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A Review and Thermal Analysis of a Heat Exchanger with Conical Cross Flow Arrangement

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Abstract— The frequently used device for transfer of heat energy is the heat exchanger which allows the heat transfer from one fluid to another. The heat exchanger with cross flow arrangement has tube-side and outside fluid flows perpendicular to each other. It often occurs that in the cross-flow arrangements due to perturbations of fluid flow and heat transfer conditions, some parts of tubes in bundle are excluded from an intensive heat transfer process. This results in the effects as if the tubes were progressively shorter in comparison to their real length and the thermally active tube bundle were conical in shape (trapezium). The thermal analysis of the cross-flow arrangement for any number of tubes in bundle with progressive changes (decrease / increase) in tube length has been carried out and presented in this paper. To solve this problem the method of the weighted mean value of outside fluid temperature was applied. The numerical results are presented in diagrams with the effectiveness of tube side fluid versus the effectiveness of the outside fluid. The outside liquid flows one time in direction with narrowing cross-section that results in increasing of its velocity, and the second time it flows in opposite direction where the stream expands and liquid slows down its velocity. It was found that for assumed equal number of transfer units (NTU) of a heat exchanger for these both cases the second one leads to higher values of effectiveness.

Keywords — Conical Cross-Flow, Effectiveness, Number of Transfer Unit, Relative Deviation, Heat Transfer Coefficient

1. Introduction

For an effective heat transfer process, the cross - flow heat exchangers have found great demand in technology. This type of heat apparatus is gaining wide applications in industry and other domains e.g. in air conditioning, food storage etc. There are still many unsolved problems related to the thermal theory of these exchangers which are to be clarified. Thus, it needs a continuous development in the theory of these heat exchangers for the purpose of applications by the designer. The first papers regarding thermal theory of cross - flow arrangement were published by Smith [1] and Hansen [2]. Further development was done mainly by Braun [3] and Nicole [4].

Recently, the analysis of temperature fields and the effectiveness of heat transfer in a codirected cross flow arrangement for any number of tubes in bundle has been carried out by Bes and Roetzel [5] on the basis of a new and exact method developed by them in paper [6] and described in paper [7]. The point of differences between mentioned papers persists in an assumption regarding the overall heat transfer coefficient that in paper [7] was taken as constant and in paper [5] it can vary from tube row to tube row. In both cases length of tubes was considered to be constant.

Now and then it happens that in the bundle of tubes, due to perturbation of fluid flow- and heat transfer conditions, these tubes are partly excluded from an intensive heat transfer process. Then, it works as if the tubes were progressively shorter in comparison to their real length and as if the thermally active tube bundle were conical in shape.

The purpose of this paper is an extension of the analysis developed by Bes and Roetzel [5] to the case of the any length of tubes in bundle for cross-flow arrangement. A solution to the problem is also presented by using the method developed in paper [6]. Moreover, due to change in length of tubes, the alternations in the overall heat transfer coefficients and as a consequence in the number of transfer units (ntu) for any tube are taken into account.

2. Literature Review

The effects of the conical ring turbulator inserts on the heat transfer rate and friction factor are experimentally investigated by Promvonge [8]. It is found that the ring to tube diameter ratio and the ring arrays provide a significant effect on the thermal performance of the test tube. The experimental results demonstrate that the use of conical ring inserts leads to a higher heat transfer rate than that of the plain surface tube.

Yan et al [9] investigated numerical simulation of conical tube bundles. It is observed that the effect of structural parameters on heat transfer characteristics. fluid flow characteristics inside tube of different cross section. Also investigated result shows that cone angle cross - section has significant effect on heat transfer. In addition to this, helical pitch has little influence on heat transfer enhancement. It is also seen that the secondary fluid becomes intensive along the tube due to increase of tube curvature. Secondary fluid flow contains four contours and flow direction of each contour are different due to this heat transfer rate increases.

Shinde and Danger [10] performed experiments on cone shaped helical coil heat exchanger at various Reynolds number. The main focus is on a comparison between cone and simple helical coil heat exchanger taking pitch, height and length of both coils constant. It is found that for cone shaped coil heat transfer rate are up to 1.38 times higher than simple helical coil. Also, overall heat transfer coefficient, convective heat transfer and Nusselt number increases when fluid flow rate increases.

