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Derivatives Demystified: Unmasking Conceptual Difficulties

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Abstract

This case study is related to "conceptual difficulties in learning derivative" that explores the difficulties of conceptual understanding of derivative. The study is based on data from purposively selected 40 students of grade XI at Makwanpur Multiple Campus, Makwanpur. The data were collected through the CPUBT and interview guidelines. Four problems were asked in the CPUBT which was constructed based on APOS theory. Based on test result, three respondents were selected for interview and the in-depth interview was conducted. The information collected through the interview was systematized and analyzed according to general inductive method. The finding of this study indicates that students had weak concept to understand the derivative as a rate of change, unable to understand the clear geometrical meaning of derivative, unable to make exact sense of limit necessary to study the derivative. Hence, it can be concluded that both teacher and students should focus the conceptual teaching and learning of derivative and pre knowledge of derivative such that they could easily understand the concept of derivative. Thus, teacher should change their teaching style so they can make their classroom very fruitful and learning derivative become a meaningful which avoids the rote learning.

Keywords: derivative, conceptual understanding, difficulties, difficulties in conceptual understanding

Introduction

The study of calculus, particularly the concept of derivatives serves as central role in calculus and so in mathematics education. It has so many applications in different fields of academia such as engineering, physics, economics and many more. Among other things the derivatives measure the steepness of a function, slope of a

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tangent line to a curve at a given point, the rate of change of output relative to input, and helps in finding critical points of a graph. The derivative can also be used as a tool to model the behavior of changing quantities such as population dynamics, finding velocity and acceleration of moving object and others. Therefore, having a solid understanding of derivative is important. Despite their importance, students often encounter significant conceptual barriers when learning derivatives and calculus is seen as a difficult subject because of weak presentation and understanding (Park, 2012).

Teaching and learning derivative is not a easy task. The students feels calculus as one of the difficult subjects, they have difficulties in various terms and language such as limit, tends to, approaches, as small as we please etc., handling quantifiers, symbolic representation, consequent student preference for procedural method rather than conceptual understanding thus to overcome such difficulties active learning method, symbolic manipulation software, computer programming are very effective (Tall 1992). Students, have lack in conceptual understanding, were not successful in analyzing derivative function, haven't interpreted the graph of the derivative function with original function and this case may be the result of the traditional teaching methods (ORHUN, 2012).

Misconception can be defined as the perception (conception) that is far from the consensus of the experts' perceptions for a specific subject (Zembat, 2010). The word "derivative" is colloquially used for both "derivative at a point" and "derivative of a function," which may confuse students about whether the word "derivative" refers to (a) a point-specific value or (b) a function so student's misconception about derivative is also related to their thinking about the tangent line on graphical situation, misconception on tangent line to the curve should intersect the curve only one at a tangency point and students tried to find the equation of tangent line of a curve, struggle to connect between the equation and the graph of tangent line because it intersects the curve at other point also besides the tangency point when they extended the line (Park, 2012). Students, are unable to use the operation with the definition of the derivative, experience difficulty in doing the geometric representation of the derivative and hence the misconception rooted from the thinking derivative of a function as the derivative at a specific point, not knowing the symbolic representation, graphical representation and difficulty to construct a relation between the slopes, tangent and normal hence if they tried to understand the average rate of change and instantaneous rate of change and connect it to the slope of a tangent then they would become clear in the understanding of derivative concept (Kaplan and et al, 2015). Students have some misconception on notion of function (Tarmizi, 2010). students have some wrong concept about slope, students who used the ratio-of-totals approach for slope interpretation often went on to interpret the derivative similarly. A ratio-of-totals $\left(\frac{y}{x}\right)$ instead of the ratio-of-differences ($\Delta y/\Delta x$) approach to slope interpretation was the dominant incorrect reasoning (Tyne, 2016).

