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Bayesian Framework of Exponentiated Odd Lomax Exponential Distribution: Application to Student Satisfaction and Academic Achievement Data

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Abstract: *Bayesian inference updates prior beliefs with new evidence providing a mathematically rigorous, quantified measure of uncertainty. The aim of this study is to derive the Bayesian framework for the EOLE distribution using a Gamma prior. It subsequently derives a Bayesian regression model for the EOLE distribution by utilizing Gamma and Normal distributions as priors.*

The proposed model was empirically validated through a dataset of student satisfaction and academic achievement. Data from 628 respondents were collected via random sampling across 32 campuses at eight universities, utilizing a survey technique. Parameters of the proposed distribution were estimated using the MLE technique under the classical framework and the MCMC technique under the Bayesian framework, along with its regression model. For Bayesian inference, diagnostic tests of MCMC chains included trace plots, ergodic mean plots, autocorrelation, and prior and posterior predictive checks under graphical assessment, as well as effective sample size and $\hat{R}$ under numerical assessment. The estimated parameters from the Bayesian model and its regression model were consistent with the MLE technique. Hence, the Bayesian model of EOLE distribution is an alternative model for the analysis of educational data.

Keywords: Bayesian modelling, Exponentiated odd Lomax exponential, Markov chain Monte Carlo, Academic achievement, Students satisfaction

1 Introduction

Probability distributions are mathematical tools that help us to measure uncertainty in data across diverse disciplines, including engineering, social sciences, and reliability analysis. Because data structures are becoming more complex and diverse in the modern era, researchers are continuously involved in developing and creating the novel probability distributions to assess uncertainty in data. These new distributions are crucial to enhance the precision of empirical models, which form the foundation for subsequent statistical analyses and inference [22, 26]. A distribution that extends or modifies an existing model by introducing additional parameters is referred to as a generalized distribution. A common feature of these generalized distributions is the inclusion of one or more parameters, especially either shape, scale, or location. Generally, while at least three parameters are necessary to capture essential distributional features, four are typically sufficient for meaningful generalization. In specialized applications, however, a fifth or sixth parameter may be incorporated to further enhance model flexibility [3]. Likewise, the Bayesian framework of the proposed distribution also provides quantification of uncertainty. It also framework for statistical inference, quantifying uncertainty through the use of probability distributions. This methodology systematically refines knowledge by integrating empirical data to update a prior probability distribution into a posterior probability distribution via Bayes' theorem [33]. Largely enabled by computational advances -particularly the development of Markov Chain Monte Carlo (MCMC) methods-the Bayesian approach has garnered significant attention and widespread application across diverse statistical disciplines over the past five decades [15, 18]. In Bayesian analysis, all posterior model parameters are treated as random variables, which can be utilized to predict future events. In the literature, Bayesian analytical tools have been applied across a diverse range of methodologies in higher education research. In terms of applications, Bayesian network models have been used in the behavioral analysis of both postgraduate student satisfaction and customer satisfaction [24, 28]. Similarly, Bayesian structural equation modelling has been applied to study university student satisfaction [5]. Furthermore, Bayesian analysis of generalized linear models has been employed to estimate overall satisfaction [25], while Bayesian belief network models have been used to the predict and assess student learning outcomes [23]. Hence, Bayesian tools are powerful for inference within the framework of distribution theory. However, while different Bayesian approaches have been utilized to address various research questions, the Exponentiated Odd Lomax Exponential (EOLE) distribution has not yet been employed as a Bayesian framework to estimate the parameters of educational datasets. Therefore, the aim of this study is to use a Bayesian framework of the EOLE distribution to estimate the parameters of educational data (i.e., academic achievement and student satisfaction) and to compare the results with the classical approach. The proposed distribution was derived by Dhungana and Kumar [13], and its cumulative density function (CDF) is given by

$$F(x;\alpha,\lambda,\theta,\delta) = \int_0^{(e^{\alpha x}-1)^\theta} \frac{\lambda}{\delta}\left\{1+\frac{t}{\delta}\right\}^{-(\lambda+1)} dt = 1-\left[1+\frac{1}{\delta}\{e^{\alpha x}-1\}^\theta\right]^{-\lambda}. \tag{1}$$

The corresponding probability density function (PDF) is

$$f(x) = \frac{\alpha\theta\lambda e^{\alpha x}(e^{\alpha x}-1)^{\theta-1}}{\delta}\left[1+\frac{1}{\delta}(e^{\alpha x}-1)^\theta\right]^{-(\lambda+1)}, \quad x\geq 0,\ \alpha,\lambda,\theta,\delta>0. \tag{2}$$

The likelihood function of proposed model is

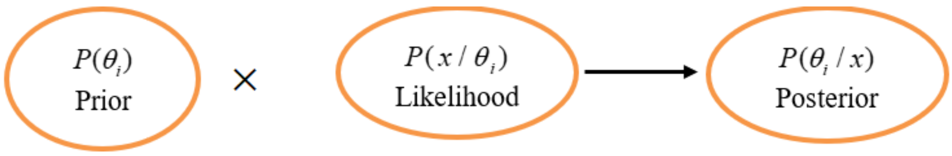
$$L(\alpha,\theta,\lambda,\delta \mid \underline{x}) = \left(\frac{\alpha\theta\lambda}{\delta}\right)^n e^{-\alpha n\bar{x}}\prod_{i=1}^{n}(e^{\alpha x_i}-1)^{\theta-1}\left[1+\frac{1}{\delta}\left(e^{\alpha x_i}-1\right)^\theta\right]^{-(\lambda+1)}. \tag{3}$$

This article is organized to move systematically from theoretical development to empirical validation. Section 2 establishes the Bayesian framework for the proposed distribution, including the derivation of the posterior distribution and the formulation of the regression model. Section 3 outlines the data analysis procedures, while Section 4 provides a detailed analysis of the datasets and presents the empirical results. Section 5 contextualizes these findings within the existing literature and discusses their significance. Finally, section 6 concludes the study by synthesizing the primary findings and highlighting their implications for future research and practical applications.