The experimental investigation on thermal analysis of conical coil heat exchanger is presented by Purandare et al [11]. The observations recorded were leads to analysis the heat exchanger for heat transfer and pressure drop characteristics at different flow rate of cold and hot fluid. The various parameters (heat transfer coefficient (hi), Nusselt number (Nu) effectiveness (ε) and friction factor (f)) are estimated for the heat exchanger. The analysis indicates that, Nu and f are the functions of flow rates, tube diameter, curvature ratio and cone angle. Nu increase with increase in tube side flow rate whereas it reduces with increase in shell side flow rate, increase in cone angle and increase in tube diameter. The empirical correlations are proposed to bring out the physics of the thermal aspects of the conical coil heat exchangers.

Kumar and Singh [12] compared the heat transfer in cone shaped helical coil and simple helical coil. The pitch, height and length of both the coils are kept same for comparative analysis. The calculations have been performed for the steady state condition and experiments were conducted for different flow rates in laminar and turbulent flow regime. It was observed that the effectiveness of the heat exchanger for the cone shaped helical coil is more than that for the simple helical coil. Results show that the heat transfer rates for the cone shaped helical coil are comparatively higher than that of the simple helical coil.

Momale et al [13] studied the performance of conical helical coil heat exchanger by using Computational Fluid Dynamics (CFD) technique. For this study two shell geometry such as straight shell and conical shell are used. It is found that there is much improvement in heat transfer as the more shell fluid comes in contact with the tube fluid when conical shell instead of helical shell is used. The pressure drop increases with conical shell arrangement.

Bhirdi et al [14] studied the effect of taper angle on the performance of cone shaped helical coil heat exchanger. In this work, the amount of heat transfer between cold water and hot water is studied with the change in taper angle of the conical coil. It is observed that the angle of the coil has a positive influence on the amount of heat transfer in the heat exchanger. The overall heat transfer coefficient of the heat exchanger is higher for conical shaped heat exchangers and it increases with the decrease in the inclination angle. Therefore, thermal effectiveness of the heat exchanger increases with the conical shaped heat exchanger with the decrease in the inclination angle of the coil.

Koirala [15] presented thermal model in which the fluid volume behind the window of each baffle so-called “dead volume” rendering a stagnation of flow taken into account and change of temperature of the shell side fluid during its flow from tube to tube is considered. The thermal analysis of the cross- counter flow arrangement for any number of tube in bundle with progressive changes in tube length has been carried out. The numerical computations with different tube and baffle numbers are performed. It is found that at the beginning the effectiveness of the heat exchanger increases distinctly with increasing tube number and baffle number. But after certain number of tube and baffles the increasing rate weakens and it becomes very small. Due to the presence of “dead volume” there is reduction in the thermally active heat transfer surface area which leads to a smaller value of Number of Transfer Units (NTU). Hence, the results of computation obtained in this paper with the consideration of the “dead volume” will provide lower value of the effectiveness than that of without the consideration of it.

Ehsan et al [16] examined the transformation of cone fins into truncated cone fins under low laminar forced convection heat transfer, focusing on their effects on heat transfer and steam condensation in a staggered heat sink arrangement. The results showed that using a truncated conical fin instead of a conical fin results in decrease of the heat transfer, the convection heat transfer coefficient, and the steam condensation rate. An increased tip diameter to base diameter ratio further reduces these parameters due to less contact area between the fin base and the plate surface.

Hundiwale et al [17] focused on designing and analyzing a shell and helical cone coil heat exchanger, highlighting its ability to reduce unit size compared to a standard shell-and-tube heat exchanger operating under the same thermal load. The experimental setup included a shell and helical cone coil configuration, utilizing diesel engine exhaust as the hot gas source and tap water as the cold fluid in a counter flow arrangement. The investigation revealed that helical cone coils extracted 15% to 20% more heat compared to conventional straight tubes, demonstrating improved effectiveness and compactness of the heat exchanger.

The existing literature extensively cover experimental and numerical data on conical and helical coil heat exchangers. However, there is a noticeable gap in research regarding conical tube bundle and shell heat exchangers from thermodynamic perspectives, particularly in the context of overall heat transfer coefficients, number of heat transfer units and therefore thermal effectiveness. Hence, the primary objective of this paper is to calculate the thermal effectiveness of shell-integrated conical tube bundle cross - flow heat exchanger. The aim is to optimize the contact area between the tube side and the shell-side fluid with the goal of achieving superior heat recovery compared to traditional cross - flow heat exchangers.

