

Quantum Hall Physics in Three-Dimensional Weyl Semimetals: A Perspective Beyond Two Dimensions

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Abstract

The quantum Hall effect (QHE) discovered in 1980 has been understood as a quintessentially two-dimensional phenomenon: the quantization of Landau levels in a two-dimensional electron gas yields Hall conductance plateaus at integer multiples of e^2/h . The advent of Weyl semimetals – three-dimensional topological materials characterized by linearly dispersing band crossings (Weyl nodes) and topologically protected Fermi arc surface states – has changed this picture fundamentally. In these systems, a true three-dimensional quantum Hall effect (3D QHE) is achieved via the Weyl orbit mechanism: electrons perform cyclotron orbits that embrace both surfaces of a macroscopic slab by tunneling through chiral bulk Landau levels, leading to quantized Hall conductance in a bona fide three-dimensional crystal. In this perspective review, we summarize theoretical basis and experimental evidence of 3D QHE. We develop the Landau quantization formalism for Weyl and Dirac semimetals, derive the Weyl orbit quantization condition and its thickness dependence, and review experimental observations in Cd_3As_2 , ZrTe_5 , and acoustic metamaterial analogs. We critically discuss the effect of disorder, Weyl node tilt and inter-node scattering on the robustness of the plateau. We highlight the major unsolved questions, including the separation of 3D bulk QHE from surface QHE contributions, the role of electron correlations in chiral Landau levels and the theoretical description of 3D QHE in amorphous topological systems.

Keywords: Quantum Hall Effect; Weyl Semimetal; Fermi Arc; Chiral Landau Level; Weyl Orbit; Three-Dimensional Topology; Hall Conductance; Berry Curvature; Topological Semimetal

Introduction

The 1980 discovery of the integer quantum Hall effect (IQHE) by von Klitzing, Dorda, and Pepper [1] in a two-dimensional electron gas (2DEG) formed at a Si-SiO₂ interface produced Hall resistance plateaus at $R_{xy} = h/ne^2$ for integer n , with a precision approaching one part in 10^9 . This astonishing reproducibility- independent of sample geometry, impurity concentration, or measurement configuration- signaled a topological origin. The explanation came from Thouless, Kohmoto, Nightingale, and den Nijs (TKNN) [2], who showed that the integer n is the first Chern number of the occupied band manifold, obtained by integrating the Berry curvature of Bloch states over the Brillouin zone. Hall conductance thereby became the first experimentally accessible topological invariant in condensed matter physics.

The physical mechanism underlying quantization is the bulk-boundary correspondence. When the Fermi energy lies in the gap between Landau levels, the bulk is insulating but the sample edges carry chiral one-dimensional conducting channels, each contributing exactly e^2/h to the transverse conductance. These edge channels propagate without backscattering, as any attempt to reverse an electron's direction requires crossing the insulating bulk, an energetically forbidden process at the relevant Fermi energy. This results in a perfectly quantized Hall conductance and zero longitudinal resistance at the plateau.

For decades, the QHE was considered a purely two-dimensional effect. In three spatial dimensions, a uniform magnetic field B is applied along the $\hat{z}$ direction, which quantizes the transverse motion, leaving the momentum component k_z as a good quantum number. Landau levels become dispersive Landau bands that always intersect the Fermi surface, preventing the bulk gap needed for quantization. This argument appeared to conclusively restrict QHE to quasi-2D geometries such as heterostructures, thin films, and quantum wells.

Weyl semimetals overturned this expectation [3, 4, 5]. These are three-dimensional materials in which valence and conduction bands cross at isolated points in the Brillouin zone—the Weyl nodes—where the dispersion is linear and chiral. Each Weyl node carries a topological charge (chirality) $\chi = \pm 1$, defined as the Berry flux through a surface enclosing it in momentum space. The non-zero chirality is protected by the absence of any symmetry that could gap out the crossing, and by the fact that the total chirality of all nodes in the Brillouin zone must sum to zero (the Nielsen-Ninomiya theorem). The topological charge has a direct physical consequence: the existence of Fermi arc surface states, which are open contours in the surface Brillouin zone connecting the projections of Weyl nodes of opposite chirality. Unlike the closed Fermi loops of ordinary metals, these arcs cannot be removed by any perturbation that does not annihilate the bulk Weyl nodes.

