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A Multi-Objective Investment Problem by Fuzzy Hyperbolic & S-Curve Membership Functions

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Abstract

Purpose: *In a typical investment problem, the investor wants to maximize the profit or revenue associated with it while simultaneously he desires to minimize time and risks associated with his investments. The fulfilment of this obligation creates a concept of multiobjective problems in which all the associated constraints need to be fulfilled along with the desired obligations associated with it (Revenue, Time and Risk). Conflict may arise in multi-objectives and can be best handled by fuzzy sets. We construct membership functions for the objective functions and formulate a fuzzy optimization problem for allocation of investments.*

Methods: *A comparative approach in solving a Fuzzy approach to multi-objective investment problem (MO-IP) using hyperbolic and S-curve membership functions.*

Results: *The results from this study indicated that optimization through the S-Curve function outperformed the hyperbolic Functions. Using S-curve membership function is one of the most idealistic approaches.*

Conclusion: *From an investor's perspective, this model emerged as a valuable tool for making more informed decisions amid the uncertainties that precede such decisions. Indeed, the presented demonstration serves as a simplified illustration of a real-world financial problem*

Keywords: Multi-objective, Optimization, Investment, Fuzzy sets, Hyperbolic & S-curve membership functions.

I. Introduction

The uncertainty inherent in the typical investment selection problem revolves around determining whether the available alternatives will efficiently yield the anticipated return or align with the fundamental Keynesian theory of money, specifically the speculative purpose. The concept of optimal allocation of funds to achieve a desired profitable return was first introduced by (Markowitz ,1952) through the theory of Optimal Portfolio Selection. Markowitz proposed quantifying risk by minimizing variance and quantifying revenue by maximizing expected value. However, controversies arose regarding the inclusion of skewness in the theory. Samuelson (1970) demonstrated the significance of skewness, showing that investors

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prefer higher skewness when anticipated value as well as variance equal. Consequently, numerous researchers modified the traditional mean-variance model to the mean-variance-skewness model, which concurrently optimizes conflicting objectives.

To address these challenges, multi-objective programming is widely employed. Every investment is associated with objectives such as minimizing portfolio-related risk and time while maximizing associated revenue. This implies an interconnection among time, risk, and revenue, making it impractical to isolate each factor, given their overlapping nature. For instance, pursuing higher revenue may entail accepting higher risk, requiring either an extended period (for capital market instruments) or a shorter duration (for money market instruments) to mature. To address the inherent complexity of these factors, (Bellman & Zadeh,1970) suggested the concept of fuzzy decision-making. Zimmermann (1978) was the first to introduce a multi-objective fuzzy linear programming problem. In real-life scenarios, problems often involve multiple objectives. Multi-objective optimization is popular approach in investment decision as it simultaneously allows to consider revenue, time and risk objectives simultaneously. The multiple objectives can be conflicting and best handled by non-linear membership functions. In non- linear hyperbolic function when one is better with respect to a goal, one tends to have a smaller marginal rate of satisfaction (Sakawa, 1993). Investment is a financial decision characterized by highly unstable environment that is constantly changing so non- linear approach is better. Thus, non- linear fuzzy membership function represents imprecise aspiration levels of a set of decision makers.

This paper presents a comparative approach to solving Multi-Objective Investment Selection Problems (MOISP) using a fuzzy non-linear membership function like hyperbolic (Bit, 2004; Verma et al.,1997) and S-curve membership functions. The coefficients of objective functions are transformed into two distinct types: Non-Linear (Hyperbolic) Membership functions Atanassov (1986), and through the utilization of the S-Curve membership function (Yu et al.,2008). Using non-linear membership provides more flexibility to model uncertainty compared to linear membership functions (Kumar et al., 2021) resolves the conflicting nature between the objectives. The main novel contribution of the proposed model for investment selection is:

- Does the fuzzy model with non-linear membership functions with minimum risk and time provide a maximum degree of satisfaction?
- Does the model also maximizes the returns simultaneously?