3. The Analytical Method

For the stream flowing at the outside crosswise to the bundle of tubes, Braun [3] analyzed the problem of thermal effectiveness of the tube bundles by solving the system of partial differential equations. Nicole [4] collected and discussed numerous variants which deal with the thermal effectiveness of different flow arrangements and different connections of the fluids flowing inside the tubes. The analytical method to solve the problem under consideration presented in this paper is based on a method purposed by Bes and Roetzel [5], [7].

3.1. Assumptions

For the simplification of the analytical problem following considerable assumptions are made:

The fluid thermal conductivity and the overall heat transfer coefficient are constant for each tube, but they can vary from tube to tube in the bundle. The heat capacity rate of each flow is constant at every part of the heat exchanger and it is not dependent on temperature. The temperature of the tube-side fluid changes continuously along the path of the flow. The flow of fluid outside of the tubes is turbulent. Due to well mixing of the fluid the temperature of the tube-side fluid is uniform at any cross-section of the tube. The temperature of the fluid outside the tube changes in both directions: along the perimeter of the tube as well as along the tube axis. Neither stream undergoes a change of phase. The temperature of the outside fluid at the inlet of the heat exchanger is uniform. But this condition can be replaced by another weaker condition for which any temperature at the inlet of the tubes is assumed to be known. No external work is done on the system and effect of gravitational potential energy is neglected. Heat losses from the system are negligible.

3.2. Energy Balance for Fluids Flowing Inside and Outside a Single Tube

A simplified model of the flows in a single tube selected from the bundle is shown in figure 1. Let one consider a single tube **i** selected from the bundle in any one sector of the heat exchanger.

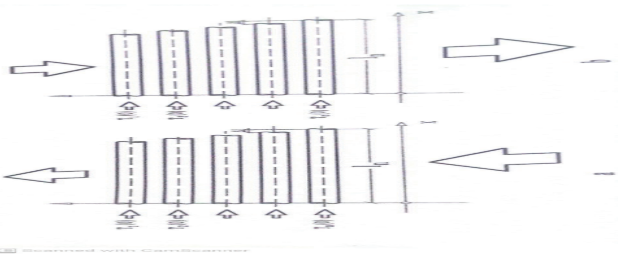


Fig. 1. Schematic representation of "conical" cross - flow with (a) decreasing cross - section and (b) increasing cross - section for the outside fluid.The temperature distribution along the flow paths of streams inside and outside a tube can be calculated after setting up the energy balance equations and by using relation for the heat flux transferred between the fluids. Finally, the energy balance for the tube-side fluid **1** flowing in the **i** th tube has the following form:

$$-\frac{C_1}{n}dt_{1,i} = k_i(t_{1,i}-t_{2,i})b_0dx \qquad (1)$$

where **C** is the heat capacity rate, **n** is the number of tubes in a bundle, **t** is the temperature in Kelvin, **k** is the overall heat transfer coefficient, b_0 is the perimeter of a tube, **x** is the coordinate in the direction of tube length. Also, the numbers **1** and **2** represent tube side and fluid and outside fluid.

The energy balance for the fluid outside the tube is given by

$$C_2(t_{2,i-1/2}-t_{2,i+1/2})\frac{dx}{l_i} = k_i(t_{1,i}-t_{2,i})b_0dx \qquad (2)$$

The local temperatures $t_{2,\ i-1/2}$ and $t_{2,i+1/2}$ of the fluid flowing outside the tube, at the conventional border dividing the thermal influence of the tubes **i-1, i** and **i+1** respectively, are necessary in this equation. An introduction of dimensionless quantities such as: $s.T_i$,

ϑ_i , $ntu_{1,i}$ in equation (1) leads to the set of differential equations

$$-\frac{dT_i}{ds} = ntu_{1,i}\frac{l_1}{l_i}(T_i - \vartheta_i) \quad i = n, n-1, \ldots\ldots 2,1. \quad (3)$$

where ***T***, ϑ, **s** and **ntu** are dimensionless temperature of tube side fluid, dimensionless temperature of outside fluid, dimensionless path of fluid for the first tube and the number of transfer units for a single tube respectively.