Potter, Kimchi, and Vishwanath [6] recognized that Fermi arcs and chiral bulk Landau levels, taken together, enable a fundamentally new type of cyclotron orbit—the Weyl orbit—that traverses both surfaces and the bulk of a three-dimensional slab. An electron on the top Fermi arc propagates under the Lorentz force until it reaches a Weyl node projection, enters the bulk through a chiral zeroth Landau level, traverses the sample thickness, emerges on the bottom surface Fermi arc, completes that arc segment, and returns through the bulk via the other node. This 3D trajectory encloses a real-space area that is quantized by a Bohr-Sommerfeld condition, leading to true Landau quantization and Hall conductance plateaus in a macroscopic 3D sample.

Since this prediction experimental groups have identified 3D QHE signatures in Cd_3As_2 [7], ZrTe_5 [8], EuMnBi_2 and acoustic Weyl metamaterials [9]. Theoretical studies have improved the Weyl orbit picture, characterized the impact of disorder and tilt on the robustness of the plateaus, and generalized the formalism to multi-Weyl semimetals and amorphous systems [10]. This review synthesizes these developments into a coherent account. In sections 2 and 3, theoretical and methodological foundations are developed. Section 4 discusses main results. Section 5 and Section 6 present the open questions and conclusions.

Theoretical Framework

Landau Quantization in Two-Dimensional Systems

The elementary theory of QHE starts with an electron of effective mass m^* confined in two dimensions in a perpendicular magnetic field $B = B\hat{z}$.

In the Landau gauge $A = (0, Bx, 0)$, the single-particle Hamiltonian is:

$$H = (p_x^2 + (p_y - eBx)^2) / 2m^* \quad (\text{Eq. 2.1})$$

The eigenvalues are the Landau levels $E_n = \hbar\omega_c(n + 1/2)$ for $n = 0, 1, 2, \dots$, where $\omega_c = eB/m^*$ is the cyclotron frequency. Each Landau level is highly degenerate, with degeneracy per unit area $n_L = eB/h$. When the Fermi energy E_F lies between the n th and $(n+1)$ th levels, precisely $n + 1$ levels are completely filled. The TKNN formula then gives:

$$\sigma_{xy} = (e^2/h) \times (1/2\pi) \int \int_{BZ} \Omega_n(k) d^2k = n(e^2/h) \quad (\text{Eq. 2.2})$$

where $\Omega_n(k) = \partial_x A_y(k) - \partial_y A_x(k)$ is the Berry curvature of band n , and $A_\alpha(k) = -i\langle u_n(k) | \partial_\alpha | u_n(k) \rangle$ is the Berry connection. The integer on the right-hand side is a topological invariant—the first Chern number—that counts how many times the Berry curvature integrates to 2π over the Brillouin zone. Its quantization is exact and independent of any microscopic details of the band structure, provided the spectral gap at E_F remains open.

The Weyl Semimetal: Hamiltonian and Topology

Near a Weyl node of chirality $\chi = \pm 1$ located at momentum k_0 , the low-energy Hamiltonian takes the form:

$$H_\chi(k) = \chi \hbar v_F \sigma \cdot (k - k_0) \quad (\text{Eq. 2.3})$$

where $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices and v_F is the Fermi velocity. The energy spectrum is $E_\pm = \pm \hbar v_F |k - k_0|$, a linear Dirac cone. The topological charge of the node is the integrated Berry flux:

$$C_\chi = (1/4\pi) \oint_S \hbar(k) \cdot dS = \chi \quad (\text{Eq. 2.4})$$

where $\hbar(k) = \nabla_k \times A(k)$ is the Berry curvature field, which resembles a magnetic monopole of strength χ located at the Weyl node. The Nielsen-Ninomiya theorem requires that all Weyl nodes come in pairs of opposite chirality, so the sum of all topological charges in the Brillouin zone vanishes: $\sum_i C_i = 0$.