Conceptual knowledge is a network of associations of mathematical procedures, integrated with the mathematical principles and procedural knowledge may be defined as as a “sequence of actions "or skills (Heibert & Carpenter, 1992). Instruction with Geogebra had positive effect on students’ scores regarding conceptual knowledge and their overall scores and there was no significant difference between experimental and control group students’ scores regarding procedural knowledge (FatihOcal, 2017) . If any one of the big ideas (rate of change, slope of tangent, limit) is ignored, the concept of derivative may not be fully understood relationally (Zulal Sahin, 2015). Some of the difficulties are caused by teacher's approach of just wanting them to memorize formula's without explaining the meaning of the formula's (Sello Makgakga, 2012). Imbalance between procedural knowledge and conceptual knowledge is a key factor that contributes to their weakness in knowledge for graphing functions from information about the derivative so to increase the conceptual understanding we have to reduce the procedural knowledge in our teaching and learning (Abbey, 2008).

Representation of the content in a various way (numeric, symbolic, graphical) makes the deeper conceptual understanding of the derivative, it helps to relational understanding of concept and blocks the compartmentalized knowledge (Shatila et al., 28th ICTCM). Research has shown that students struggle with connecting graphical and algebraic representations and with converting symbolic information to graphical information (Abbey, 2008). Teachers lack of linkage between new mathematical concept and previously learned mathematics structure, lack of regular assessment system of school are main causes of difficulties in learning mathematics hence mathematics teachers, students, school administration, parents play a vital role in making effective and productive learning of mathematics (Acharya, 2017).

Research in mathematics education consistently highlighted, misconception, conceptual and procedural understanding, struggle to connect definition with intuitive meaning how to overcome from the misconception (Abbey, 2016). Which remain to study to find difficulties of the students and the causes of misconceptions, fragmented understanding, and a lack of confidence in applying derivatives to real-world problems. Also there are the lack of study about the difficulties in understanding of derivative in the context of Nepal. From my experience of teaching the high school, students are memorizing the derivative formula and rote the process of finding derivative and they feel difficult while asking application problem. So, the present study, seeks to explore these struggles in depth. By examining the concept of rates of change, geometrical meaning and concept of limit, the research aims to uncover the root causes of conceptual difficulties. In doing so, it aspires to provide insights that can inform teaching strategies, curriculum design, and learning interventions. Ultimately, the goal is to bridge the gap between procedural competence and conceptual understanding, enabling students to view derivatives not as an obstacle, but as a powerful mathematical tool.

Methods and Materials

The case study design was adopted for this study in the area Makawanpur District, the case was the students of the grade XI of education and science faculties' of Makwanpur Multiple campus. According to the purpose, 40 students were selected as sample by purposive sampling. For the data collection, the CPUBT (Conceptual and Procedural Understanding Based Test) and interview guidelines were used.

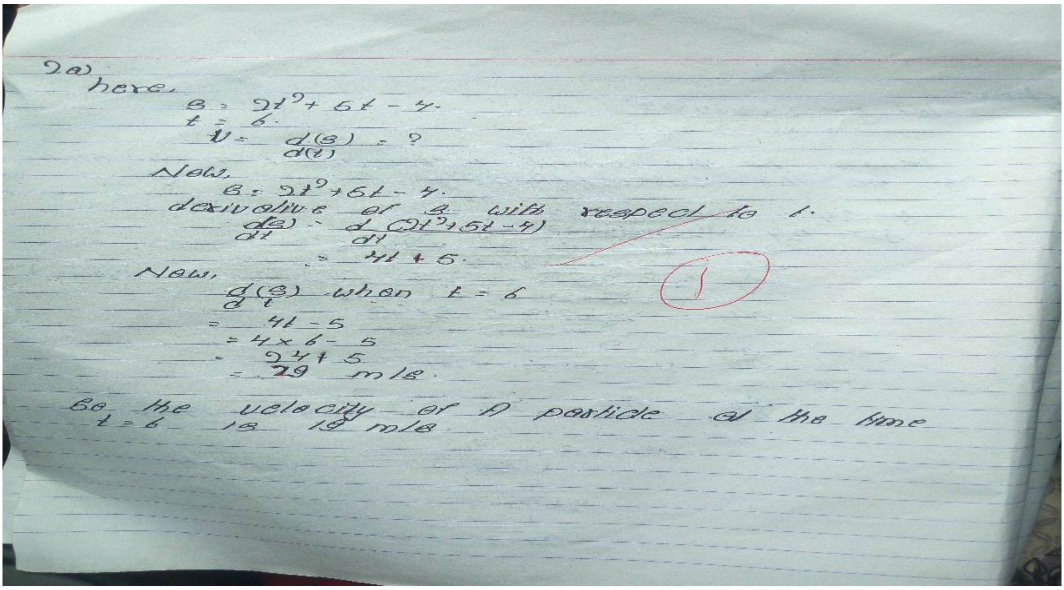
The CPUBT test was made based on APOS (Action, Process, Object and Schema) theory and given to the students, and the result was measured using rubric. After conducting the test, I analyzed the test result and on the basis of it I selected the three respondents who had weak performance having less score in test and conducted in-depth interview with them. The general inductive approach was used for analysis of the data obtained from interview i.e. the interview was recorded transcribed and coded.