Similarly, linking the equations (2) and (3) as well as introducing the dimensionless parameters T_i, ϑ_i, $ntu_{2,i}$, one gets an equation

$$\vartheta_{i-1/2} - \vartheta_{i+1/2} = ntu_{2,i}(T_i - \vartheta_i) \quad (4)$$

Now, there are two equations (3) and (4) with three unknown variables ϑ_i , $\vartheta_{i\pm 1/2}$ and T_i which have to be found. These equations cannot be solved without additional information about the dimensionless temperature ϑ_i between the boundary values of $\vartheta_{i-1/2}$ and $\vartheta_{i+1/2}$. This kind of information can be obtained from the dimensionless mean temperature ϑ_i of the outside fluid which is defined as the weighted average of the temperatures $\vartheta_{i+1/2}$ and $\vartheta_{i-1/2}$ described in the paper [7] as

$$\vartheta_i = \omega_i\vartheta_{i-1/2} + (1-\omega_i)\vartheta_{i+1/2} \quad (5)$$

where ω is a weighing factor for mean temperature of outside fluid and takes values between **0.5** and **1**.

The weighing factor determines to which of the outside dimensionless temperatures i.e. $\vartheta_{i-1/2}$ or $\vartheta_{i+1/2}$ the temperature ϑ_i is closer. The derivation and evaluation of the weight-parameter ω_i is presented later on. Thus, at this stage for further analysis it is temporarily allowed to assume that the value of ω_i, is known.

Eliminating the temperatures ϑ_i from equations (4) and (5) yields

$$\vartheta_{i+1/2} - \mu_i\vartheta_{i-1/2} = (1-\mu_i)T_i \quad (6)$$

where $\mu_i = 1 - ntu_{2,i}/(1+\omega_i ntu_{2,i})$ (7)

After eliminating the temperature ϑ_i from equations (3) and (5), the use of the equation (6) gives

$$-\frac{dT_i}{ds} = B_i(T_i - \vartheta_{i+1/2}) \quad (8)$$

where $B_i = (l_1/l_i)ntu_{1,i}/(1+\omega_i ntu_{2,i})$ for i = n, n-1, .2, 1. (9)

Now, let us consider a cross-section of any selected tube and the fluid belonging to it. Due to well mixing of the inside fluid across the tube its temperature at any location between ***x*** and ***x+dx*** is locally constant. Indeed, the slice of the tube under consideration with the appropriate volume of fluids should be treated as a small recuperator with the heat transfer surface area equal to ***bo.dx***.

If in this simple recuperator the temperature of one fluid is constant, then for solving the problem the flow direction of this inside-fluid is irrelevant. This is an essential fact in the theory of heat exchangers. Thus, the mean difference of temperature should be calculated as the logarithmic mean of the temperature differences at the inlet and outlet. This leads to the relation:

$$\vartheta_i - T_i = \frac{(\vartheta_{i+1/2} - T_i) - (\vartheta_{i-1/2} - T_i)}{\ln\left(\dfrac{\vartheta_{i+1/2} - T_i}{\vartheta_{i-1/2} - T_i}\right)} \quad (10)$$

Evaluating the equation (4) for $ntu_{2,i}$ and the equation (6) for μ_i, and putting these two parameters in the equation (10), we get,

$$\mu_i = e^{-ntu_{2,i}} \quad (11)$$

By eliminating μ_i from equations (7) and (11), a required expression for the parameter ω_i of the outside fluid temperature can be derived as

$$\omega_i = \frac{1}{1-e^{-ntu_{2,i}}} - \frac{1}{ntu_{2,i}} \quad (12)$$

Now, there is a system of difference-differential equations (6) and (8) with only two unknown functions T_i and $\vartheta_{i\pm 1/2}$ which can be easily solved with help of the boundary conditions adjusted to the flow arrangement as shown in the figure 1.

3.3. *The Temperature Distribution of Tube-Side- and Outside Fluid*

In order to calculate the temperature distribution of the fluids flowing inside and outside the tubes, one refers to the set of the difference-differential equations (6) and (8) which was derived in section 3.2. According to the mathematical way of classification, it is a system of **2n** linear difference-differential equations with constant coefficients. The set of such types of difference – differential equations can be solved by using two different ways: either by the Laplace transformation or by the recursive method of integration i.e. integration of the differential equations one by one. In this paper the second way is chosen.