A minimal two-node model on a cubic lattice with Weyl nodes at $k_z = \pm k_0$ is described by:

$$H(k) = t \sin(k_x a) \sigma_x + t \sin(k_y a) \sigma_y + [m - t_z (\cos(k_z a) - \cos(k_0 a))] \sigma_z \quad (\text{Eq. 2.5})$$

where t, t_z are hopping amplitudes, m controls the mass gap, and a is the lattice constant. The Weyl phase occurs when $|m/t_z| < 2$. The surface Brillouin zone of this model supports a Fermi arc connecting the points $(\pm k_0, 0)$ for any Fermi energy within the Weyl phase. The arc length $k_{\{F, \text{arc}\}} = 2k_0$ is a material parameter set by the node separation.

The Chiral Zeroth Landau Level

When a magnetic field B is applied perpendicular to the Weyl node separation (e.g., B along $\hat{y}$ and node separation along $\hat{z}$), the Landau level spectrum near node χ consists of conventional levels at $E_n = \pm \hbar \omega_c \sqrt{n}$, $n = 1, 2, 3, \dots$, and a unique zeroth Landau level:

$$E_0(\chi, k_z) = -\chi \hbar v_F k_z \quad (\text{Eq. 2.6})$$

This chiral zeroth Landau level is the key topological feature of the Weyl semimetal under a field. Unlike all higher levels, which form $\pm$ pairs, the zeroth level is unpaired and disperses with a definite sign of group velocity $v_z = dE_0/(\hbar dk_z) = -\chi v_F$. A node with chirality $\chi = +1$ carries a zeroth Landau level that propagates in the $-z$ direction, while $\chi = -1$ propagates in the $+z$ direction. This spectral asymmetry—a direct manifestation of the chiral anomaly—means that the zeroth Landau level can carry a net current along the field direction without dissipation, even in the bulk.

The Weyl Orbit and Bohr-Sommerfeld Quantization

Consider a Weyl semimetal slab of thickness L along $\hat{y}$, with B applied along $\hat{y}$ and Weyl nodes separated along $\hat{z}$. An electron on the top Fermi arc at position $z = z_1$ propagates along the arc (in the x -direction, driven by the Lorentz force) until it reaches the projection of the node at $z = +k_0$. It then enters the bulk through the chiral zeroth Landau level of chirality $\chi = +1$, which propagates in the $-z$ direction. After traversing the bulk thickness L , it emerges on the bottom surface at $z = -k_0$, follows the bottom Fermi arc to $z = -k_0$, re-enters the bulk through the $\chi = -1$ chiral level (propagating in the $+z$ direction), and returns to the top surface. The complete orbit encloses an area A_W in the x - z plane:

$$A_W = (k_{F,\text{arc}} / eB) \times \hbar L = k_{F,\text{arc}} \times L \times \ell_B^2 \quad (\text{Eq. 2.7})$$

where $\ell_B = \sqrt{\hbar/eB}$ is the magnetic length. The Bohr-Sommerfeld quantization condition for the orbit area gives the quantized Landau levels of the Weyl orbit:

$$e B A_W / \hbar + \gamma = 2\pi N, \quad N = 1, 2, 3, \dots \quad (\text{Eq. 2.8})$$

where γ is a geometric Berry phase. Substituting Eq. 2.7 into Eq. 2.8 gives the magnetic field values at which Hall plateaus appear:

$$B_N = N \times (\hbar k_{F,\text{arc}}) / (e L) \quad (\text{Eq. 2.9})$$

This is the defining prediction of the Weyl orbit model: the plateau positions scale as N/L , so the inter-plateau spacing $\Delta B = \hbar k_{F,\text{arc}} / (eL)$ decreases with sample thickness L . Thicker samples therefore show more closely spaced plateaus. The Hall conductance at the N th plateau takes the value:

$$\sigma_{xy} = N (e^2/h) / (k_{F,\text{arc}} L / 2\pi) \quad (\text{Eq. 2.10})$$

The factor $k_{F,\text{arc}} L / 2\pi$ accounts for the three dimensionality of the orbit. When $L \rightarrow 0$, the plateau values diverge, which is in accordance with a 2D surface Hall effect; for macroscopic L , the plateau values are much smaller than e^2/h . This scaling is a unique signature that distinguishes 3D QHE from any surface-only contribution.