2 Materials and Methods

2.1 Bayesian model

The Bayesian model is defined as prior distribution for the model parameters $\alpha,\lambda,\theta,\delta>0$ multiply with the likelihood function, then after formed the posterior distribution



In the Bayesian approach, parameters are treated as random variables that follow a certain probability distribution, known as the prior distribution [12, 34]. However, there is no unique method for selecting the best prior [31]. The prior distributions are commonly classified into different types, such as diffuse, non-informative, informative, and conjugate priors, depending on the level of prior knowledge about the population parameter. Among the various types of prior distributions, a weakly informative prior, specifically the Gamma distribution (which is bounded between zero and infinity), has been employed in this study following prior predictive checking [4, 33]. The reason for selecting the Gamma prior is that it is strictly positive, making it suitable for modelling parameters such as rates, precisions, and scale parameters that cannot take negative values. In addition, it is flexible in shape, which provides computational convenience due to its conjugacy properties in Bayesian inference. Therefore, the prior distribution for each parameter is

$$\lambda \sim G(k_1,\vartheta_1);\theta\sim G(k_2,\vartheta_2);\alpha\sim G(v_3,\vartheta_3) \text{ and } \delta\sim G(v_4,\vartheta_4)$$

$$p(\lambda) = \frac{e^{-\lambda\vartheta_1}\lambda^{k_1-1}\vartheta_1^{k_1}}{\Gamma(k_1)}, \qquad \lambda, k_1, \vartheta_1 > 0. \tag{4}$$

$$p(\theta) = \frac{e^{-\theta\vartheta_2}\theta^{k_2-1}\vartheta_2^{k_2}}{\Gamma(k_2)}, \qquad \theta, \vartheta_2, k_2 > 0. \tag{5}$$

$$p(\alpha) = \frac{e^{-\alpha\vartheta_3}\alpha^{k_3-1}\vartheta_3^{k_3}}{\Gamma(k_3)}, \qquad \alpha, k_3, \vartheta_3 > 0. \tag{6}$$

$$p(\delta) = \frac{e^{-\delta\vartheta_4}\delta^{k_4-1}\vartheta_4^{k_4}}{\Gamma(k_4)}, \qquad \delta, \vartheta_4, k_4 > 0. \tag{7}$$

Then posterior probability distribution is

$$p(\alpha, \theta, \lambda, \delta/x) \propto (\alpha, \theta, \lambda, \delta/x).p(\alpha)p(\theta)p(\lambda)p(\delta) \tag{8}$$

The posterior distribution represents our updated beliefs about the parameters after considering the data. According to Baye's theorem yield the posterior distribution of proposed model is

$$p(\alpha, \theta, \lambda, \delta \mid x) = \frac{\alpha^{n+k_3-1}\theta^{n+k_2-1}\lambda^{n+k_1-1}e^{-(\alpha n\bar{x}+\lambda\vartheta_1+\theta\vartheta_2+\alpha\vartheta_3+\delta\vartheta_4)}}{\delta^{\,n-k_4+1}\prod_{i=1}^{4}\Gamma k_i}\prod_{i=1}^{n}\left(e^{\alpha x_i}-1\right)^{\theta-1}\left[1+\frac{1}{\delta}\left(e^{\alpha x_i}-1\right)^{\theta}\right]^{-(\lambda+1)} \tag{9}$$

2.2 Bayesian regression model

The probability density function of response variable follows the EOLE distribution is

$$f(y \mid \alpha, \theta, \lambda, \delta) = \frac{\alpha\theta\lambda}{\delta}e^{\alpha y}\left(e^{\alpha y}-1\right)^{\theta-1}\left[1+\frac{1}{\delta}\left(e^{\alpha y}-1\right)^{\theta}\right]^{-(\lambda+1)}; \tag{10}$$

where $y > 0$ is the response variable, $\alpha, \delta > 0$ are scale parameters, and $\theta, \lambda > 0$ are shape parameters. The scale parameters $\delta > 0$ depends linearity on the predictor's variable. The model specification is $y_i \sim EOLE(\alpha, \theta, \lambda, \delta_i)$, for $i = 1, 2, \ldots, n$; where, $\ln(\delta_i) = \beta_0 + \beta_1 x_i \Rightarrow \delta_i = e^{\beta_0+\beta_1 x_i}$; where,$\beta_0$ and β_1 are the intercept and slope. Similarly, likelihood function of regression model is

$$l(y \mid \alpha, \theta, \lambda, \beta_0, \beta_1, y, x) = \prod_{i=1}^{n} f\left(y \mid \alpha, \theta, \lambda, \delta_i = e^{\beta_0+\beta_1 x_i}\right) \tag{11}$$

and loglikelihood function of model is

$$l(y \mid \alpha, \theta, \lambda, \beta_0, \beta_1, y, x) = \sum_{i=1}^{n} f\left(y_i \mid \alpha, \theta, \lambda, \delta_i = e^{\beta_0+\beta_1 x_i}\right) \tag{12}$$

In Bayesian analysis, we assign the prior distribution to all unknown parameters and compute their posterior distribution. Therefore, the Gamma and normal distribution have been chosen as the prior distribution.

$$\lambda \sim G(k_1, \vartheta_1); \quad \theta \sim G(k_2, \vartheta_2); \alpha \sim G(k_3, \vartheta_3); \beta_0 \sim N(\mu, \sigma^2); \beta_1 \sim N(\mu, \sigma^2).$$

The posterior distribution is the proportional to the likelihood times to the prior distribution which is

$$p(\alpha, \theta, \lambda, \beta_0, \beta_1 \mid y, x) \propto l(y \mid \alpha, \theta, \lambda, \beta_0, \beta_1, x) \times p(\alpha)\,p(\theta)\,p(\lambda)\,p(\beta_0)\,p(\beta_1) \tag{13}$$

The posterior is complicated, so there is no closed form of inference. Therefore, the proposed distribution is estimated using the MCMC method, which generates samples from the posterior distribution and facilitates statistical inference when the posterior distribution is analytically intractable.

2.3 Markov chain Monto Carlo (MCMC)

MCMC is one of the most popular methods to draw the values of parameters from the appropriate distribution, and it provides a better approximation of the target posterior distribution. It is a technique of simulation. Using MCMC, we can generate samples of the posterior distribution [17, 18]. MCMC utilizes different techniques to construct the chains, such as the Gibbs sampler[20], algorithms through Stan (a probabilistic programming language)[18], and the Hamiltonian Monte Carlo (HMC) algorithm and its adaptive variant, the No-U-turn sampler (NUTS) [21]. The user-friendly interface Stan utilizes the efficient No-U-Turn Sampler (NUTS) algorithm to generate the posterior samples for this study [30]. This technique enables researchers to explore complex Bayesian statistical models.

3 Data Analysis

3.1 Sources of data

The data for this study were obtained from a survey, "Student Satisfaction and Learning Outcomes of Nepalese University Students," conducted from July 2023 to December 2023 [32]. Data were collected from eight central universities in Nepal using a stratified sampling method. The universities were stratified based on the number of campuses, number of student enrollments, geographical location (inside or outside the Kathmandu Valley), and campus affiliation (affiliated and constituent). Ultimately, 32 campuses were randomly selected through a stratified allocation process from the eight selected central universities in Nepal. After selecting the campuses, one class from each campus was chosen based on availability. The criterion for class selection was that a minimum of 20 students had to be present during the data collection period to ensure adequate representation for each campus. Before data collection, the research team explained the purpose of the study to the students. During the data collection period, researchers assured the anonymity and confidentiality of all students who participated voluntarily in the research. The research team collected data using a group-administered questionnaire.