This paper is structured in the manner described below in the first section, the basic ideas of fuzzy theory are introduced.In second section delves into the linear methodology for solving MOISP. Subsequently, the third section explores the innovative non-linear methodology for MOISP. The fourth section focuses on the novel S-Curve methodology proposed for solving MOISP. Finally, the fifth section entails a comprehensive comparison of all the aforementioned methodologies. The concluding section synthesizes the findings, providing a final comparison of the efficacy of each methodology.

II. Reviews

Markowitz (1952) was the first to work in portfolio management where he quantified trade-off between return and risk. In his ownright, Roy created a series of Markowitz-like efficient portfolios. The two analyses differed primarily in that Markowitz suggested letting the investor select a desired portfolio from the efficient frontier rather than recommending a particular portfolio, and he required only non-negative investments. However, controversies arose regarding the inclusion of skewness in the theory, with several researchers (Markowitz,1952; Arditti,1967; Samuelson, 1970) suggesting its relevance due to the existence of non-normally distributed investment portfolios (Pornchai et al.,1997; Arditti,1971; Deng et al.,2005). Samuelson (1970) demonstrated the significance of skewness, showing that investors prefer higher skewness when anticipated value as well as variance equal. Consequently, numerous researchers (Simkowitz & Beedles,1978; Konno et al., 1993;

Konno & Suzuki,1995; Mishra et al., 2007) modified the traditional mean-variance model to the mean-variance-skewness model, which concurrently optimizes conflicting objectives Some research papers have tried to extend the classic model of portfolio selection (Drut, 2010 & Canal, 2011). Other method where a portfolio selection problem is first converted to a mathematical model when the risk and anticipated returns of a portfolio have been determined (Zhang et al., 2020). To address these challenges, multi-objective programming is widely employed. Over the last two decades, researchers have explored multi-objective problems in the framework of choosing investments and making decisions. In recent work (Azarberahman et al., 2025) to determine optimal portfolio used strength Pareto evolutionary algorithm (SPEA-II) and traditional methods found that SPEA-II generate the portfolios with higher returns and lower risk. Zahedirad et al. (2025) solved project portfolio optimization and contractor selection using goal programming.

The incorporation of fuzzy set theory (Tanaka & Asai,1984; Rommelfanger,1989; Chakraborty et al., 2011) in decision-making processes provides a more flexible framework, allowing for solutions that better accommodate real-world complexities. Peykani et al. (2021) proposed a fuzzy multi-period multi-objective portfolio optimization in uncertain data. The model is capable of being used under data ambiguity constraints as budget constraint, cardinality constraint, and bound constraint. Banerjee et al. (2024) proposed a multi-criterion decision-making model using fuzzy approach in TODIM and genetic algorithm for optimal portfolio apportionment. Jana et al. (2014) suggested a multi-objective approach for solving the fuzzy portfolio problem. This research focused on compiling a fuzzy portfolio using large-scale data without considering short-selling. Zolfaghari et al. (2021) proposed a multi-objective fuzzy mixed integer programming, a heuristic approach, and a meta-heuristic to select and schedule projects portfolio by using resources, cash flow, delay cost and project robustness. Metaheuristic algorithms that have investigated solution spaces for portfolio optimization are Genetic Algorithms (GA) and Particle Swarm Optimization (PSO). But these algorithms' shortcomings include poor convergence rates and an unsatisfactory balance between exploration and exploitation, which calls for the development of novel models (Ashrafzadeh et al., 2023; Bedoui et al., 2023). As per literature a lot of work has been done on linear multi-objective portfolio model non- linear membership functions in fuzzy objective yet to be explored. Moreover, assuming accurate input data and linear relationships by the Mean-Variance model makes it not so effective in managing the nonlinearities and uncertainties in financial markets (Chen et al., 2020; Mansour et al., 2019).The novel contributions of the paper are fuzzy model with nonlinear membership function to solve a fuzzy multi objective optimization problem.