Axion Electrodynamics

The axion term in the effective action describes the low-energy electromagnetic response of a Weyl semimetal with two nodes separated by momentum $2b$ along $\hat{z}$:

$$S_{\text{ax}} = (\alpha / 4\pi^2) \int d^4x \theta(x) \mathbf{E} \cdot \mathbf{B} \quad (\text{Eq. 2.11})$$

where $\alpha = e^2/\hbar c$ is the fine-structure constant and $\theta(x) = 2b_1 z$ is the axion field, linearly varying in the node-separation direction. The gradient of θ on the surfaces of the slab gives rise to a surface anomalous Hall conductivity:

$$\sigma_{xy}^{\text{surf}} = (e^2/h) (2b / 2\pi) \quad (\text{Eq. 2.12})$$

The formula indicates that the anomalous Hall effect of a Weyl semimetal is determined by the momentum-space node separation $2b$, with a quantization unit of e^2/h per momentum slice of width $2\pi/a$. The axion description is exact in the low-energy limit and provides a rigorous field-theoretic foundation for understanding the 3D QHE in magnetic Weyl semimetals, where time-reversal breaking leads to the anomalous Hall effect and, in the presence of an external field, the Weyl orbit quantization.

METHODS

Kubo Formula and Linear Response Theory

The Hall conductivity σ_{xy} is rigorously computed from the Kubo formula for linear electromagnetic response.

We derive the Hall conductivity σ_{xy} from the Kubo formula for the linear electromagnetic response. For a system with Hamiltonian H and single-particle eigenstates $|n\rangle$ at energies E_n , the zero-frequency, zero-temperature Hall conductance is:

$$\sigma_{xy} = (ie^2\hbar / \text{Vol}) \sum_{\{n \neq m\}} [f(E_n) - f(E_m)] \langle n | v_x | m \rangle \langle m | v_y | n \rangle / (E_n - E_m)^2 \quad (\text{Eq. 3.1})$$

where $v_\alpha = \partial H / (\hbar \partial k_\alpha)$ are velocity operators and $f(E)$ is the Fermi-Dirac function. In the clean limit at zero temperature this reduces to the integral of the Berry curvature over occupied states:

$$\sigma_{xy} = (e^2 / h) \sum_{n(\text{occ.})} \int d^2k / (2\pi)^2 \Omega_n^z(k) \quad (\text{Eq. 3.2})$$

In the presence of a quantizing magnetic field the summation over k -states is replaced by a summation over Landau level indices N and the remaining wavevector k_z . If the Fermi energy is inside the Landau gap, the contribution of the chiral zeroth Landau level is topologically quantized, $\sigma_{xy} = \pm e^2/h$ per Weyl node pair. For large B , corrections from higher Landau levels fade away.

Recursive Green's Function Transport Calculations

The Landauer-Büttiker approach is the most tractable numerical technique for disordered Weyl semimetal slabs. The sample is described by a finite tight-binding lattice of size (WXLXM) attached to ideal semi-infinite leads. The retarded Green's function of scattering region is:

$$G^r(E) = [E \cdot I - H_{\{\text{scat}\}} - \Sigma_L(E) - \Sigma_R(E)]^{-1} \quad (\text{Eq. 3.3})$$

where H_{scat} is the Hamiltonian of the disordered sample region and $\Sigma_{\{L,R\}}$ are the self-energy matrices of the left and right leads, computed analytically from the lead Green's functions. The transmission matrix T is obtained from:

$$T = \Gamma_L G^r \Gamma_R G^a \quad (\text{Eq. 3.4})$$

where $\Gamma_{\{L,R\}} = i(\Sigma_{\{L,R\}} - \Sigma_{\{L,R\}}^\dagger)$ are the broadening matrices and $G^a = (G^r)^\dagger$. The four-terminal Hall resistance is obtained from the Büttiker formula as a function of the partial transmission coefficients T_{ij} between voltage and current probes. This method correctly accounts for disorder-induced localization, inter-node scattering, and finite-size effects in the structure of the Hall plateau. The standard calculation is a recursive evaluation of G^r layer by layer and has a computational cost of $O(W^2 M)$.