Starting with the differential equation (8) for **i = n** and by using assumption that the outside fluid temperature at the inlet to the heat exchanger equals unity i.e. $\vartheta_{n+1/2}(s) = 1$, it can be proved as in the paper [5] that step-by-step procedure of integration of equations (9) and (11) leads to the following relation for the tube side fluid temperature as:

$$T_i(s) = 1 - c_i e^{-B_s s} - \sum_{j=i+1}^{j=n} a_{i+1,j} b_{i,j} c_j (1-\mu_j) e^{-B_j s}$$

for i = n, n-1, ,……………, 2, 1. (13)

where auxiliary parameters

$$a_{i+1,j} = \prod_{k=i+1}^{k=j-1} (B_k - \mu_k B_j) / (B_k - B_j \quad (a_{i+1,i+1} = 1 \text{ is defined}) \quad (14)$$

and $b_{i,j} = B_i / (B_i - B_j)$ (15)

Similarly, the general equation for the outside fluid temperature can be written as

$$\vartheta_{i+1/2}(s) = 1 - \sum_{j=i+1}^{j=n} a_{i+1,j} c_j (1-\mu_j) e^{-B_j s} \quad (16)$$

for i = n, n-1, ……, 2, 1.

The unknown parameters c_i in equations (13) and (16) are integration constants which will be found with the help of boundary conditions. Using the boundary condition for the dimensionless temperature of tube side fluid at the entrance to every tube which according to assumption T is equal to zero i.e.

T_i (s = 0) = 0. for i = n, n-1, …………, 2, 1. (17)

Putting these n boundary conditions from the equation (17) in the equation (13) lead to a recursive formula for the determination of all n integration constants as

$$c_i = 1 - \sum_{j=i+1}^{j+n} a_{i+1,j} b_{i,j} c_j \left(1-\mu_j\right) \quad (18)$$

for i = n, n-1, ……, 2, 1.

with the starting value c_n = 1.

3.4. Overall Heat Transfer Coefficient for Outside Fluid in Single Tube

Let us consider the conical cross - flow of fluids in any sector of the heat exchanger under consideration in which fluid **1** flows inside of parallel tubes and fluid **2** flows outside of tubes as shown in figure 1. The amount of fluid passing the bundle of tubes from outsides is distributed uniformly and it is the same for each tube row. But due to decrease/increase in the length of tubes row to row in the conical cross - flow arrangement its velocity increases/decreases as shown in the figure 1a and 1b respectively. This causes the stepwise change in the local heat transfer coefficients for the fluid **2** flowing outside of the tubes as well as in the overall heat transfer coefficients from tube to tube. The local heat transfer coefficients $\boldsymbol{\alpha_{2,i}}$ of the outside fluid in each tube can be calculated by using the relation derived in papers [18], [19] for turbulent heat transfer as

$$\alpha_{2,i} = \alpha_{2,n} \left(\frac{l_n}{l_i} \right)^{0.8} \quad (19)$$

The heat transfer coefficients for the **ith** tube can be now determined by using the equation (19) when heat transfer coefficients for the **nth** tube $\boldsymbol{\alpha_{2,n}}$ and length ratios $\mathbf{1_n/1_i}$ for all tubes are known.

Similarly, the relations for overall heat transfer coefficients $\boldsymbol{k_i}$ is defined from the following well known relation derived in paper [19] as

$$\frac{1}{k_i} = \frac{1}{\alpha_1} + \frac{1}{\alpha_{2,i}} \quad (20)$$

Combination of the equations (19) and (20) leads to required relation for overall heat transfer coefficients $\boldsymbol{k_i}$ for each tube **i** in the conical cross - flow arrangement of any sector as

$$\frac{1}{k_i} = \frac{1}{\alpha_1} + \frac{1}{\alpha_{2,n}} \left(\frac{l_i}{l_n} \right)^{0.8} \quad (21)$$

Thus, the overall heat transfer coefficients for **ith** tube can be now determined by using the equation (21) when heat transfer coefficients of the tube side fluid $\boldsymbol{\alpha_1}$, that of the outside fluid for the **nth** tube $\boldsymbol{\alpha_{2,n}}$ and length ratios $\mathbf{1_i/1_n}$ for all tubes are known.