III. Methodology

Fuzzy Approach Optimization using Nonlinear Functions (Zadeh,1965; Lai & Hwang,1993)

Utilizing the fuzzy programming method to address the multi-objective investment selection problem, employing Hyperbolic form for membership functions. A multi-objective investment selection problem optimization with p objectives and q constraints is typically addressed using a fuzzy Approach. In this paper, these methods are considered for a comparative analysis of two non-linear membership functions to be explored.

Algorithm of Steps

Step-1: For each objective function within the set of p objectives, solve it by substituting it into the provided constraints of the problem. Obtain the solution of the objective functions and the corresponding set of alternatives. Repeat this process for each set of objectives.

Step 2: The results derived from Step I, We identify the maximum and minimum values for each objective function using equation.

Step 3: Determine the upper cap and the lower cap of the membership function using (1) of the formulae:

$$L_k^{acc} = \min imum\big(Z_k(X_k)\big)$$

$$U_k^{acc} = \max imum\big(Z_k(X_k)\big) \quad \text{...............(1)}$$

Determination of membership function ($\mu_k\big(Z_k(X)\big)$ for the objective function is hyperbolic as represented by tanh & $Z_k(X_k)$ are the various objective functions.

A: for Maximization: -

$$\mu_k\big(Z_k(X)\big) = \begin{cases} 1 & Z_k(X) \le L_k^{acc} \\ \frac{1}{2}\tan h\left(m_k - Z_k(X)\right) + \frac{1}{2} & L_k^{acc} \le Z_k(X) < U_k^{acc} \\ 0 & Z_k(X) \ge U_k^{acc} \end{cases} \quad \text{...(2)}$$

Where

$$m_k = \frac{L_k^{acc} + U_k^{acc}}{2}$$

B: for Minimization: -

$$\mu_k\big(Z_k(X)\big) = \begin{cases} 1 & Z_k(X) \le L_k^{acc} \\ \frac{1}{2}\tan h\left(Z_k(X) - m_k\right) + \frac{1}{2} & L_k^{acc} \le Z_k(X) < U_k^{acc} \\ 0 & Z_k(X) \ge U_k^{acc} \end{cases} \quad \text{...(3)}$$

Here the maximization and minimization function will be the same as it is a quadratic function(non-linear).

Step 5: The Fuzzy formulation to MO-ISP as Single LPP is as

$$\text{Max}(\zeta) \quad \text{..................... (4)}$$

Subject to

$\mu_k\big(f_k(X)\big) \ge \zeta$; k=1,2,3,…,n.