Results and Discussion

The conceptual difficulties refer the difficulties in the meaning making of derivative as a rate of change, difficulties in the meaning making of limit and geometrical meaning of derivative and difficulties in connection between them. Difficulties in conceptual understanding are the factors that hinders in learning the basic and fundamental elements in the larger structure of the content. I used the three problems which are in the process, object and schema level questions based on APOS framework. Based on test result the answer and mistakes done by respondents were analyzed by taking an interview. Under the conceptual difficulties I categorized the three main difficulties called difficulties in meaning of rate of change & derivative, derivative as a slope of tangent line and meaning making the sense of limit.

Meaning Making of Rate of Change. This difficulty refers the difficulties related to explain how derivative refers to the rate of change between two variables and the elements that hinder to understand rate of change. The researcher analyzed the difficulties related to rate of change on the basis of concept of function, meaning of average rate of change and instantaneous rate of change. If the respondents are able to explain the above concepts, then there are no any difficulties otherwise it can be seen there is difficulty.

For finding difficulty in the meaning making of rate of change, the process level question was asked. The question was "***explain the derivative as a rate of change. A particle moves in a straight line, the distance s covered by particle in time t is given by*** $s = 2t^2 + 5t - 4$***, where the distance is measured in meter and time in second. Find the velocity, v(t) of the particle at t = 6*** ". In this problem many students leave the first part and total number of student answered the second part. One respondent did the first part and other did not. If the students were able to find the velocity at $t = 6$ with correct unit then, it is considered that they did not have any difficulty otherwise, it is considered they have difficulty. Example of one student's answer sheet was presented below:



From the above solution, the student A left the first part and solved only second part. So, we can say there is difficulty in explaining the derivative as rate of change, was not able to explain the meaning of derivative as rate of change. In this context, the discussion with respondents is given below:

I: What is the meaning of function?

Student A: The function is the relation.

Student B: The function is given in x and y.

Student C: y= f(x) is called the function.

I: What is the meaning of y=f(x) and give me one example.

Student C: I think y is given in the variable x. So y is called the function of x and y= $4x^2+5$ *is the example of function.*

I: If the function is given in s = p+5 then how can you write it in symbol?

Student C: Here, s is the function of p so it can be written as s=f(p).

Student A: y= f(x)

I: How could you explain change in function y = f(x) i.e. the change in the value of x affects the value of y?

Student C: Of course, In the function y = f(x), when the value of x changes then the value of y will be changed this is called change.

I: If the function y is given in the variable t then in which respect do we have to differentiate y?

Student C: We must differentiate with respect to t.

Student A: We differentiate with respect to x? Am I right?

I: But you differentiate the given function s with respect to t in your paper why?

Student A: Ummm.. because we have to find ds/dt so.

I: Do you know derivative is a function?

Student C: Derivative is a value, am I right?

This conversation shows that student A and student B have no clear concept of function, they had big problem in the notation of function. They were not able to explain the meaning of y = f(x). Student could not understand derivative of a given function is again a function. In this regards, Park (2012) and Tarmizi (2010) stated that the student might be confused in understanding of derivative is a function or a point specific and they misunderstood in notion of function. They didn't know why they are differentiating the function with respect to t, had lack of concept about the meaning of "with respect to", "differentiate", didn't care about in dependent and independent variable of given function. The same conclusion is made by the study of Tall (1992). Thus we can conclude that student have difficulty in the meaning making of words or phrases like "Differentiating" & "with respect to" and various terms and language and they just memorize the process.

