Completeness of the Paranormed Difference Sequence Space Defined by the Orlicz Function

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Abstract
The idea of difference sequence space was initiated by Kizmaz in 1981 and it was frequently studied then by many mathematicians like Et and Kolak, Asma, Alsaidi, Faried and Bakery, Pahari, Ghimire, Paudel etc. In this present article we have tried to find the completeness property of the sequence space $\ell_M(X, \Delta, \bar{\alpha}, P, L)$

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1. Introduction

Sequence space, a space of sequences $\bar{x} = (x_k)$ which is linear with respect to coordinatewise addition and multiplications

$$(x_k) + (y_k) = (x_k + y_k)$$
$$\lambda(x_k) = (\lambda x_k).$$

for any two sequences $\bar{x} = (x_k)$ and $\bar{y} = (y_k)$, and for any scalar $\lambda \in \mathbb{R}$ or $\mathbb{C}$ The space of all sequences is denoted by ω.

Sequence spaces play a very important role in contemporary mathematics. Especially in analysis, such as: approximation theory, summability theory, operator theory, frame theory, Schauder basis, structure theory of topological linear spaces These spaces form the very basis of mathematical theory, in particular by fertilizing functional analysis with new notions and concepts to work with.

Sequence spaces emerged in early 1900's along with the development of Banach space and summability theory. Works towards Cesaro summability, Holder's sum gave a fruitful contribution to sequence space. David Hilbert (1906) introduced ℓ_2 space, which leads to the space of squaresummable sequences:

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$$\ell_2 = \left\{\bar{\zeta} = (\zeta_k): \sum_k |\zeta_k|^2 < \infty\right\}$$

Stephen Banach then introduced the absolutely p-summable sequence space ℓ_p in 1932 as

$$\ell_p = \left\{\bar{\zeta} = (\zeta_k): \sum_k |\zeta_k|^p < \infty\right\}$$

where $1 \leq p < \infty$. This sequence space was just the generalization of ℓ_2 space which is the turning point towards the development of sequence spaces.

The classical sequence spaces well-known in mathematical area are the null sequence space c_0, space of convergent sequences c, space of all bounded sequences ℓ_∞ and the space of all absolutely p-summable sequences ℓ_p. Maddox in 1980, generalized the classical sequence spacces to the vector valued sequences spaces $c_0(X), c(X), \ell_\infty(X)$ and $\ell_p(X)$. Many more attempts have been made to generalize the notion of scalar-valued sequence spaces to the space of vector-valued sequences. Some of them are the researchers, including Maddox, Malkowski and Rakocevic[19], Parashar and Choudhary[21], Ruckle[27], etc. They have significantly contributed to the theory of sequence and function spaces. Literature on sequence space theory can be found in standard functional analysis textbooks and monographs, including Basar[4], Kamthan and Gupta[13], Lindenstrauss and Tzafriri[17], Rao and Ren[26] etc. Many researchers, including Bala[3] Bhardwaj and Bala[5], Khan[14], Kolk[16], Srivastava and Pahari[28], Tripathy and Sen[31], have studied the properties of generalized sequence spaces by using the Orlicz function, which is applicable to a wide range of sequence spaces.

Władysław Orlicz (1936)[17] introduced Orlicz sequence space, generalizing ℓ_p space using Young function ϕ, which is denoted by ℓ_ϕ. The space ℓ_ϕ includes ℓ_p as the special case.

In 1971, Lindenstrauss and Tzafriri[17] introduced Orlicz sequence space:

$$\ell_M = \left\{\bar{\zeta} = (\zeta_k) \in \omega: \sum_k M\left(\frac{|\zeta_k|}{\eta}\right) < \infty \text{ for some } \eta > 0\right\}$$

by using Orlicz function M.

Kizmaz(1981)[15] introduced and studied the difference sequence spaces from these sequence spaces with elements as the difference of consecutive terms of the sequences.