Shubnikov-de Haas Oscillation Analysis

Experimentally, we extract the Fermi arc length $k_{\text{F,arc}}$ from Shubnikov-de Haas oscillations in longitudinal resistance R_{xx} as a function of $1/B$. The oscillation period $\Delta(1/B)$ is connected to the extremal cross-sectional area A_F of the Fermi surface in a plane perpendicular to B by the Onsager relation:

$$\Delta(1/B) = 2\pi e / (\hbar A_F) \quad (\text{Eq. 3.5})$$

In the case of a Weyl orbit, $A_F = k_{\text{F,arc}} \times (2\pi/L) \cdot N$, and the oscillation period in $1/B$, in conjunction with the ARPES determined or theoretically computed arc lengths, give $k_{\text{F,arc}}$. The amplitude of the SdH oscillations is temperature dependent and can be described by the Lifshitz-Kosevich formula, from which the cyclotron mass m_c and the quasiparticle scattering time τ can be extracted, giving indirect information about the disorder strength and phase coherence length relevant for the stability of the Hall plateau.

ARPES Characterization of Fermi Arc States

Angle-resolved photoemission spectroscopy (ARPES) directly images the electronic structure of the surface Brillouin zone with a resolution in k of the order of $0.01 \text{ \AA}^{-1}$ and an energy resolution of about 5 meV. We identify the Fermi arc as a non-closed intensity contour at the Fermi level in the surface Brillouin zone connecting Weyl node projections of opposite chirality. For quantitative comparison with transport, the length $k_{\text{F,arc}}$ determined by ARPES at zero field and room temperature must be extrapolated to low-temperature, high-field conditions. Spin-resolved ARPES can also be used to further characterize the spin texture of the arc states, which can confirm their topological character by the winding of the spin vector along the arc.

RESULTS

Hall Plateau Structure and Thickness Dependence

The most diagnostic prediction of the Weyl orbit model is the scaling of the plateau positions with thickness (Eq. 2.9). Theoretical studies of tight-binding Weyl semimetal models have confirmed well-defined quantized plateaus of the Hall conductance σ_{xz} when the magnetic field is along the direction of the Weyl node separation, with the N th plateau appearing at B_N to N/L [11]. Consistent with the orbit quantization picture, the width of each plateau in magnetic field decreases

with increasing sample thickness. The number of visible plateaus in the experimentally accessible field range grows linearly with L , providing a clear spectroscopic signature.

Investigations of tilted magnetic fields [12] demonstrate an additional effect of the chiral zeroth Landau levels. When the field is tilted by an angle θ with respect to the node separation axis, the transverse group velocity emerges for the chiral Landau bands and the edge states will be spatially displaced. This causes an intrinsic offset in σ_{xy} proportional to $\sin(\theta)$. For $\theta >$ a critical angle $\theta_c \approx \arctan(1/\pi)$, the sign of σ_{xy} flips and the edge states jump from one side of the sample to the other. This angular dependence has no analogue in the 2D QHE and provides a clear experimental test of the chiral Landau level mechanism.

For multi-Weyl semimetals with topological charge J , the zeroth Landau level carries J -fold degeneracy and each Weyl orbit encloses J times the standard area. The resulting Hall plateaus appear at $\sigma_{xy} = J N e^2/h \times (2\pi/k_{F,arc})L$, amplifying the Hall signal relative to the $J = 1$ case by a factor of J . This amplification is potentially useful for detection of 3D QHE in thin film geometries where the individual plateau signals are small.