Further discussion on the basis of answer of question number 2a) can be presented below:

I: Can you tell me how you were able to solve the second part? you found ds/dt and put t=6 why you did it?

Student A: Such type of questions is in textbook & I practiced it.

I: What is the meaning of ds/dt= 29 m/s can you explain it?

Student: ummm..No.

Hence, student did not understand the meaning of rate of change, they could not connect the concept of rate of change in the application problems, couldn't understand the meaning of ds/dt or notation of derivative. There is difficulty to understand notational representation of derivative.

All of participants left the first part of the question, were unable to explain the concept of derivative as a rate of change. Student A and Student B left the question, but student C answered about rate of change. In this context the interview can be presented as below:

I: Do you have any idea about rate of change?

Student A: Well I don't know. I was absent when my teacher taught this chapter.

Student B: Let y= f(x) be the continuous function then by the definition of function, y changes while x will change. If Δx *and* Δy *be the small change in x and y respectively* *then limiting values of* $\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x)-f(x)}{\Delta x}$ *is known as the derivative as a rate of change.*

Again to make the confirmation about the students understanding of meaning of rate of change I asked further questions. The example of face- to -face interview is mentioned as, first I asked "Are you sure this is a rate of change?" The respondent B replied *"I am not sure but I write it from the textbook. I am not very clear about the rate of change".* After that I asked "*Do you have any idea about average rate of change and instantaneous rate of change?"* Respondent replied *"No idea sir"*. In the same manner I asked "Okey", "you wrote $\frac{\Delta y}{\Delta x}$ is a derivative as a rate of change. Did you mean $\frac{\Delta y}{\Delta x}$ is a derivative? Is there difference between $\frac{\Delta y}{\Delta x}$ and dy/dx?", Then respondent replied *"umm…well I am confused but dy/dx also called derivative."*

Students feel very much difficult in recognizing the symbolic representation of average rate of change and instantaneous rate of change. They did not able to differentiate between the symbols $\frac{\Delta y}{\Delta x}$ and dy/dx i.e. respondents didn't know the average rate of change and instantaneous rate of change. Same result is found in the study of Tyne (2016). Thus, from the above facts we can conclude that the students had much difficulty explaining the derivative as a rate of change, they just depending on their textbooks and teachers and they just memorized the process without understanding the conceptual meaning. Thus, there is difficulty in symbolic representation.

For further investigation, I asked "let us check your answer sheet. For the question 2a), you answered the second part well, but you did not write the answer of first part where you had asked explain derivative as a rate of change. Why?" Then, student A replied that *" I cannot explain it. I don't know anything about it. I think these things are not asked in our exam"*. Again I asked "what is represented by the symbol $\frac{\Delta s}{\Delta t}$ and ds/dt?" then, student A replied that *"don't No"*.

When I asked student A then he answered meaning of speed well but he did not relate the speed as average rate of change. Although Student A answered second part of the questions 2a), he did not relate instantaneous rate of change. In the questions students were asked to find velocity at t= 6 sec. All of the participants answered the velocity at t=6 seconds is ds/dt = 29 m/s, which is They didn't know the difference between average speed and instantaneous speed. Hence, student have difficulty in meaning making of rate of change concept.

Again, to find out why they felt the derivative is difficult, I asked "*So tell me why do you feel so difficult?"* then student B replied that

> *I feel it very difficult I don't know why but our teacher never goes to such detail and he just told something about chapter at the beginning of the chapter then he directly goes to the exercise, and I think there was also our mistake because we did not attend the class regularly. So, I don't know anything about the average rate of change and instantaneous rate of change.*

On the same question, student A replied that "*Sir I was absent in the classroom and we always read for passing exam".* Again I asked, "Did you repeat he lesson at your home when your teacher finished the lesson." Then students A and B replieds *"No".*

The above discussion justified the fact that the derivative becomes difficulty cause by teacher and student. Teacher's interest of students procedural understanding is the main causes which reduces conceptual understanding & students were memorized what they learn. (Makgakga, 2012).