3.5. Number of Transfer Unit for Outside Fluid in Single Tube

Due to the change in the overall heat transfer coefficients as well as change in the tube lengths from tube to tube the number of transfer units in the case of the outside-fluid also differs from tube to tube. As given in the reference [20] the relation between number of transfer units of whole exchanger (*NTU*) and that of a single tube (ntu) for the outside fluid is given by

$$NTU_2 = \sum_{i=1}^{n} ntu_{2,i} = \sum_{i=1}^{n} \frac{k_i b_0 l_i}{C_2} \quad (22)$$

Combining this relation with the equation (21), we get following relatio

$$NTU_2 = \frac{\alpha_1 b_0 l_n}{C_2} \sum_{i=1}^{i=n} \frac{\frac{l_i}{l_n}}{1 + \frac{\alpha_1}{\alpha_{2,n}} \left(\frac{l_i}{l_n} \right)^{0.8}} \quad (23)$$

After simple algebraic manipulations using the equations (21), (22) and (23), we can write:

$$ntu_{2,n} = NTU_2 \bigg/ \left\{ 1 + \left(1 + \frac{\alpha_1}{\alpha_{2,n}}\right) \sum_{i=1}^{i=n-1} \left(\frac{l_i}{l_n}\right) \bigg/ \left[1 + \frac{\alpha_1}{\alpha_{2,n}} \left(\frac{l_i}{l_n}\right)^{0.8} \right] \right\} \quad (24)$$

for the n^{th} tube

and

$$ntu_{2,i} = ntu_{2,n} \left(1 + \frac{\alpha_1}{\alpha_{2,n}}\right)\left(\frac{l_i}{l_n}\right) \bigg/ \left[1 + \frac{\alpha_1}{\alpha_{2,n}} \left(\frac{l_i}{l_n}\right)^{0.8} \right]$$

for the tubes i = n – 1, n-2, ……..,2,1. (25)

The relations for the tube length ratios l_i/l_n can be expressed in term of given parameter **κ** are:

$$\frac{l_i}{l_n} = 1 - (n-i)\kappa \quad (26)$$

where $\boldsymbol{\kappa = \Delta l / l_n}$ is the ratio of tube length difference to length of $\mathbf{n^{th}}$ tube known as *relative deviation*.

Finally, the number of transfer units for all the tubes can be determined by using the equations (24) and (25) with the help of the equation (26).

3.6. Effectiveness of Heat Exchanger

The heat exchanger effectiveness is defined as the ratio of the actual rate of heat transfer to the maximum possible rate of heat exchange between the fluids. The maximum possible rate would be obtained in a counter flow heat exchanger of infinite heat transfer area. Moreover, the effectiveness depends on the heat capacity rate of the fluids. The maximum heat transfer can never be attained in practice. To evaluate the performance of a heat exchanger the calculation of its effectiveness is used [21]. For the flow arrangements under consideration the thermal effectiveness is equal to the arithmetical mean value of the tube side fluid temperatures at the outlet of the heat exchanger as given in the paper [5]:

$$P_1 = \frac{1}{n} \sum_{i=1}^{i=n} T_i(s) \Big| s = l_i / l_1 \quad (27)$$

Thus, by using the equation (13) in the equation (27), we get

$$P_1 = 1 - \frac{1}{n} \sum_{i=1}^{i=n} \left[c_i e^{-B_i l_i / l_1} - \sum_{j=i+1}^{j=n} a_{i+1,j} b_{i,j} c_j (1 - \mu_j) e^{-B_j l_i / l_1} \right] \quad (28)$$

The effectiveness $\mathbf{P_2}$ is calculated with the help of the following simple relation:

$$P_2 = P_1 R_1 \quad (29)$$

where R_1 is the heat capacity rate ratio of the tube side fluid.

The relations (28) and (29) allow us for construction of diagrams: the effectiveness of tube-side fluid versus the effectiveness of outside fluid in the way as reported in the paper [21].