In 1997, Et and Colak[7] introduced the Δ^m sequence spaces as:

$$\ell_\infty(\Delta^m(X)) = \left\{\bar{\zeta} = (\Delta\zeta_k) \in X: \sup_{k\geq 1} \|\Delta^m \zeta_k\| < \infty\right\}$$
$$c(\Delta^m(X)) = \left\{\bar{\zeta} = (\Delta\zeta_k): \exists l \in X: \|\Delta^m \zeta_k - l\| \to 0 \text{ as } k \to \infty\right\}$$
$$c_0(\Delta^m(X)) = \left\{\bar{\zeta} = (\Delta\zeta_k) \in X: \|\Delta^m \zeta_k\| \to 0 \text{ as } k \to \infty\right\}$$

where,

$$m \in \mathbb{N}, \Delta^0 \bar{\zeta} = (\zeta_k), \Delta\bar{\zeta} = (\zeta_k - \zeta_{k-1})$$
$$\Delta^m \bar{\zeta} = (\Delta^m \zeta_k) = (\Delta^{m-1}\zeta_k - \Delta^{m-1}\zeta_{k-1})$$

and so that,

$$\Delta^m \zeta_k = \sum_{r=1}^{m} (-1)^r \binom{m}{r} \zeta_{k+r}$$

The spaces $Z(\Delta^m(X))$ are Banach spaces together with norm

$$\|\bar{\zeta}\|_\Delta = \sum_{k=1}^{m} \|\zeta_k\| + \|\bar{\zeta}\|_\infty$$

where $Z = \ell_\infty, c_0$, and c.

Asma (2000)[2], Alsaedi (2003)[1], In 2013, Ganie and et. al[10], Many researchers, including Subramanian (2008)[30], Faried and Bakery (2009)[8], Ghimire and Pahari[11], Pahari[20] etc. have used Orlicz sequence space to construct the generalized form of different difference sequence spaces. They investigated the structural features and inclusion relationships between the spaces.

In 2019, Basar and Yesilkayagil[4] studied paranormed sequence spaces generated by infinite matrices. in 2023, Paudel, et. al[22] introduced the difference sequence spaces of fuzzy numbers by using the Orlicz function M, for any sequence $\mathbb{F} = (\mathbb{F}_k) \in \omega^F$ as

$$\mathbb{S}(\mathbb{F}, M, \alpha, P) = \left\{\mathbb{F}: \sum_k M\left(\frac{\|\alpha_k \Delta \mathbb{F}_k\|^{p_k}}{\eta}\right) < \infty \text{ for some } \eta > 0\right\}$$

and

$$\mathbb{S}(\mathbb{F}, M, \alpha, P, G) = \left\{\mathbb{F}: \sum_k M\left(\frac{\|\alpha_k \Delta \mathbb{F}_k\|^{p_k/G}}{\eta}\right) < \infty \text{ for some } \eta > 0\right\}.$$

where $\sup_k p_k = G$. They also discussed the containment relationship between these spaces.

Ganie A. H.[10] developed the notions of fuzzy sets and fuzzy set operations. Paudel and et al [22][23][24], Pokharel[25] studies the various properties of sequence space using fuzzy concept.

For any sequence of strictly positive real numbers $\bar{\alpha} = (\alpha_k), \bar{\beta} = (\beta_k)$, and for any sequence of complex numbers $P = (p_k), Q = (q_k)$, Kaphle et. al.[12] generalized the sequence space $\ell_M(X, \bar{\lambda}, P)$ of Banach space X-valued sequences defined by Orlicz function M defined by Shrivastava and Pahari[28] and defined the sequence space $\ell_M(X, \Delta, \bar{\alpha}, P)$ as follows:

$$\ell_M(X, \Delta, \bar{\alpha}, P) = \left\{\bar{\xi} = (\xi_k) \in \omega(X): \sum_k M\left(\frac{\|\alpha_k \Delta_m \xi_k\|^{p_k}}{\eta}\right) < \infty \text{ for some } \eta > 0\right\}$$

If for each $k, p_k = 1$ then $\ell_M(X, \Delta, \bar{\alpha}, P)$ becomes $\ell_M(X, \Delta, \bar{\alpha})$ and if for each k, $\alpha_k = 1$, $\ell_M(X, \Delta, \bar{\alpha}, P)$ becomes $\ell_M(X, \Delta, P)$.