All the above discussion justified the fact that the students had difficulty in understanding function notation, derivative as a rate of change, derivative as a function and to understand the phrases like "differentiating", "with respect to". They just memorized a procedure, and could not verbalize meaning behind what they calculated, unable to explain the meaning of ds/dt = 29 m/s. Thus, teacher should be very careful and he/she must explain the derivative as a rate of change in detail. There is a lack of multiple representations of same thing. The symbolic form and verbal form play important role to understand the concept of derivative. Students could not tell the meaning of $\frac{\Delta y}{\Delta x}$ & the meaning of $\frac{dy}{dx}$. So, researcher found that lack of conceptual explanation and multiple representation harms to students to understand derivative as a rate of change conceptually (Aspin wall and Shaw, 2002).

Meaning Making on Slope of Tangent at a Given Point. This difficulty refers the difficulty on the graphical understanding of derivative as a slope of tangent, understanding meaning of slope, finding slope when the derivative function was given & choosing the correct derivative function of given function with reason. If the students can do all the above concept, then there is no difficulty otherwise it should be considered that there was difficulty. Students have misconceptions rooted from not knowing the symbolic representation, graphical representation and difficulty to construct a relation between the slopes, tangent and normal (Kaplan & et al, 2015).

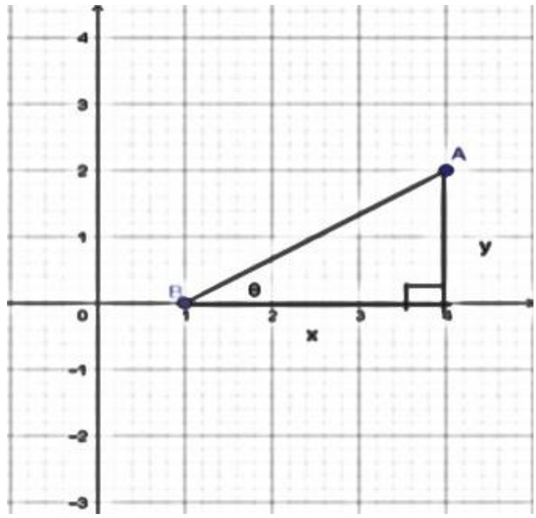
I asked the questions 2b) to measure difficulty related graphical understanding of derivative i.e. derivative as a slope of a tangent line, which was ***"Define differential coefficient of a function f(x) at a point and interpret it geometrically. Compute the slope***

of a function $y = (x^2 + 3)^2$ ***at*** $x = 2$***".*** This problem was process level question. None of my respondents answered this problem. Based on the problem first I had taken an interview about slope which can be presented below:

I: What is the slope of straight line?

Student A: Slope of straight line is the $Tan\theta$

I: Good. Ok tell me can we say slope of given straight line is y/x in the figure alongside?



Student A: Sure sir $Tan\theta = \frac{p}{b}$ *so.*

Student B: $\text{Slope}(\text{Tan}\theta) = \frac{y_2 - y_1}{x_2 - x_1}$.

I: Can we say $\frac{\Delta y}{\Delta x}$ *is the slope of line AB?*

Student B: The slope is the y/x then how is it possible?

Student A &C: Agree with B.

I: Do you know what is Δy

Student A, B and C: No.

Above discussion shows student had difficulty in understanding the meaning of Δy and therefore the concept of slope. They could not relate the slope of line as ratio of difference, were not able to say what the Δx and Δy represent, had incorrect reasoning about slope is the ratio of totals ($\frac{y}{x}$). They couldn't connect the concept of slope as the ratio of difference ($\frac{\Delta y}{\Delta x}$) than ratio of totals ($\frac{y}{x}$). The same conclusion was founded in the research of Tyne (2016). Thus, students had difficulty in symbolic representation of difference in variable x and y.

Majority of the students leave the questions 2b) so researcher and student's conversation about tangent and secant can be presented below:

I: Do you have any idea about tangent and secant?

Student A: Ummm.. tangent is line that touches the curve in one point and no idea about secant.

Student B&C: I think secant is line that touches the curve at two points.

I: Can we say tangent is limiting form of secant?