A total of 878 questionnaires were distributed to the students, of which 770 responses were received, yielding an 87.6 percent response rate. After excluding incomplete or straight-line responses related to our study variables, a final sample of 628 valid responses were retained for the study. The collected data were entered into EpiData 3.1 and exported to R 4.4.0 [27] for data analysis.

3.2 Measurement of variables

3.2.1 Academic achievement (AA)

In this study, academic achievement (performance-based) was reported by students based on their preceding semester or annual examination marks, recorded either in SGPA or percentage. Prior to analysis, all academic achievement measures were standardized to a common scale. Specifically, scores originally recorded as percentages (0–100%) were transformed into a 4.0 SGPA scale to ensure uniformity. This transformation was performed as: SGPA = (Percentage / 100) $\times$ 4 as a scale variable.

3.2.2 Students satisfaction (SS)

Student satisfaction (perception-based) was measured using a structured questionnaire comprising 19 items on a five-point Likert scale ranging from strongly disagree to strongly agree. These items were designed to measure students' perceived satisfaction across different dimensions. These aspects cover students' perception of their satisfaction, which includes the teachers' role, campus facilities, institutional brand/image, administrative support, curriculum effectiveness, examination system support, class environment, and teaching pedagogy. From these responses, a composite score was computed based on the mean score of all items in a continuous scale variable representing the overall satisfaction of the students.

4 Results

4.1 Exploratory data analysis

The median value of academic achievement was 3.20, with a minimum of 2.0 and a maximum of 4.00. The mean is slightly lower than the median suggesting a left-skew in the distribution. Likewise, the average student satisfaction score was 3.00, with a minimum score of 1.0 and a maximum score of 5.0 (Table 1). The exploratory data analysis revealed that the academic achievement scores were left-skewed. This was

Table 1: Descriptive characteristic of data set

Characteristics	**Minimum**	**Q_1**	**Mean**	**Median**	**Q_3**	**Maximum**
Academic achievement	2.00	2.96	3.17	3.20	3.48	4.00
Student satisfaction	1.00	2.60	3.00	3.00	3.40	5.00

evidenced by a greater dispersion in the bottom 25% of the data (from Q1 to the minimum) compared to the top 25%. In contrast, student satisfaction scores were approximately symmetrical, despite the presence of outliers. Therefore, both datasets violated the normality assumption, as evidenced by the left-skewed distribution (AA) and the presence of notable outliers in both boxplots (Figure 1).

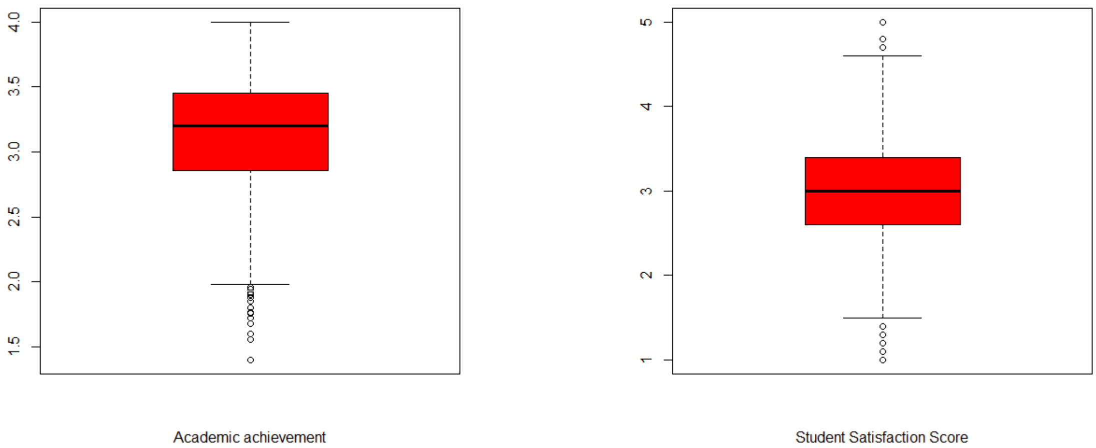


Figure 1: Box plot of AA (left panel) and SS (right panel).

The Total Time on Test (TTT) utilized to visualize the cumulative hazard function over time. The concave shape of the academic achievement plot indicates a decreasing hazard rate, while the shape of the student satisfaction plot suggests [an increasing/constant] hazard rate. This divergence in hazard behavior underscores the necessity for a highly flexible probabilistic model. Given its flexibility in capturing various hazard rate patterns, the EOLE distribution provides a justified and reliable framework for modelling the observed data (Figure 2).

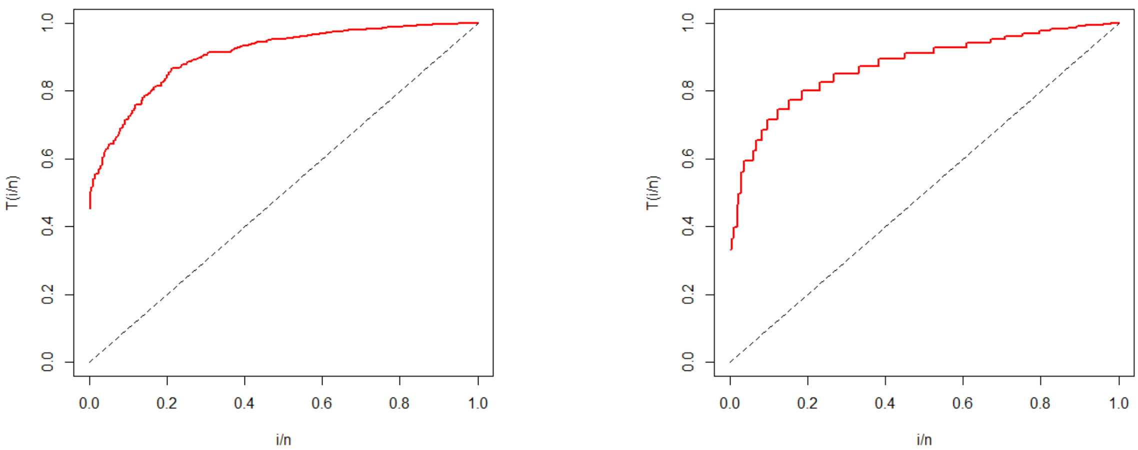


Figure 2: TTT plot of AA (left panel), and SS (right panel).