Also if $\alpha_k = p_k = 1$ the class is denoted by $\ell_M(X, \Delta)$.

2 Preliminaries and Definitions

In this section we define some of the definitions used in this article.

2.1 Orlicz Function

Every continuous, non-decreasing, convex function $M: \mathbb{R}^+ \to \mathbb{R}^+$which also satisfies;

1. $M(t) = 0$ if $t = 0$,
2. $M(t) > 0$,

3. $M(t)$ approaches to ∞, as t approaches to ∞
is said to be an Orlicz function. For an Orlicz function M,

$$M(\eta t) \leq \eta M(t), \eta \in (0,1)$$

If for each non-negative , $\exists k > 0$:

$$M(2t) \leq kM(t)$$

then M is said to meet the Δ_2-condition .
$\Delta_2 -$ condition is a growth restriction on an Orlicz function M. If we replace the convexity property of M by the inequality

$$M(x + y) \leq M(x) + M(y),$$

The function M is called modulus function.

2.2 Paranorm

A paranorm on a linear space X is a function $g: X \to \mathbb{R}$, satisfying :
$PN_1. g(\Theta) = 0$
$PN_2. g(-x) = g(x) \forall x \in X$
$PN_3. g(x_1 + x_2) \leq g(x_1) + g(x_2) \forall x_1, x_2 \in X$
PN_4. (Continuity of scalar multiplication) For any sequence (λ_n) of scalars satisfying $\lambda_n \to \lambda$ and for any sequence $(x_n); x_n \in X$ satisfying $\lim_{n\to\infty} g(x_n - x) = 0$, implies

$$\lim_{n\to\infty} g(\lambda x_n - \lambda x) = 0.$$

In this case, (X, g) is referred as a Paranormed space. The paranormed space (X, g) is said to be complete paranormed space if every Cauchy sequence in X is convergent to a limit point say $x \in X$.

3 Main Results

In this section we establish some of the properties of above mentioned classes.
Throughout the research, we denote

$$\gamma_k = \frac{q_k}{p_k} \text{ and } \delta_k = \left|\frac{\alpha_k}{\beta_k}\right|^{p_k}$$

Lemma 1 For any sequence $(p_k); p_k > 0, p_k < \infty$ with $\sup p_k = L, D = \max\{1, 2^{-L}\}$ we have
i. $|\xi + \zeta|^{p_k} \leq D\{|\xi|^{p_k} + |\zeta|^{p_k}\}$;
$ii. |\alpha|^{p_k} \leq \max\{1, (\alpha)^L\}$.
Kaphle et.al.[12] proved the following relations:
Theorem 1 $\ell_M(X, \Delta, \bar{\alpha}, P) \subset \ell_M(X, \Delta, \bar{\beta}, P) \Leftrightarrow \inf_k \delta_k > 0$.
Theorem 2 $\ell_M(X, \Delta, \bar{\beta}, P) \subset \ell_M(X, \Delta, \bar{\alpha}, P) \Leftrightarrow \lim \sup_k \delta_k < \infty$.
Theorem 3 $\ell_M(X, \Delta, \bar{\alpha}, P) = \ell_M(X, \Delta, \bar{\beta}, P) \Leftrightarrow 0 < \lim_k \inf t_k < \lim_k \sup t_k < \infty$.
Theorem 4 The space $\ell_M(X, \Delta, \bar{\alpha}, P)$ is linear $\Leftrightarrow \sup_k p_k < \infty$.
Theorem 5 $\ell_M(X, \Delta, \bar{\alpha}, P, L)$ forms a linear space over $\mathbb{C}$.
Theorem 6 If $\inf_k p_k = l > 0$ then $\ell_M(X, \Delta, \bar{\alpha}, P, L)$ is a paranormed space with paranorm

$$G^*(\bar{\xi}) = \inf\left\{\left(\eta > 0: \sum_k M\left(\frac{\|\alpha_k \Delta_m \xi_k\|^{\frac{p_k}{L}}}{\eta}\right) \le 1\right\}\right.$$

Proof:

The properties PN_1. and PN_2. holds easily.