Student A: No idea.

I: Can you tell me the slope of tangent?

Student A: I think $\frac{\Delta y}{\Delta x}$.

I: Now can we write slope of tangent = $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ *?*

Students A: Of course. Yes I understand, which is dy/dx also. Aaaa… it means derivative represents slope of tangent, right?

I: Excellent, Very good conclusion.

Above interview justified the students had misconception about tangent that touches in only one point of the curve, did not know the relationship between the tangent line and secant line, had more difficult to interpret derivative as a slope of tangent geometrically (Park, 2012). Thus students had lack of conceptual understanding of limit and they could not relate the limit with derivative, not able to relate the concept that when we take limit in slope of secant then slope of secant becomes the slope of tangent. This shows they had more difficulty in the understanding of derivative as a slope of tangent and difficulty in symbolic representation.

For further investigation to find out the causes of difficulty in understanding the geometrical meaning of derivative I asked "Tell me why you feel so difficult? Did your teacher explain this?" Then, Student A replied:

> *Sir, we always focused on our examination, and our teacher told us some little bit but not in detail like that. Also, there is our mistakes too because I couldn't attempt class regularly. Every Friday we go to our home and we leave our class when we are returning.*

On the same question the student C replied that:

> *Sir firstly I don't know what is slope and what is tangent, the reason I think I did not take optional mathematics in class IX and X and also you know which type of teacher we have in our school. Our teacher never taught derivative using graph.*

The above conversation shows that the main reason of the student's weak understandings was being irregular in the class room, focusing the procedural knowledge by teacher, lack of pre knowledge about slope of straight line etc. Student had so much difficulty in understanding geometrical meaning of derivative because of their teacher and themselves. Teacher's lack of explanations and focus on conceptual understanding about the derivatives makes the difficulty to the students. Thus, students had more difficulty in the understanding of derivative as a slope of tangent, teacher should change their teaching style and should focus on the conceptual understanding of derivative rather than the procedural understanding of derivative.

The entire student left the question number 3 unanswered. The students were asked to find the slope from the graph of derivative function. In the same context the interview with the students can be presented as below:

I: Do you know the derivative of given function is again the function?

Student A: Don't know. Derivative is value isn't it.

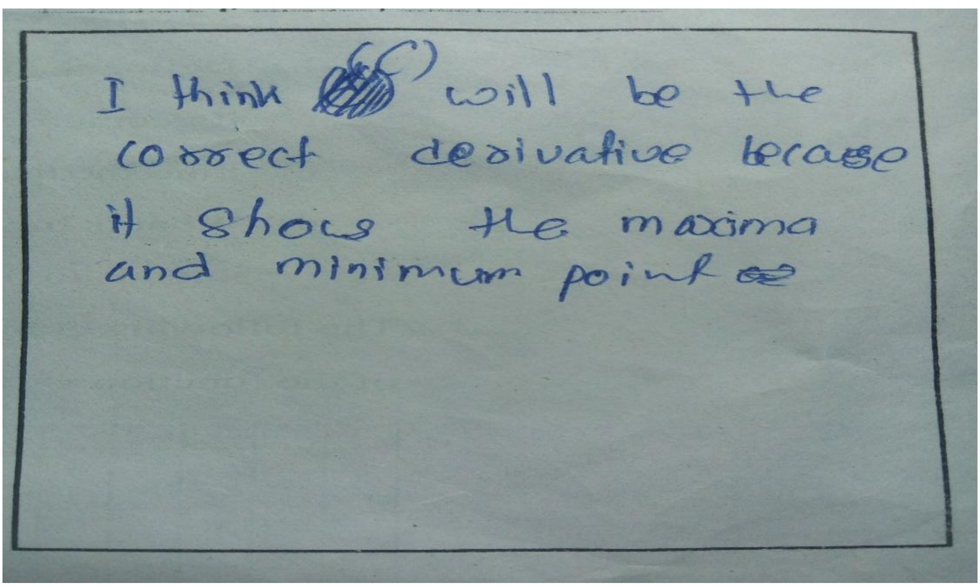
Student B &C: No idea.

I: Can we say the derivative function is function of slope of tangent?