Experimental Observations in Cd_3As_2

Cadmium arsenide (Cd_3As_2) is a three-dimensional Dirac semimetal-equivalent to two Weyl semimetals related by combined time-reversal and inversion symmetry-with exceptionally high electron mobility exceeding $3 \times 10^6 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ at low temperature. Its Dirac points at $k_z = \pm k_D$ are each protected by a C_4 rotational symmetry, and the Fermi arc length $k_{F,arc} \approx 2k_D \approx 0.05 \text{ \AA}^{-1}$ as measured by ARPES.

Tang et al. [7] fabricated Hall bar devices from Cd_3As_2 slabs of thickness $L = 10\text{--}80 \text{ nm}$ using focused ion beam milling and performed magnetotransport measurements at $T = 30 \text{ mK}$ in fields up to 45 T . Clear Hall plateaus appeared at values of R_{xy} consistent with Eq. 2.9, with plateau positions shifting systematically with L as $B_N \propto N/L$. The plateau positions measured in transport agreed to within 15% with predictions using $k_{F,arc}$ from ARPES, and the SdH oscillation period in $1/B$ confirmed the Weyl orbit cross-section. Crucially, R_{xx} at the Hall plateaus was suppressed but non-zero-with typical values $R_{xx}/R_{xx,0} \sim 0.1\text{--}0.3$ -consistent with a residual longitudinal conductance from the dispersive Landau bands, distinguishing 3D from 2D Hall physics.

ZrTe₅ and the Quasi-1D Chiral Landau Level

ZrTe₅ is a quasi-2D layered material with van der Waals interlayer coupling, exhibiting a 3D Dirac semimetal state at low temperature with very low carrier density $n \sim 10^{16} \text{ cm}^{-3}$. Under moderate fields ($B \sim 1\text{--}4 \text{ T}$) at $T = 2 \text{ K}$, the single 3D Dirac cone enters the lowest Landau level regime. Tang et al. [8] observed a quantized Hall plateau in bulk ZrTe₅ crystals at $R_{xy} = h/e^2$, attributed to the contribution of two chiral zeroth Landau levels (one per Dirac node of each chirality) each giving e^2/h . The plateau was accompanied by a strongly suppressed R_{xx} and was stable over a range $\Delta B \sim 1 \text{ T}$, consistent with the Landau level gap $\hbar\omega_c \sim 5 \text{ meV}$ at $B = 2 \text{ T}$.

The observation in ZrTe₅ is different from Cd_3As_2 in an important way: the plateau is observed in a bulk crystal of macroscopic thickness ($\sim 100 \text{ }\mu\text{m}$) where the Weyl orbit quantization condition

(Eq. 2.9) would demand B_N values well below the experimentally accessible field range. The quantization in ZrTe_5 thus stems from the bulk chiral Landau level directly instead of the Weyl orbit: when B is strong enough to drive the whole system into the lowest Landau level, the zeroth Landau level forms a 1D quantum wire along the field direction with the Hall response quantized. This shows that there are two different physical mechanisms that can give rise to 3D QHE signatures: Weyl orbit quantization (which dominates in thin films and semimetals with short arc lengths) and chiral Landau level quantization (which dominates in bulk single crystals at extreme quantum limit).

Disorder Robustness and the Role of Inter-Node Scattering

Numerical results by Li and Liu [13] with the recursive Green's function method on disordered Weyl semimetal slabs showed that 3D QHE is robust against onsite disorder (Anderson disorder, potential strength W) up to a critical disorder strength $W_c \sim \hbar\omega_c$, after which inter-node scattering becomes important. The crucial difference is between intra-node disorder (which scatters electrons within a single chiral Landau level without changing k_z much) and inter-node disorder (which connects nodes of opposite chirality, gapping the chiral level and destroying topological protection).

Interestingly, in the presence of disorder, the longitudinal conductivity σ_{xx} shows non-monotonic behavior as a function of W : it initially decreases as disorder localizes some bulk states, then increases sharply when inter-node scattering opens a conduction channel across the Weyl gap. The simultaneous tracking of σ_{xy} and σ_{xx} as functions of disorder therefore provides a clean diagnostic for the 3D QHE regime.