For PN_3. define a set for $\bar{\xi} \in \ell_M(X, \Delta, \bar{\alpha}, P, L)$

$$A(\bar{\xi}) = \left\{\left(\eta > 0: \sum_k M\left(\frac{\|\alpha_k \Delta_m \xi_k\|^{\frac{p_k}{L}}}{\eta}\right) \le 1\right\}\right.$$

Consider two sequences $\bar{\xi}$ and $\bar{\zeta} \in \ell_M(X\Delta, \bar{\alpha}, P, L)$.

Let $\eta_1 \in A(\bar{\xi})$ and $\eta_2 \in A(\bar{\zeta})$.

Since M is convex,

$$\sum_k M\left(\frac{\|\alpha_k \Delta_m (\xi_k + \zeta_k)\|^{\frac{p_k}{L}}}{\eta_1 + \eta_2}\right)$$

$$\le \frac{\eta_1}{\eta_1 + \eta_2} \sum_k M\left(\frac{\|\alpha_k \Delta_m \xi_k\|^{\frac{p_k}{L}}}{\eta_1}\right) + \frac{\eta_2}{\eta_1 + \eta_2} \sum_k M\left(\frac{\|\alpha_k \Delta_m \zeta_k\|^{\frac{p_k}{L}}}{\eta_2}\right)$$

Then we can see that

$$\eta_1 + \eta_2 \in A(\bar{\xi} + \bar{\zeta})$$

So that

$$G^*(\bar{\xi} + \bar{\zeta}) \le \eta_1 + \eta_2$$

Which implies that

$$G^*(\bar{\xi} + \bar{\zeta}) \le G^*(\bar{\xi}) + G^*(\bar{\zeta})$$

That is PN_3. holds.

Now it remains to prove PN_4. For this take

$$\bar{\xi}^{(n)} = \left(\xi_k^{(n)}\right) \in \ell_M(X, \Delta, \bar{\alpha}, P, L)$$

with $G^*\left(\bar{\xi}^{(n)}\right) \to 0$ and (λ_n) such that $(\lambda_n) \to \lambda$.

Now,

$$G^*\left(\lambda_n \bar{\xi}^{(n)}\right) = \inf\left\{\left(\eta > 0: \sum_k M\left(\frac{\left\|\alpha_k \lambda_n \Delta_m \xi_k^{(n)}\right\|^{\frac{p_k}{L}}}{\eta}\right) \le 1\right\}\right.$$

$$= \inf\left\{\left(\eta > 0: \sum_k M\left(\frac{|\lambda_n|^{\frac{p_k}{L}} \left\|\alpha_k \Delta_m \xi_k^{(n)}\right\|^{\frac{p_k}{L}}}{\eta}\right) \le 1\right\}\right.$$

$$\le \inf\left\{\left(\eta > 0: \sum_k M\left(\frac{g^{\frac{p_k}{L}} \left\|\alpha_k \Delta_m \xi_k^{(n)}\right\|^{\frac{p_k}{L}}}{\eta}\right) \le 1\right\}\right.$$

where $g = \sup_n |\alpha_n|$.