Student A: No idea.

From the above discussion, the researcher concluded that the student had no idea about the graphical understanding of derivative function and they were very surprised when I told them the derivative is a function. No one can explain the derivative function gives the slope of tangent of original function at given value of x. Students were not able to interpret the graph of derivative function (ORHUN (2012). Thus, student had more difficulty in the graphical understanding of the derivative function & finding the slope of original function when derivative function was given.

In the question number 4, it was asked to choose the correct graph of derivative function from given graph of original function with correct reason. If students were able to choose correct graph with correct reasoning, then it can be considered as having no difficulty otherwise it is considered having difficulty. They were unable to choose the correct graph of derivative of given graph with correct reason. For example, one of the answers of students can be presented below:



Above answer sheet showed that students were unable to choose the correct graph of derivative of given function with reason. Student A chose the graph c) and he gave the reason that graph c) shows the maxima and minima. But the reason given by student A is incorrect. Students had lack of graphical understanding of derivative function, unable to understand the increasing (decreasing) function, stationary point and sign of slope of tangent at given interval. So there is difficulty in the graphical understanding of derivative function and original function.

The interview based on questions 4 can be presented as below:

I: What is increasing and decreasing function? Is there any relationship in increasing (or decreasing) function and slope of tangent at a given point?

Student C: No idea.

Student A&B: Sorry.

I: Do you know interval and stationary point?

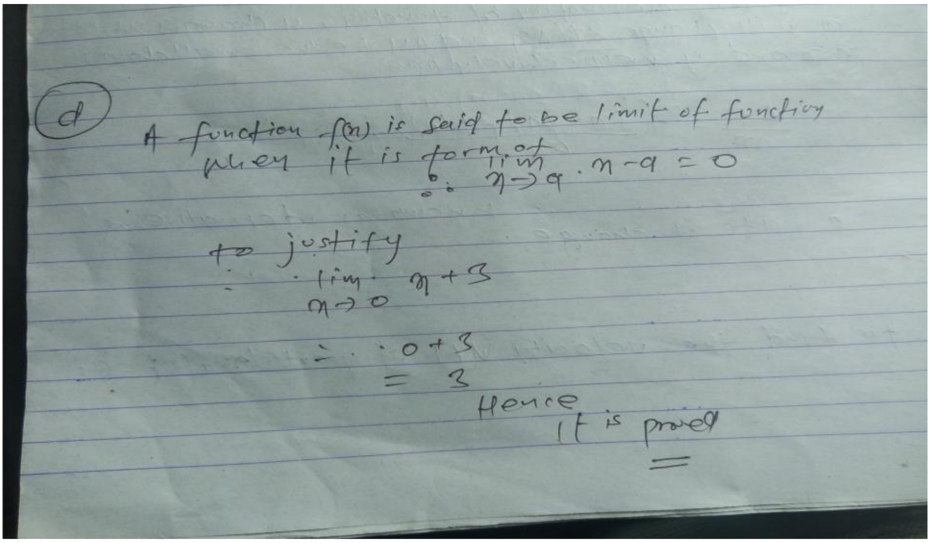
Student A: Not clear.

The above discussion shows that, students had very much difficulty in choosing the correct graph. My entire respondent felt much difficulty in graphical understanding of derivative function and original function. They could not explain sign of slope when graph of function is increasing or decreasing and unable to tell about the stationary point. The study of ORHUN (2012) students hadn't interpreted the graph of the derivative function. Hence, students have difficulty in understanding the derivative of given function.

All the above discussion justified the fact that student had so much difficulty in understanding derivative as a slope of the tangent line. Students are struggling to understand the concept of tangent, misconception about tangent which touches a curve at a point rather than it is a limiting form of secant, they have difficulties in, limit, various terms and language such as limit, tends to, approaches, as small as we please etc., have lack of graphical understanding of the derivative function. As a result of the study by Acharya (2017) teachers lack of linkage between new mathematical concept and previously learned mathematics structure is also one of the reason that students felt difficult to understand the concept which is the same conclusion here. Teacher's approach of just wanting from students to memorize formula's without explaining the meaning of the formula's is again one of the key things which affects students understanding of derivative conceptually (Sello Makgakga, 2012).