The Hall conductivity is due to the chiral zeroth Landau levels crossing the gap, and is robust to disorder scattering for an intermediate number of layers along the magnetic field, as confirmed numerically. Interestingly, in the presence of disorder, the longitudinal conductivity σ_{xx} exhibits a nonmonotonic dependence on W , first decreasing as the disorder localizes some bulk states, then increasing sharply as a conduction channel opens across the Weyl gap due to inter-node scattering. A concomitant measurement of σ_{xy} and σ_{xx} as a function of disorder is therefore a clean diagnostic for the 3D QHE regime.

DISCUSSION

Three-Dimensional versus Two-Dimensional Hall Physics

The 3D QHE and the usual 2D QHE share the same unit of quantization, e^2/h , and the same topological origin in the Chern numbers of occupied states. But they are basic four differences. First, the majority of a 2D QHE system is completely insulating at the plateau ($R_{xx} \rightarrow 0$), but the 3D system always has a finite longitudinal conductance from the dispersive chiral Landau bands. 2. In 2D, plateau quantization is universal (same for all samples with a given plateau index) whereas in 3D it depends on sample thickness L by Eq. 2.10. Third, the 2D QHE is robust to all smooth deformations of the Hamiltonian that preserve the gap, while the 3D QHE is robust only to disorder that does not induce inter-node scattering. Fourth, 2D QHE plateaus are metrologically precise (part-per-billion reproducibility), while 3D QHE plateaus have corrections of order $\hbar v_F / (e^2 L)$ that are non-negligible for realistic sample thicknesses.

Distinguishing 3D Bulk QHE from Surface QHE

A key experimental question is to distinguish genuine 3D QHE from a case in which top and bottom surfaces separately enter a 2D Hall state. Both scenarios produce R_{xy} plateaus, and for thin samples the two can have similar magnitudes. The definitive discriminants are: (i) thickness scaling of plateau positions, $B_N \propto N/L$ versus B_N constant for surface QHE; (ii) Weyl orbit SdH periodicity in R_{xx} vs. $1/B$, which is absent for a surface-only mechanism; and (iii) direct spatial imaging of current flow distribution across the sample cross-section, which in 3D QHE should show current flowing through the bulk chiral Landau level rather than being confined to the top and bottom surfaces. No experiment has yet achieved (iii), making it the most pressing near-term goal.

Open Theoretical Challenges

Several theoretical questions remain open. The role of electron-electron interactions in the chiral Landau level has not been systematically studied. In 2D systems, strong interactions in partially filled Landau levels produce the fractional QHE. In the 3D case, the chiral zeroth Landau level is a 1D dispersing channel with a much lower effective density of states, and whether fractional or non-Abelian topological phases can arise in the strongly correlated limit is an open question. The description of 3D QHE in amorphous Weyl systems, recently suggested by theoretical studies [10], is particularly intriguing: without any lattice periodicity, Bloch's theorem and the k -space Chern number description break down, yet topological Landau quantization appears to survive. Developing a real-space topological framework for amorphous 3D QHE is an important theoretical priority.

CONCLUSION

The three-dimensional quantum Hall effect in Weyl and Dirac semimetals represents a genuine extension of one of condensed matter physics's most precise and topologically rich phenomena into a new dimensional regime. The Weyl orbit mechanism, based on the interaction of Fermi arc surface states and chiral bulk Landau levels, offers a consistent physical explanation which is theoretically solid and experimentally validated in Cd_3As_2 and ZrTe_5 . The axion electrodynamics provides a nice field-theoretic complement to the semiclassical orbit picture.

The interesting thing about 3D QHE is not that it is a higher dimensional analogue of 2D QHE, in crucial ways it is not, but that it shows how topological protection can work in a bulk 3D system without the need for a 2D confining geometry. The chiral Landau level, which conducts current through the bulk without backscattering, is a direct signature of the topological charge of the Weyl node, manifesting itself as a plateau in the Hall resistance in a macroscopic laboratory measurement. The remaining experimental and theoretical questions - spatial current imaging, the role of correlations, amorphous systems - are expected to add to our understanding of how topology manifests in real materials far beyond the quasi-2D paradigm of the original quantum Hall discovery.

Conflict of Interest

The authors declared no conflict of interest.

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