If we let $t = \max(g, 1)$, we have

$$G^*\left(\lambda_n \bar{\xi}^{(n)}\right) \le \inf\left\{\eta > 0: \sum_k M\left(t^{\frac{p_k}{L}} \frac{\left\|\alpha_k \Delta_m \xi_k^{(n)}\right\|^{\frac{p_k}{L}}}{\eta}\right) \le 1\right\}$$

If we take $\frac{\eta}{t} = r$, then

$$G^*\left(\lambda_n\bar{\xi}^{(n)}\right) \leq \inf\left\{tr > 0: \sum_k M\left(\frac{\left\|\alpha_k\Delta_m\xi_k^{(n)}\right\|^{\frac{p_k}{L}}}{r}\right) \leq 1\right\}$$
$$= tG^*\left(\bar{\xi}^{(n)}\right)$$

which shows that $G^*\left(\lambda_n\bar{\xi}^{(n)}\right)$ approaches to 0 as $n \to \infty$

Let $\lambda_n \to 0$ as $n \to \infty$ and $\bar{\xi} \in \ell_M(X, \Delta, \bar{\alpha}, P, L)$. Then for any positive ϵ less than unity we can find $N \in \mathbb{Z}^+$for which

$$|\lambda_n| \leq \epsilon \forall n \geq N$$

Since,

$$\inf_k p_k = l > 0, |\lambda_n|^{p_k/L} \leq \epsilon^{l/L} \text{ for all } n \geq N.$$

Now,

$$M\left(\frac{\|\lambda_n\alpha_k\Delta^m\xi_k\|^{\frac{p_k}{L}}}{\eta}\right) \leq \sum_k M\left(\frac{|\lambda_n|^{\frac{p_k}{L}}\left\|\alpha_k\Delta_m\xi_k^{(n)}\right\|^{\frac{p_k}{L}}}{\eta}\right)$$
$$\leq \sum_k M\left(\frac{\epsilon^{\frac{l}{L}}\left\|\alpha_k\Delta_m\xi_k^{(n)}\right\|^{\frac{p_k}{L}}}{\eta}\right)$$

So, if $\eta \in A\left(\epsilon^{l/L}\bar{\xi}\right)$ then $\eta \in A\left(\lambda_n\bar{\xi}\right)$.

That is,

$$A\left(\epsilon^{l/L}\bar{\xi}\right) \subset A\left(\lambda_n\bar{\xi}\right)$$

Since,

$$\inf\left\{\eta: \eta \in A\left(\epsilon^{l/L}\bar{\xi}\right)\right\} = \epsilon^{l/L}\inf\{\eta: \eta \in A(\bar{\xi})\}$$

we can get

$$\inf\left\{\eta: \eta \in A\left(\lambda_n\bar{\xi}\right)\right\} \leq \epsilon^{l/L}\inf\{\eta: \eta \in A(\bar{\xi})\}$$

which shows that $G^*\left(\lambda_n\bar{\xi}^{(n)}\right) \leq \epsilon^{l/L}G^*(\bar{\xi})$ for every positive integer $n \geq N$, i.e. $G^*\left(\lambda_n\bar{\xi}^{(n)}\right)$ approaches to 0 as $n \to \infty$.

Consequently, PN_4. is proved and so $\ell_M(X, \Delta, \bar{\alpha}, P, L)$ formed a paranormed space.

Theorem 7$\ell_M(X, \Delta, \bar{\alpha}, P, L)$ together with paranorm G^* is complete.

Proof:

Take $\bar{\xi}^{(i)} = \left(\xi_k{}^{(i)}\right)$ as a Cauchy sequence which belongs to $\ell_M(X, \Delta, \bar{\alpha}, P, L)$. Consider a fixed positive number r such that $M(r) \geq 1$. Then for any positive ϵ we can get $N \in \mathbb{Z}^+$which is greater than or equal to1 with

$$G^*\left(\bar{\xi}^{(i)} - \bar{\xi}^{(j)}\right) < c, \text{ where, } c = \frac{\epsilon}{r}, \forall i \geq N, j \geq N$$