4.2 Parameter estimation and model validation

The parameters of the proposed distribution were estimated via MLE, implemented using the optim function with the L-BFGS-B algorithm in R[27]. Subsequently, the goodness-of-fit of the proposed model was evaluated for each dataset using the Kolmogorov-Smirnov (KS), Anderson-Darling (AD), and Cramér-von Mises (CvM) tests. The results yielded p-values > 0.05 for each datasets, indicating that the proposed model provides a statistically adequate fit to the empirical data (Table 2).

Table 2: Estimated parameter values (standard error) and test values (p-value)

Parameters	**Academic achievement**	**Student satisfaction**
$\hat{\alpha}$	0.1348 (0.0191) ***	0.20463 (0.0983) ***
$\hat{\theta}$	7.6885 (0.2892) ***	4.4772 (0.50180) ***
$\hat{\lambda}$	14.4181 (8.5364) ***	2.75209 (0.9155) ***
$\hat{\delta}$	0.1908 (0.2184) ***	1.73646 (4.7397) ***
-LL	**-310.1029**	**-649.7269**
KS test - D(p-value)	0.04011 (0.1498)	0.00027 (0.999)
AD test - A(p-value)	1.24070 (0.2525)	1.3389 (0.2201)
CVM test - Ω^2 (p-value)	0.15094 (0.3868)	0.24952 (0.189)

*** *Significance at* < 0.001*, parentheses of parameters indicate the standard error.*

In addition, the adequacy of the proposed model was evaluated graphically by comparing the empirical distribution with the fitted cumulative distribution function. As illustrated in Figure 3, the proposed distribution demonstrates a satisfactory alignment with the empirical data.

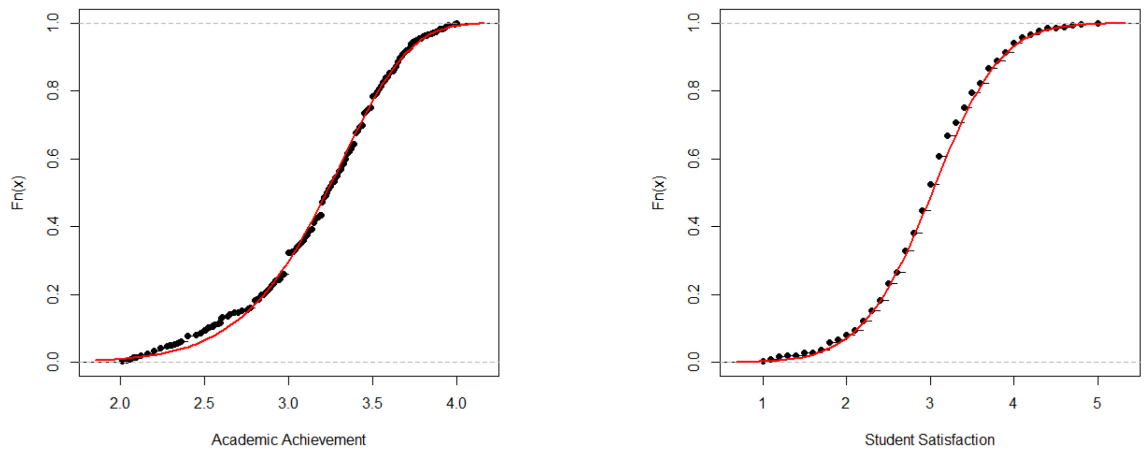


Figure 3: Empirical vs theoretical fitted CDF of AA (left panel); and SS (right panel).

4.3 Bayesian analysis of exponentiated Odd Lomax exponential distribution

4.3.1 Prior and posterior predictive checking

In Bayesian analysis, independent gamma distributions were used as informative priors, with specific hyperparameter values $\alpha, \lambda, \theta, \delta \sim G(3.1, 1.5)$ is data: academic achievement (SGPA scores) and student satisfaction. Following this, prior predictive checks were conducted, and after applying the likelihood function, posterior predictive checks were performed. The results, as shown in Figures 4 and 5, indicated that the prior predictive values fell within the posterior predictive range of AA and SS. This suggests that the model was consistent with the prior beliefs and assumptions, providing a good fit and capturing the data (for details see [33]).

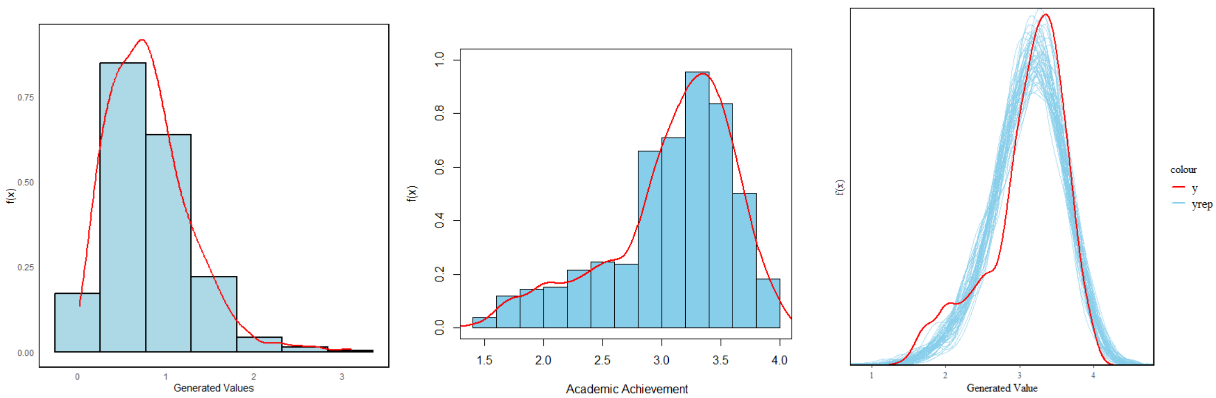


Figure 4: Plot of prior predictive checking (left panel); likelihood (center Panel); and posterior predictive checking (right panel) of academic achievement.