Where ζ is degree of satisfaction of a fuzzy set

Fuzzy Optimization using S-Curve Membership Functions

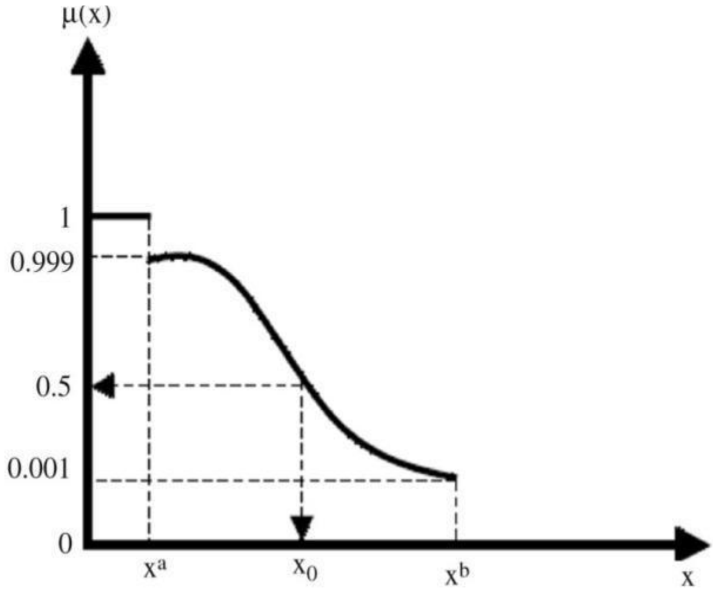
Modified S-Curve

This approach involves the transformation of a fuzzy multi-objective linear problem (FMOLP) into a crisp mathematical programming formulation. In this method, a modified S-Curve (Pandian, 2005) is applied as the membership function to all fuzzy objective functions. The S-curve membership function is a particular case of a logistic function with specific B, C and α values. These values must be described in a heuristic-experimental manner by the decision maker (DM). The Modified S-curve membership is a specific instance of the logistic function with certain values for parameters B, C, and α. It is crucial to calculate these values appropriately. The modified membership values of the solution are deemed suitable and proper if they belong to the interval , irrespective of the nature and shape of the associated membership functions. The membership function for modified s-curve can be denoted as:

$$\mu_k(x) = \begin{cases} 1 & x < x^a \\ 0.999 & x = x^a \\ \frac{B}{1+Ce^{ax}} & x^a < x < x^b \\ 0.001 & x = x^b \\ 0 & x > x^b \end{cases} \quad \text{... (5)}$$

Figure 1

S Curve Membership Function



So, defining the membership function $\mu_k(Z_k(x))$ *for the objective functions* ($Z_k(x)$) using the above s-curve membership function(Figure 1) is

$$\mu_k(Z_k(x)) = \begin{cases} 1 & Z(x) < L_k^{acc} \\ 0.999 & Z(x) = L_k^{acc} \\ \dfrac{B}{1 + Ce^{\frac{Z(x)-L_k^{acc}}{U_k^{acc}-L_k^{acc}}}} & L_k^{acc} < Z(x) < U_k^{acc} \\ 0.001 & Z(x) = U_k^{acc} \\ 0 & Z(x) > U_k^{acc} \end{cases} \quad \ldots (6)$$

Where B, C are constants with crisp values.

Torabi and Hassini's Fuzzy Programming Solution Method

There are various approaches for solving multi-objective linear programming (MOLP) models, and there is a growing trend in utilizing fuzzy programming approaches. Fuzzy approaches have the capability to explicitly measure the satisfaction degree for each objective function. (Torabi & Hassini, 2008) introduced a single-phase fuzzy programming approach:

$$\text{Max}\,\lambda(x) = \gamma\lambda_0 + \gamma)\sum_k \theta_k \mu_{z_k}(x)$$

Such that:

$$\lambda_0 \leq \mu_{z_k}(x)$$

$$\lambda_0, \gamma \,\epsilon\, [0,1], k1,2, \dots, n \quad \textbf{... (7)}$$

$$\lambda_0 = min(\{\mu_{z_k}(x)\}), \gamma = 0.5$$

$_k$ and γindicates the analogous influence of k[th] objective function and coefficient of comparison respective.$\theta_k$is ecided by decision-maker such that$\sum \theta_k = 1$. For this approach, we have taken the following values of $(\theta_1 = .4, \theta_2 = 0.3, \theta_3 = 0.3)$.

Algorithm

Step 1: We first describe the modified S-curve membership functions (equation (6)) for all fuzzy objective functions and fuzzy inequality constraints.

Step 2: We do mathematical formulations of the Fuzzy Multi-Objective Linear Programming (FMOLP) model for the Multi-Objective Investment Selection Problem (MOISP).

Step 3: Formulations of the FMOLP problem are converted to an equivalent single-objective using Torabi and Hassini's fuzzy programming approach using equation (7).

Step 4: The optimal solutions of the single-objective nonlinear problem are found by using LINGO or TORA.

IV. Results and Discussion

Mr. Mehra, a retired IAS officer has recently received his retirement benefits in terms of his Provident Funds. He seeks his own judgment skill, he decides to invest in some investment alternative. Each of that investment alternative is assigned with specific Risk, Maturity Time & Yield or (Maximum Revenue). The various list of alternatives along with each associated factor in time, risk and yield is given in the table 1.