4. Numerical Results and Discussion

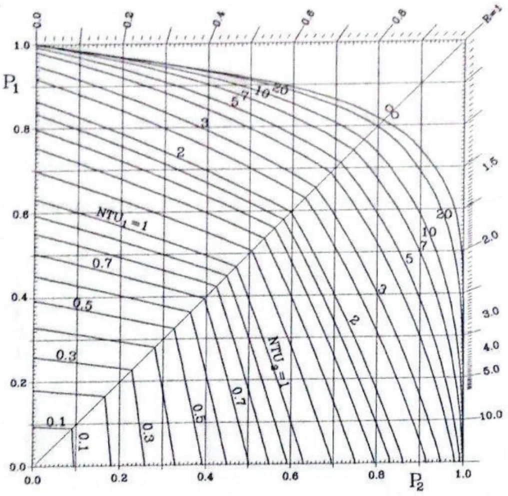
Numerical results obtained from the theory developed in this paper are illustrated in figures 2, 3 and 4 for the number of tubes n = 10 and deviation of the flow expressed by relative deviation i.e., decrease/increase of the tube length, κ = +0.03, −0.03 and 0 respectively. The effectiveness of the tube side fluid P_1 is plotted against the effectiveness of the outside fluid P_2 to demonstrate how effective a heat exchanger will behave over a range of the Number of Transfer Units NTU and heat capacity rate ratio R. On the upper boundary the values of the heat capacity rate ratio R_1 for the tube side fluid are shown on the right boundary the values of the heat capacity ratio R_2 for the outside fluid are presented. This means that all points on a line represents a particular value of R. Above the diagonal only NTU_1 - curves for the tube side fluid and below the diagonal NTU_2 - curves are drawn. In order to compare the results of the heat exchanger under consideration (figures 2 and 3), numerical result for pure cross - flow heat exchanger with same length of tubes i.e. relative deviation, κ = 0 is shown in figure 4. To validate the results of this project, the result obtained by Koirala [19] with same parameters is used where the effectiveness of heat exchanger with conical cross - flow arrangement with one pass and n = 10 rows with decreasing cross - section in the direction of the outside fluid flow and relative deviation κ = +0.03 together with curves of log mean temperature difference correction factor F is shown in the figure 5. From the diagrams of the effectiveness P_1 and P_2 one can abstract the followings:

- The numerical results have shown that for the values of relative deviation, κ = +0.03, - 0.03 and 0, the diagrams of effectiveness P_1 against P_2 (figures 2, 3 and 4) show almost symmetry referred to a line where heat capacity rate ratio R = 1.

It is clear from these diagrams that the effectiveness of the heat exchangers with the flow arrangement under consideration (i.e., conical cross - flow) increases with the value of relative deviation for $\kappa = +0.03$ as observed in the figure 2 and decreases distinctly with the value of relative deviation for $\kappa = -0.03$ as seen in the figure 3 as compared with the value of relative deviation for $\kappa = 0$ as illustrated in the figure 4.

It should be emphasized that the consideration of conical cross - flow arrangement in heat exchangers results in the reduction of its effectiveness for increasing values of relative deviation, $\kappa > 0$

. This is because of the decrease in the tube length from row to row there is the reduction in the thermally active heat transfer surface area which dominates the increase in the local heat transfer coefficient. This leads to the smaller value of *NTU* (Number of Transfer Units) in comparison with the reference value of NTU_o for pure cross - flow arrangement with the same length of tubes (i.e., $\kappa = 0$). Moreover, when the velocity of the outside fluid grows up due to shortness of the tube length from row to row (i.e. for the higher value of relative deviation κ), the effectiveness of the exchanger decreases further. On the contrary, the consideration of conical cross - flow arrangement in heat exchangers results in the slightly enhancement of its effectiveness for decreasing values of relative deviation, $\kappa < 0$.