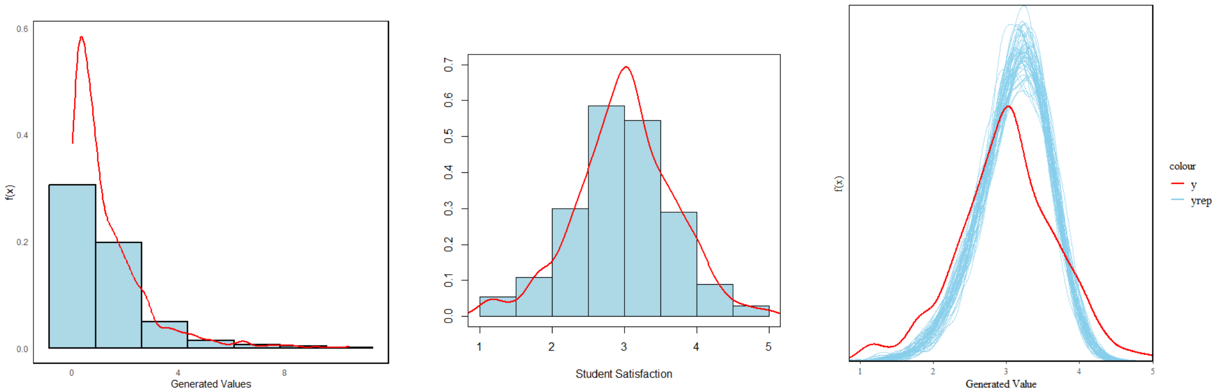


Figure 5: Plot of prior predictive checking (left panel); likelihood (center Panel); and posterior predictive checking (right panel) of student satisfaction.

4.3.2 Model diagnostics

The posterior distribution proved to be complex, making direct posterior inference challenging. Consequently, we employed the MCMC technique using the rstan package [30]. We specified prior distributions as independent gamma distributions with predefined hyperparameters. The model was executed to generate four Markov chains, each consisting of 5,000 samples, with the first 1,000 samples used for warmup. Therefore, the posterior sample drawn from chain 1 $(\alpha_1^{(j)}, \lambda_1^{(j)}, \theta_1^{(j)}, \delta_1^{(j)})$, $j = 1, 2, \ldots, 5000$ to chain 4 $(\alpha_4^{(j)}, \lambda_4^{(j)}, \theta_4^{(j)}, \delta_4^{(j)})$, $j = 1, 2, \ldots, 5000$. Initially, we extracted the posterior samples and then tested for convergence using trace plots, ergodic mean plots, and autocorrelation plots of each variables AA and SS. Additionally, we measured the effective sample size (`n_eff`) and $\hat{R}$.

4.3.3 Convergence diagnostics

Prior to scrutinizing parameter estimates or engaging in further inference, it is prudent to inspect sequential (dependent) realizations of the parameter estimates and visualize them. The figure illustrates the sequential manifestation of model parameters, revealing oscillations around a horizontal line without any trend. This observation suggests that the MCMC was most likely to be sampling from the stationary distribution and exhibiting satisfactory mixing properties (Figures 6 and 7).

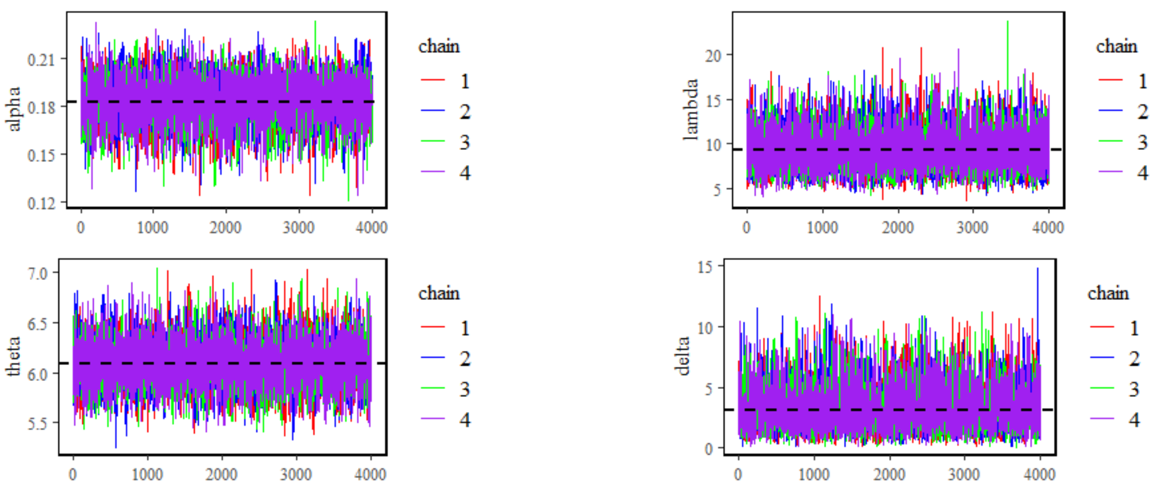


Figure 6: Sequential realization of the parameters α, λ, θ, and δ of academic achievement.

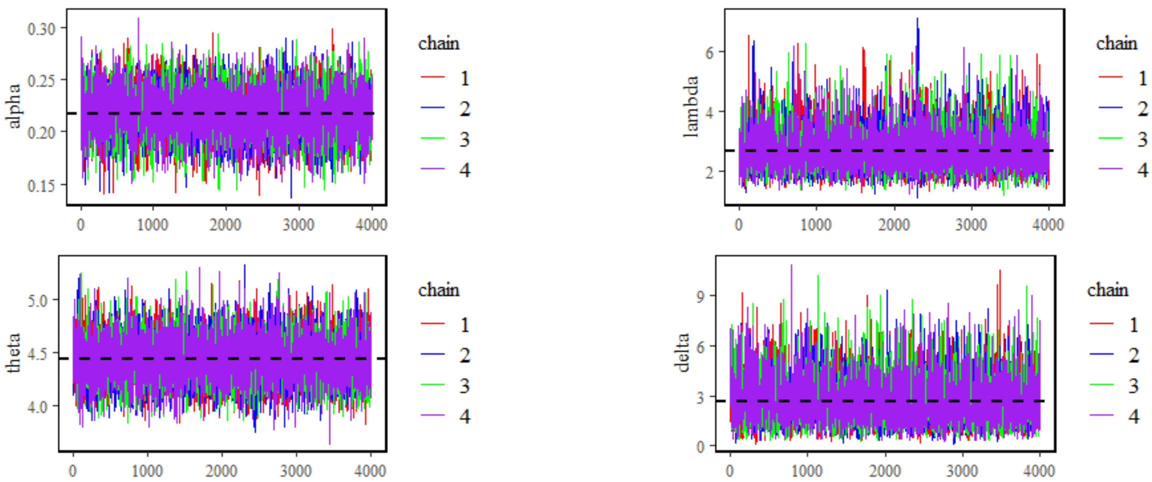


Figure 7: Sequential realization of the parameters α, λ, θ, and δ of student satisfaction.

4.3.4 Running mean (Ergodic mean) plot

The goal of the running mean plot is for the line to eventually become flat (or level off), indicating that the average value is no longer changing significantly with new samples. This stable average is the ergodic mean, and reaching it confirms the chain has successfully sampled from the target distribution of the MCMC (Figures 8 and 9).