Understanding and Sense Making on Limit. This difficulty refers to difficulty in defining a limit of a function and meaning of tends to & other symbols and phrases. The concept of limit is one of the three big ideas (rate of change, slope of tangent, limit) in understanding derivative (Zulal Shah 2015). To find out the difficulties in concept of limit researcher has presented the question number 2d) in the CPUBT test and the difficulties were seen in the answer sheet and some interview questions.

The problem presented in the CPUBT was follows: ***"what is limit of a function and explain how the limit of a function y = x+3 as x tends to 0 is 3"***. In this problem student A and student C did not give any answer on their solution sheet but student B gave the definition of limit but he had not explain about how the limit of given function is 3. The definition given by student B presents below



Analyzing the above definition of limit, it shows they are struggling to define a limit of a function. Students, unanswered the second part of the problem, were unable to explain how the limit of a function x+3 is 3 when value of x tends to 0. Thus, they had no conceptual understanding about right - hand limit and left- hand limit, they were struggling to understand the meaning of tends to. To confirm the difficulties related to the concept of limit researcher has conducted the interview with all respondents. The interview taken by researcher with student B can be mentioned as below:

I: What is limit of a function? Is a limit of a function is again a function or particular value?

Student B: I don't know.

I: But you write limit of a function is again a function in your answer sheet?

Student B: I wrote it carelessly.

I: Can you find the limit of a function f(x) = x+3 as x tends to 0.

Student B: Sure. The limit of given function is 3.

I: But how?

Student B: Put the value of x=0 in given function.

From the above discussion, the students were unable to define limit of a function, understand the limit of a function is always a value. Student B has the misconception that the limit is again a function, and it has a form. This shows students had procedural knowledge they found that the limit, but they were not able to explain how? Same as the study of Abbey (2008) shows students' preference for procedural knowledge, resulting from their prior success with those procedural skills, inhibits their conceptual learning. Further discussion can be presented below:

I: If then, could you tell me the limit of a function, $f(x) = \frac{x^2-4}{x-2}$ as $x \to 2$ *?*

Student B: It is 0/0 not determined.

I: What is the meaning of x tends to 2. Do you think x tends to 2 means, put the value of x exactly 2 on given function?

Student B: Yes we find the limit of a function in the same way but it is 0/0 I couldn't understand.

Here, Student B had misconception about the meaning of "tends to". Students thought that x tends to 2 means value of x is exactly 2. This shows they had lack of conceptual understanding of limit, and they had been focused on procedural understanding. When I asked to find limit of function $f(x) = \frac{x^2-4}{x-2}$ as $x \to 2$ then they become very much surprised. All these facts justified that the students did not understands the symbols, phrases like "tends to" and other conceptual understanding.

From the above interview I concluded that students have very much difficulty about the concept of limit of a function. They have a misconception about limit of a function is a particular value or the limit of a function is again a function. Also students cannot explain what meaning of the symbol $x \to a$ was. The students had difficulties in, defining limit and infinite process, various terms and language such as limit, tends to, approaches, as small as we please etc., handling quantifiers, symbolic representations, and consequently student's preference for procedural method rather than conceptual understanding These are all because of lack of conceptual understanding.

Conclusion

From the analysis, interpretation, the findings are that students have weak concept to understand the derivative as rate of change. They are not able to understand the clear geometrical meaning of derivative and also unable to make the exact sense of limit necessary to understand derivative. From these findings, I concluded that the students have more difficulty in conceptual understanding of derivative which may discourage the students from taking the study of derivative easily. Based on above findings, main difficulties are, the students more focusing on procedural understanding, lack of multiple representation of derivative, lack of graphical understanding of function, lack of pre knowledge, weak performance of teachers etc. Hence, it can be concluded that the teacher is focusing only on procedural understanding of derivatives and students becomes exam oriented, which makes derivative as one of the difficult topic in Mathematics education. Thus, both teacher and students should focus the conceptual learning and pre knowledge such that they can easily understand the concept. Thus, teachers should change their teaching style so they can make their classroom very fruitful and learning derivative becomes meaningful which avoids the rote learning.

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