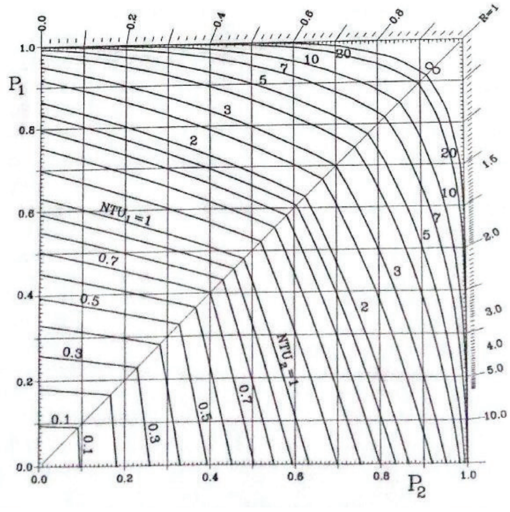
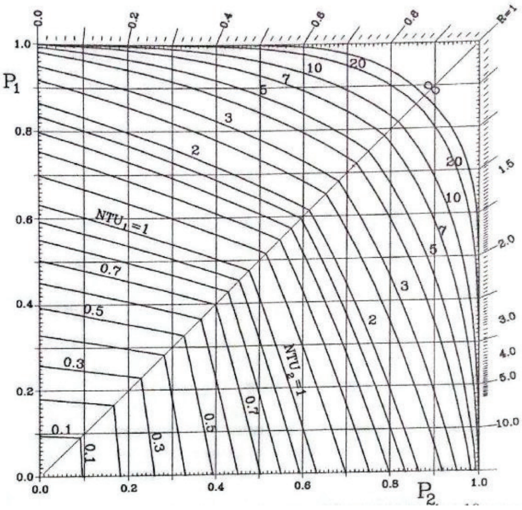


Fig. 2. Effectiveness of heat exchanger with conical cross - flow arrangement with one pass and n = 10 rows with decreasing cross- section in the direction of the outside fluid flow (see fig. 1a) and relative deviation κ = +0.03.

Fig. 3. Effectiveness of heat exchanger with conical cross - flow arrangement with one pass and n = 10 rows with increasing cross- section in the direction of the outside fluid flow (see fig. 1b) and relative deviation κ = −0.03.



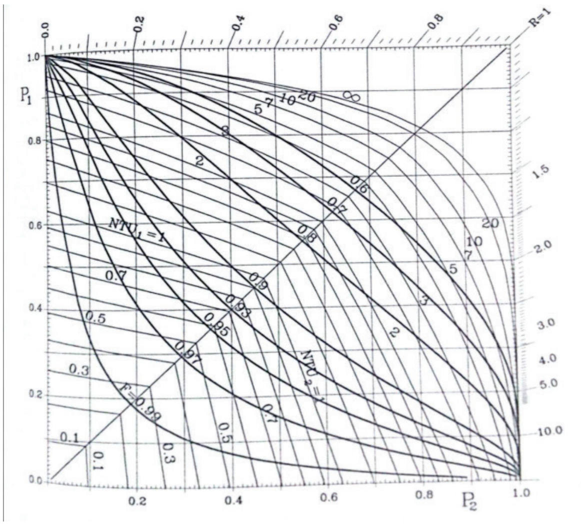


Fig. 4. Effectiveness of normal cross - flow heat exchanger with one pass and n = 10 rows and relative deviation κ = 0.

Fig. 5. Effectiveness of heat exchanger with conical cross - flow arrangement with one pass and n = 10 rows with decreasing cross - section in the direction of the outside fluid flow and relative deviation κ = +0.03 together with curves of log mean temperature difference correction factor F [19].

Conclusions

- A new method is proposed for the calculation of the thermal effectiveness of a heat exchanger with the conical cross flow arrangement and the analytical method using the weighted mean of outside fluid inlet and outlet temperatures to pure cross - flow heat exchanger developed in [3] and [5] can also be applied to the case of "conical" cross - flow arrangement.
- Convevient explicit expressions are derived for the thermal analysis of slightly deviated "conical" cross -flow arrangement with n^{th} tube rows.
- The effectiveness P_1 and P_2 can be promptly calculated by using the relations derived in this paper for given number n of tubes in the conical cross - flow arrangement, Number of Transfer Units NTU, heat capacity rate ratio

R of the fluids, ratio of heat transfer coefficients $\alpha_1/\alpha_{2,1}$ of the fluids and ratio κ of the tube length difference to length of n^{th} tube (relative deviation).

It should be emphasized that the effectiveness of the heat exchangers with the conical cross - flow arrangement decreases distinctly with the increasing values of relative deviation i.e., $\kappa > 0$ and that increases with the decreasing values of relative deviation i.e., $\kappa < 0$. This paper gives the numerical evidences of this fact and it should not be ignored by the designer.

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