So that,

$$\sum_k M\left(\frac{\left\|\alpha_k\Delta_m\xi_k^{(i)} - \alpha_k\Delta_m\xi_k^{(j)}\right\|^{\frac{p_k}{L}}}{G^*(\bar{x}^{(i)}) - G^*(\bar{x}^{(j)})}\right) \leq 1, \forall i \geq N, j \geq N$$

Thus,

$$\sum_k M\left(\frac{\left\|\alpha_k\Delta_m\xi_k^{(i)} - \alpha_k\Delta_m\xi_k^{(j)}\right\|^{\frac{p_k}{L}}}{G^*\left(\bar{\xi}^{(i)}\right) - \bar{\xi}^{(j)})}\right) \leq M(r) \forall i \geq N, j \geq N$$

Since M is non-decreasing being Orlicz function, so

$$\frac{\left\|\alpha_k\Delta_m\xi_k^{(i)}-\alpha_k\Delta_m\xi_k^{(j)}\right\|^{\frac{p_k}{L}}}{G^*\left(\bar{\xi}^{(i)}\right)-\bar{\xi}^{(j)}\right)}\leq r$$

So,

$$\left\|\alpha_k\left(\Delta_m\xi_k^{(i)}-\Delta_m\xi_k^{(j)}\right)\right\|^{\frac{p_k}{L}}<\epsilon$$

Hence $\left(\xi_k{}^{(i)}\right)$ is Cauchy in X, $\underline{\forall k}\in\mathbb{N}$. X being complete, $\exists\xi_k\in X\colon\forall k\in\mathbb{N}$ with $\xi_k^{(i)}\to\xi_k$ as $i\to\infty$. It will be shown that $\bar{\xi}=(\xi_k)\in\ell_M(X,\Delta,\bar{\alpha},P,L)$.

Take a constant η with

$$G^*\left(\bar{\xi}^{(i)}-\bar{\xi}^{(j)}\right)<\eta<\epsilon,\forall i\geq N,j\geq N$$

Since M is nondecreasing, we have

$$\sum_k M\left(\frac{\left\|\alpha_k\Delta_m\left(\xi_k^{(i)}-\xi_k^{(j)}\right)\right\|^{\frac{p_k}{L}}\Big)}{\eta}\right)$$
$$\leq\sum_k M\left(\frac{\left\|\alpha_k\Delta_m\left(\xi_k^{(i)}-\xi_k^{(j)}\right)\right\|^{\frac{p_k}{L}}}{G^*\left(\bar{\xi}^{(i)}-\bar{\xi}^{(j)}\right)}\right)$$
$$\leq 1,\forall i\geq N,j\geq N$$

Taking limit $j\to\infty$, the above inequality becomes

$$\sum_k M\left(\frac{\left\|\alpha_k\Delta_m\left(\xi_k^{(i)}-\xi_k\right)\right\|^{\frac{p_k}{L}}}{\eta}\right)\leq 1,\forall i\geq N$$

Then,

$$G^*\left(\bar{\xi}^{(i)}-\bar{\xi}\right)=\inf\left\{\eta\colon\sum_k M\left(\frac{\left\|\alpha_k\Delta_m\left(\xi_k^{(i)}-\xi_k\right)\right\|^{\frac{p_k}{L}}}{\eta}\right)\leq 1\right\}$$
$$\leq\eta<\epsilon,\forall i\geq N.$$

Therefore,

$$\bar{\xi}^{(i)}\to\bar{\xi}\text{ as }i\to\infty$$

Then,

$$\bar{\xi}^{(i)}-\bar{\xi}\in\ell_M(X,\Delta,\bar{\alpha},P,L)\text{ for all }i\geq N.$$

So,

$$\bar{\xi}^{(N)}\text{ and }\bar{\xi}^{(N)}-\bar{\xi}\in\ell_M(X,\Delta,\bar{\alpha},P,L).$$

Hence,

$$\bar{\xi}=\bar{\xi}^{(N)}-\left(\bar{\xi}^{(N)}-\bar{\xi}\right)\in\ell_M(X,\Delta,\bar{\alpha},P,L).$$

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