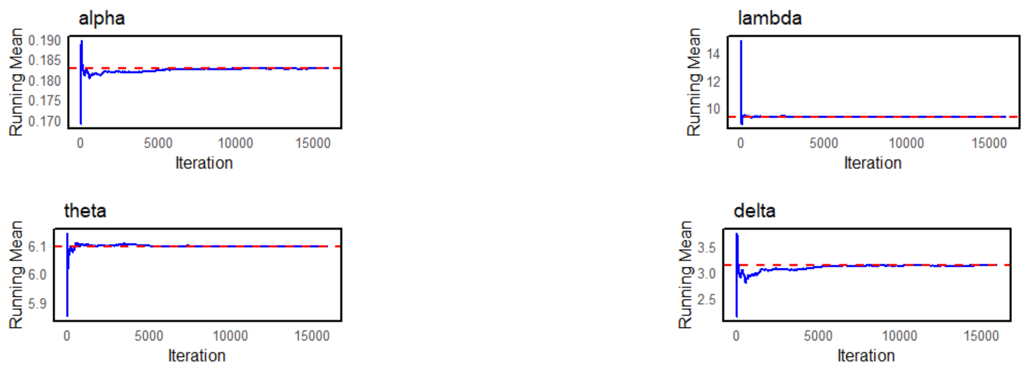


Figure 8: The Ergodic mean plot of α, λ, θ, and δ of academic achievement.

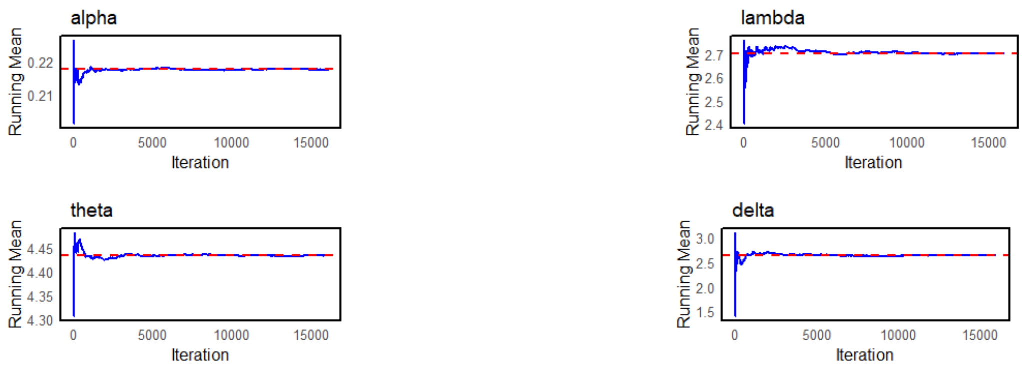


Figure 9: The Ergodic mean plot of α, λ, θ, and δ of student satisfaction.

4.3.5 Autocorrelation

The autocorrelation of samples generated by Stan diminishes to values approaching zero at approximately lag 40. Higher autocorrelation suggests that the sampler may not be effectively exploring the posterior distribution. As the value of "n" increases, both autocorrelation and the overall size of the chain decrease [4]. Because the autocorrelation was nearly negligible (close to zero), we concluded that the samples were effectively independent draws from the posterior distribution. (Figures 10 and 11).

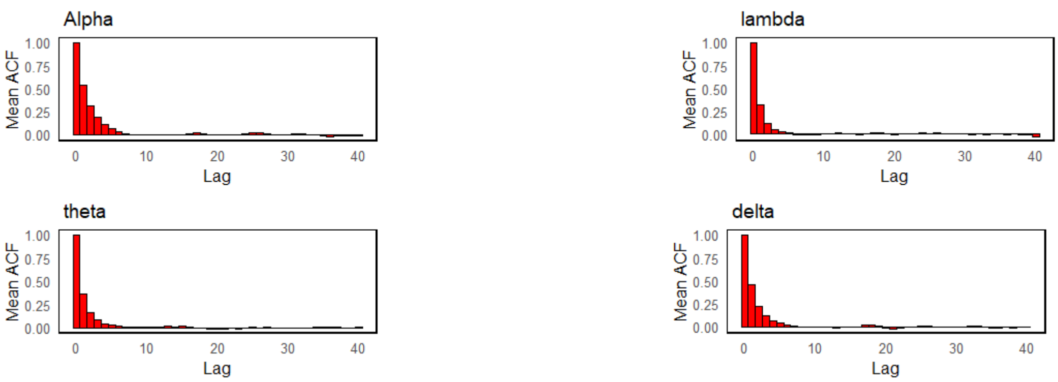


Figure 10: Auto correlation of posterior sample of α, λ, θ, and δ of academic achievement.

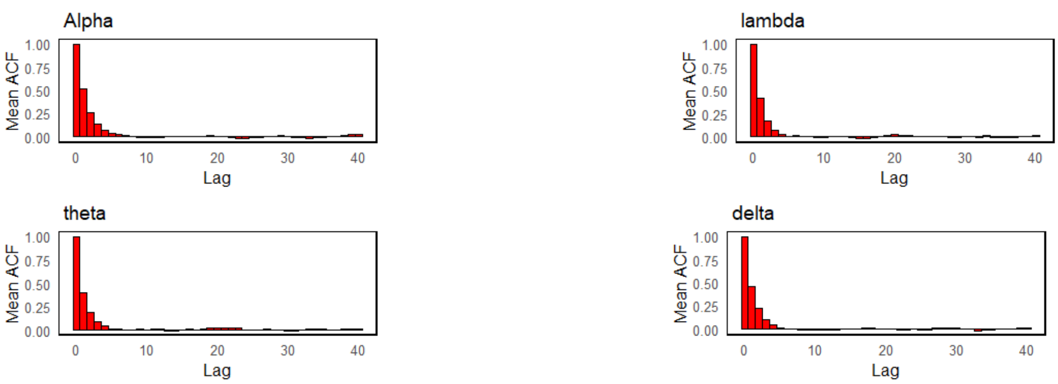


Figure 11: Auto correlation of posterior sample of α, λ, θ, and δ of student satisfaction.

4.4 Posterior analysis

Convergence statistics such as the potential scale reduction factor ($\hat{R}$) introduced by Gelman and Rubin [19] and the effective samples number (n_eff) proposed by [18] provide valuable insights into the convergence and efficiency of MCMC algorithms. A rule of thumb suggests that when ($\hat{R}$) is less than 1.1, convergence has likely been achieved. The effective samples number (n_eff) is exceeding 10% of the total sample size, indicative of sampling efficiency in the MCMC chain. The findings of n_eff and ($\hat{R}$) revealed that all parameters exhibited an effective sample size and ($\hat{R}$) values suitable for estimating the posterior distribution. The MCMC results for the proposed distribution has been summarized as, including mean, standard deviation, first quartile, third quartile, 2.5th percentile, 97.5th percentile, presented for $(\alpha_i^{(j)}, \lambda_i^{(j)}, \theta_i^{(j)}, \delta_i^{(j)})$ =1,2,. . . . . . ,5000 from chain i=1,2,3,4 (Table 3).