Table 1

Data Table

Investment Alternatives	Lenth of investment(years)	Rate of return (in 100%)	Risk	Growth Potential (in 100%)
A(X_1)	4	0.03	1	0.0
B(X_2)	7	0.12	5	0.18
C(X_3)	8	0.09	4	0.10
D(X_4)	6	0.20	8	0.32
E(X_5)	10	0.15	6	0.20
F(X_6)	3	0.06	3	0.07
Cash(X_7)	0	0	0	0

He wants to design an investment portfolio such that he can maximize the total yield or rate of return, minimize the risk associated with them and time constraint which he again wants to keep at minimum.

In mathematical terms, the above problem can be formulated as:

Maximizing return $\quad$ Max $Z_1 = 0.03x_1 + 0.12x_2 + 0.09x_3 + 0.20x_4 + 0.15x_5 + 0.06x_6 + 0.07x_7$

Minimizing time $\quad$ Min $Z_2 = 4x_1 + 7x_2 + 8x_3 + 6x_4 + 10x_5 + 3x_6 + 0.00x_7$

Minimizing risk $\quad$ Min $Z_3 = x_1 + 5x_2 + 4x_3 + 8x_4 + 6x_5 + 3x_6 + 0.00x_7$

Subject to

$4x_1 + 7x_2 + 8x_3 + 6x_4 + 10x_5 + 3x_6 + 0.00x_7 \leq 7$

$x_1 + 5x_2 + 4x_3 + 8x_4 + 6x_5 + 3x_6 + 0.00x_7 \leq 5$

$x_7 \geq 10$

$0x_1 + 0.18x_2 + 0.10x_3 + 0.32x_4 + 0.20x_5 + 0.07x_6 \geq 0.10$

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

The membership function for hyperbolic and S-curve membership function for objective function is constructed(Appendix I &II)using equations 2,3 and 7.

Formulating the FLPP using hyperbolic and S-curve membership functions using equations (5) to (7).

Max(ζ)

Subject to

$$\frac{1}{2}\tanh(0.03x_1+0.12x_2+0.09x_3+0.20x_4+0.15x_5+0.06x_6+0.07x_7-0.095)+\frac{1}{2} \geq \zeta$$

$$\frac{1}{2}\tanh(4.44- 4x_1 - 7x_2 -7x_3 -8x_4-10x_5-3x_6-0x_7)+\frac{1}{2} \geq \zeta$$

$$\frac{1}{2}\tanh(3.75- x_1 - 5x_2 -4x_3 -8x_4-6x_5-3x_6-0x_7)+\frac{1}{2} \geq \zeta$$

From Torabi Hassani Multi-objective Model using equation (8).:-

$$\text{Max } \lambda(x)=0.5\lambda_0+0.5*\sum_{k=1}^{3}\theta_k\ \mu_k(x)=0.5\lambda_0+0.5*(\theta_1\mu_1 f_1(X)+\theta_2\mu_2 f_2(X)+\theta_3\mu_3 f_3(X))$$

$$=0.5\lambda_0+0.2\ \mu_1 f_1(X)+0.15\mu_2 f_2(X)+0.15\mu_3 f_3(X))$$

$$=0.5\lambda_0+0.2*\left(\frac{1}{1+0.001001001*e^{13.813*\left(\frac{0.06-0.03x_1+0.12x_2+0.09x_3+0.20x_4+0.15x_5+0.06x_6+0.07x_7}{0.07}\right)}}\right)+0.15*\left(\frac{1}{1+0.001001001*e^{13.813*\left(\frac{4x_1+7x_2+8x_3+6x_4+10x_5+3x_6-1.88}{5.12}\right)}}\right)+0.15*\left(\frac{1}{1+0.001001001*e^{13.813*\left(\frac{x_1+5x_2+84+68+6x_5+3x_6-2.50}{2.50}\right)}}\right)$$

