329989 \div 514$, we have, Dividend = 2329989, Quotient = 4533, Remainder = 27. Bijanka of dividend 2329989 is 6, Bijanka of quotient 4533 is 6 and Bijanka of remainder is 0. Here, Bijanka of dividend = Bijanka of Divisor × Bijanka of Quotient + Bijanka of Remainder. Hence the operation is correct.

4 Conclusion

The Vedic tradition was primarily oral, and there is no concrete evidence indicating exactly when the Vedas were written or when the Vedic period began. However, it is widely believed that the Vedic era emerged around 2000 to 3000 BC. Historical accounts suggest that South Asian Region served as a significant center of knowledge during that time. In particular, mathematical concepts and findings were often expressed in the form of brief, generalized statements called “Sutras,” which could be applied universally. In the 19th century, the scholar Bharati Krishna Tirthaji studied ancient mathematics carefully for many years. After this long research, he reconstructed a mathematical system based on sixteen sutras and several sub-sutras. He stated that these sutras were taken from the Vedas.

Among these, the sutras Nikhilam, Paravartya Yojayet, and Urdhva Tiryagbhyam are particularly significant. When applied correctly, these techniques make arithmetic division extremely simple, engaging, and efficient. While the first two sutras are useful only in special cases of division, the Urdhva Tiryagbhyam sutra can be applied to any type of division process. If these techniques are integrated into the modern mathematics curriculum and practiced effectively, they can provide simple and efficient solutions to even the most complex problems. However, the real beauty and usefulness of this method can only be understood by practicing it.

References

[1] Bathiya, D., 2010, Vedic Mathematics (Made Easy), *Jaico Publishing House*, New Delhi, India.

[2] Bose, S., 2014, Vedic Mathematics, *V and S Publishers*, New Delhi.

[3] Chauthaiwale, S. M., and Kolluru, R., 2010, Enjoy Vedic Mathematics, *Sri Sri Publication*, Banglore.

[4] Chauthaiwale, S. M., and Kolluru, R., 2010, Enjoy Vedic Mathematics, *Sri Sri Publication Trust*, Art of Living, Banglore.

[5] Datta, B., and Singh, A. N., 1962, History of Hindu Mathematics: A source Book, Calcutta: *Asia Publication House*, Calcutta.

[6] Gillings, R. J., 1972, Mathematics in the Time of the Pharaohs, *MIT Press*.

[7] Hayashi, T., 1995, The Bakhshali Manuscript: An Ancient Indian Mathematical Treatise, *Groningen Oriental Studies*.

[8] Heath, T. T. L., 1956, Euclid Elements, *Dover Publications*.

[9] Katz, V. J., 1998, A History of Mathematics: An Introduction, *Addison-Wesley*.

[10] Nicholas, A. P., Williams, K., and Pickles, J. , 2010, Vertically and Crosswise, Scotland, *UK: Inspiration Books*.

[11] K. K. Parajuli, K. Jha, S. Acharya, and S. Maskey, 2019 *Connection of Paravartya Sutra with Vedic and Non-Vedic Mathematics*, in Proceeding of 4th International Vedic Mathematics Conference, United Kingdom.

[12] Patwardhan, S. K., Naimpally, A. S., and Singh, L. S., 2015, Lilavati of Bhaskaracharya, *Motilal Banarasidass*, Delhi.

[13] Singh, U., 2009, The History of Ancient and Early Medieval India (From the stone Age to 12th Century), *Pearson India Education*, Delhi.

[14] Singhal, V., 2014, Vedic Mathematics for all Ages, *Motilal Banarasidass*, New delhi.

[15] Tirthaji, B. K., 2015, Vedic Mathematics, V. S. Agrawala, Ed., *Motilal Banarasidass Pvt Ltd*, New Delhi.

[16] Williams, K., 2013, The Crowning Gem, *UK: Inspiration Books*, Scotland.

[17] Williams, K., 2009, The Natural Calculator, *UK: Inspiration Books*, Scotland.

[18] Williams, K., 2019, Discover Vedic Mathematics, *Scotland: Inspiration Books*, UK.