Table 3: Summary of posterior estimates from MCMC analysis of academic achievement and student satisfaction

Estimates	Academic achievement				Student satisfaction			
parameter	$\hat{\alpha}$	$\hat{\lambda}$	$\hat{\theta}$	$\hat{\delta}$	$\hat{\alpha}$	$\hat{\lambda}$	$\hat{\theta}$	$\hat{\delta}$
Mean	0.1825	9.2783	6.0936	3.1305	0.2174	2.6997	4.4333	2.6322
SD	0.0144	2.1816	0.2287	1.7629	0.0227	0.6667	0.2142	1.3610
$P_{2.5}$	0.1527	5.7324	5.6561	0.6589	0.1728	1.7087	4.0329	0.6840
Q_1	0.1732	7.7132	5.9368	1.8306	0.2020	2.2278	4.2859	1.6267
Median	0.1832	9.0339	6.0866	2.7959	0.2174	2.5890	4.4240	2.3863
Q_3	0.1926	10.5804	6.2469	4.1023	0.2329	3.0502	4.5767	3.3901
$P_{97.5}$	0.2088	14.1583	6.5524	7.4722	0.2605	4.3224	4.8664	5.9280
n_eff	4512.875	7851.227	6250.451	5472.243	5147.001	6509.886	6269.210	5885.331
R_hat	1.0007	1.0004	1.0007	1.0006	1.0005	1.0004	1.0004	1.0004

Standard deviation = SD; 2.5th Percentile = $P_{2.5}$*; First Quartile =* Q_1*; Third Quartile =* Q_3*; 97.5th Percentile =* $P_{97.5}$.

Furthermore, a comparative analysis was conducted between the empirical cumulative distribution function and the theoretical distributions derived from both MLE and Bayesian posterior estimates. The results demonstrate that the proposed distribution exhibits a high degree of concordance with the empirical data across both estimation frameworks, confirming the robustness of the model's parameters (Figure 12).

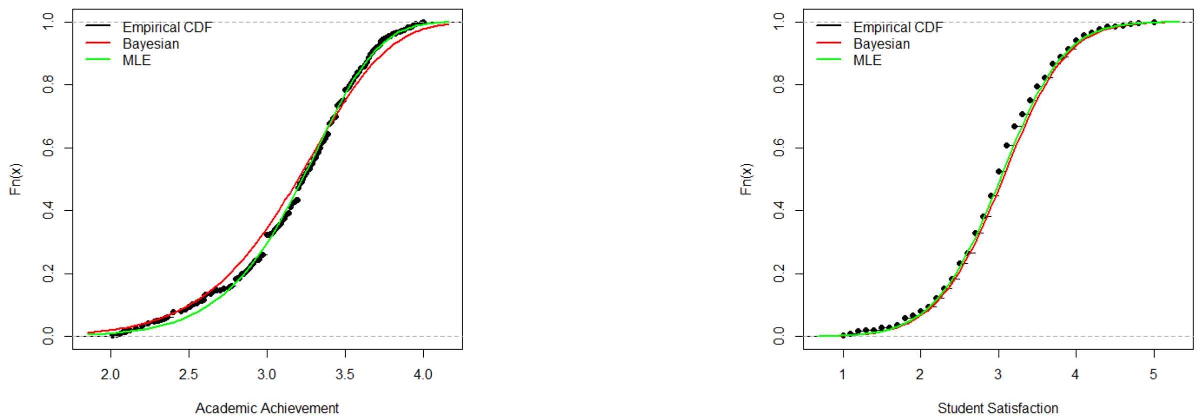


Figure 12: Comparison of empirical CDF, theoretical CDF from MLE and Bayesian estimation.

Using the MCMC method offers a distinct advantage over MLE in that it allows for the construction of reasonable interval of estimated parameters. It provides Highest Posterior Density (HPD) and creditable confidence interval of each parameter [10]. The width of the HPD interval provides a measure of uncertainty in the beliefs. A wider HPD interval indicates greater uncertainty, while a narrower HPD interval suggests more confidence in the beliefs. The estimated values of the posterior parameters, along with the 95% credible confidence intervals and 95% HPD intervals for each parameter, are presented. The findings reveal that both models provide credible posterior estimates, supporting the suitability of the Bayesian model (Table 4).

Table 4: Posterior estimated parameter with 95% credible and HPD intervals of academic achievement and student satisfaction

Parameters	$\hat{\alpha}$	$\hat{\lambda}$	$\hat{\theta}$	$\hat{\delta}$
Academic achievement				
Estimated average value	0.1825	9.2783	6.0936	3.1305
95% credible interval	0.1604–0.2098	5.0925–11.7067	5.6811–6.5325	0.8357–6.1773
95% HPD interval	0.1619–0.2113	4.7559–11.2209	5.6761–6.5251	0.5707–5.6743
Student satisfaction				
Estimated average value	0.2174	2.6997	4.4333	2.6322
95% credible interval	0.1727–0.2605	1.7086–4.3224	4.0328–4.8664	0.6839–5.9280
95% HPD interval	0.1726–0.2603	1.5937–4.0588	4.0174–4.8466	0.4195–5.3055

4.5 Bayesian regression model

A Bayesian regression framework was employed to model the relationship between student academic achievement (response variable) and satisfaction (predictors). The prior distribution of each parameter λ, θ and $\lambda \sim G(3.1, 1.5)$; and $\beta_i \sim N(0, 1)$, $i = 0, 1$ and regression model was performed by stan package from R [30]. After the perform the model, convergence of the MCMC chains was assessed visually using trace plots, running mean plot and autocorrelation of posterior distribution. All plots exhibited characteristic stationarity and good mixing, with the four independent chains overlapping substantially, indicating successful convergence to the target posterior distribution.

The model's parameter estimates demonstrated excellent convergence, with the mean and median values showing remarkable consistency across all four independent chains. This consistency was a strong indicator that the MCMC sampling algorithm had successfully identified a stable posterior distribution. The posterior distribution parameters were also consistently estimated across chains, with mean values of α and θ approximately 0.20 and 6.02, respectively. However, the shape parameter λ, with a mean of approximately 7.76, exhibited a wider credible interval (5.0-11.6), suggesting greater uncertainty in its exact value. This is a common characteristic for parameters that control the tails of a distribution. Hence, all the consistent and stable results across all chains provided strong confidence in the reliability and validity of the model's parameter estimates (Table 5).

Table 5: Posterior estimates of regression summary from MCMC analysis of academic achievement on student satisfaction

Characteristics	**Chain 1**					**Chain 2**				
	α	θ	λ	β_0	β_1	α	θ	λ	β_0	β_1
Mean	0.1953	6.0222	7.7417	1.3317	-0.0178	0.1955	6.0192	7.7709	1.3242	-0.0144
SD	0.0274	0.2963	1.6758	1.0995	0.0649	0.0281	0.3067	1.7418	1.1253	0.0665
$P_{2.5}$	0.1477	5.4266	5.0367	-0.8174	-0.1500	0.1480	5.4065	4.9911	-0.8501	-0.1447
Q_1	0.1767	5.8244	6.5144	0.6054	-0.0610	0.1754	5.8152	6.5370	0.5609	-0.0597
Median	0.1928	6.0331	7.5536	1.3128	-0.0162	0.1931	6.0245	7.5725	1.3198	-0.0135
Q_3	0.2125	6.2235	8.7540	2.0806	0.0232	0.2125	6.2323	8.8115	2.0805	0.0320
$P_{97.5}$	0.2545	6.5760	11.4720	3.4436	0.1092	0.2556	6.6143	11.6645	3.4793	0.1159
Characteristics	**Chain 3**					**Chain 4**				
	α	θ	λ	β_0	β_1	α	θ	λ	β_0	β_1
Mean	0.1975	6.0031	7.7716	1.4189	-0.0147	0.1960	6.0244	7.7591	1.3632	-0.0182
SD	0.0274	0.2909	1.6456	1.0763	0.0639	0.0269	0.2951	1.7405	1.0547	0.0657
$P_{2.5}$	0.1509	5.4214	5.0842	-0.6439	-0.1408	0.1513	5.4269	4.9493	-0.6707	-0.1461
Q_1	0.1781	5.8042	6.5748	0.6878	-0.0575	0.1772	5.8412	6.5222	0.6555	-0.0606
Median	0.1948	6.0058	7.5967	1.3998	-0.0155	0.1936	6.0371	7.5472	1.3362	-0.0194
Q_3	0.2144	6.2056	8.7716	2.1449	0.0281	0.2115	6.2256	8.8079	2.0535	0.0260
$P_{97.5}$	0.2578	6.5562	11.5430	3.5262	0.1113	0.2547	6.5772	11.6155	3.5074	0.1115

SD = Standard deviation; $P_{2.5}$ = 2.5th percentile; Q_1 = First quartile; Q_3 = Third quartile; $P_{97.5}$ = 97.5th percentile.

MCMC sampling successfully converged, as evidenced by consistent parameter estimates across chains, indicating a stable posterior distribution (Table 6).

Table 6: Posterior estimated parameter with 95% credible and HPD intervals Bayesian regression model

Parameters	$\hat{\alpha}$	$\hat{\theta}$	$\hat{\lambda}$	$\hat{\beta}_0$	$\hat{\beta}_1$
Estimated average value	0.1961	6.017	7.761	1.359	-0.01629
95% credible interval	0.1494–0.2557	5.4193–6.5798	5.0080–11.5742	-0.7625–3.4963	-0.1454–0.1122
95% HPD interval	0.1455–0.2510	5.4132–6.5708	4.6595–11.1000	-0.7385–3.5124	-0.1420–0.1153

5 Discussion

In Bayesian analysis, the MCMC technique employs different probability distributions for parameter estimation. For instance, Chaudhary and Kumar [8, 9] estimated the parameters using the MCMC technique of Bayesian analysis of the Gompertz extension distribution and the Perks distribution. Similarly, Alsadat et al. [2] introduced and applied the extended odd Weibull power Lomax (ExOW-POLO) distribution, deriving its reliability and statistical properties while employing both frequentist and Bayesian estimation methods, including the MCMC technique for parameter estimation. In recent studies, Almongy et al. [14] used the unit inverse exponentiated Lomax distribution to compare and estimate its parameters with traditional and Bayesian approaches. Almongy et al. [1] also adopted a similar approach, utilizing the odd Lomax inverted Nadarajah-Haghighi (OLINH) distribution to compare the performance of frequentist and Bayesian parameter estimation methods. Hence, in this research, a Bayesian approach to parameter estimation was used, and compared with MLE technique. The findings are consistent with the MLE and procedure are similar with available literatures. Furthermore, this study verified the assumptions of stationary, convergence, autocorrelation, and prior and posterior predictive checks. Therefore, Bayesian approach for the proposed model is well established, which is standard practice in Bayesian research. Likewise, other scholars have utilized similar approaches to assessing the model's predictive performance on educational data. For instance, the Unit Lindley Mixed Model [6] and Bayesian network models [11] have been used for predicting student behavior. Similarly, the Bayesian evaluation and assessment model [7] and the Bayesian hierarchical beta regression model [16] have been instrumental in enhancing student learning and quantifying teaching quality. While, Sheng et al. [29] utilized Bayesian analysis to determine student satisfaction and identify influential factors. Building on these methodological frameworks, the results of the Bayesian regression analysis in this study indicate successful convergence of the MCMC sampling. This finding is evidenced by consistent parameter estimates across chains, which suggests a stable and reliable posterior distribution that align with Gelman et al. [17]. Hence, this successful MCMC convergence validates the sampling technique by establishing a stable posterior distribution, thereby ensuring the robustness of the parameter estimates.

6 Conclusion

This study derived a Bayesian model and a Bayesian regression model based on a gamma prior; and a combination of gamma and normal priors with the EOLE likelihood function, respectively. This framework is specifically address non-normal, complex, and skewed data, providing a highly robust analytical structure for educational research. The model parameters were estimated through MLE and MCMC techniques for classical and Bayesian approach, respectively. To provide empirical strength, two distinct datasets-academic achievement and student satisfaction are used for model validation. Both graphical and numerical assessments confirm the validity of the model and the accuracy of the findings. The estimated parameters for both the Bayesian model and the Bayesian regression model showed close agreement with the MLE estimates. This convergence underscores the consistency of the results and establishes the Bayesian EOLE model as a powerful and reliable alternative for the analysis of data.

The practical implication for future research will be the development of a dedicated R package integrated with the rstan environment, which will facilitate comprehensive data modelling and analysis in a more accessible and efficient for researchers. Furthermore, the expansion of micro-level studies will be essential for evaluating the performance of academic institutions, generating valuable insights that will enhance evidence-based decision-making.

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Conflicts of Interest
The authors declare that they have no conflicts of interest.

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