Such that:

$$\lambda_0\leq\frac{1}{1+0.001001001*e^{13.813*\left(\frac{0.06-0.03x_1+0.12x_2+0.09x_3+0.20x_4+0.15x_5+0.06x_6+0.07x_7}{0.07}\right)}}$$

$$\lambda_0\leq\frac{1}{1+0.001001001*e^{13.813*\left(\frac{4x_1+7x_2+8x_3+6x_4+10x_5+3x_6-1.88}{5.12}\right)}}$$

$$\lambda_0\leq\frac{1}{1+0.001001001*e^{13.813*\left(\frac{x_1+5x_2+84+68+6x_5+3x_6-2.50}{2.50}\right)}}$$

$$\lambda_0 \in [0,1].$$

The above two formulations are solved using Tora 2.0 and results are displayed in Table 2 and Table3. In Table 3 we observe that s-curve function give better results then non-linear hyperbolic function (Table 2). The z_1 value for nonlinear function was better than s-curve functions and S-curve gave better values for z_2 and z_3. So, a tradeoff of z_1 with respect to z_2 and z_3 is acceptable. Compared to fuzzy approach by Kaur et al. (2024) using fuzzy linear membership function the results of S-curve give better results (Figure2). The degree of satisfaction for S-curve is 0.9991 compared to linear curve where degree of satisfaction was 0.62. Also, degree of satisfaction by fuzzy hyperbolic membership functions is 0.50 which is lower than linear approach and S-curve membership functions used (Figure 2).

Table 2

Results of Fuzzy Approach Using Non-Linear Function

Objective Function	Decision Variable	
$Z_1 = 11.082$	$X_1 = 0.4930262$	
	$X_2 = 0, X_3 = 0$	$\zeta = 0.5006114$
$Z_2 = 9.8322$	$X_4 = 0.4062128$	
$Z_3 = 2.524$	$X_5 = 0, X_6 = 0, X_7 = 0$	

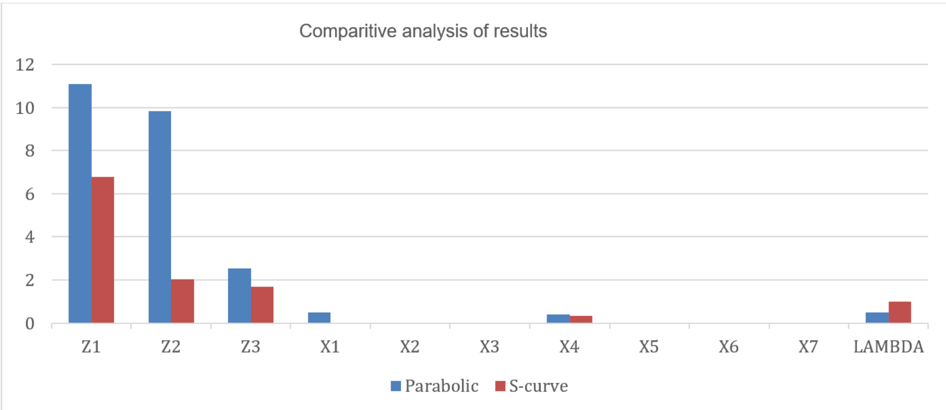
Table 3

Results of Fuzzy Approach Using S-Curve Function

Objective function	Alternatives	
$Z_1 = 6.7758$	$X_1 = 0, X_2 = 0, X_3 = 0$	
$Z_2 = 2.032$	$X_4 = 0.33879$	$\lambda_{s-curve} = 0.9991680$
$Z_3 = 1.6935$	$X_5 = 0, X_6 = 0, X_7 = 0$	

Figure 2

Graphical Representation of Objective Function Values, Order Allocation and Lambda



V. Conclusion and Implication

The study, titled An Analysis of Multi-Objective Investment Selection Problem by Fuzzy Approach employed the methodology of the use of Non-Linear and S-Curve functions was a novel approach introduced in this study. The results from this study indicated that optimization through the S-Curve function outperformed the Non-Linear Function.

From an investor's perspective, this model emerged as a valuable tool for making more informed decisions amid the uncertainties that precede such decisions. Indeed, the presented demonstration serves as a simplified illustration of a real-world financial problem. In actual, real-world problems are multifaceted and are influenced by various macroeconomic factors. The limitation of the study is that the type of membership function and fuzzy min operator may not be suitable for any real-life examples.

As more parameters and additional constraints are considered, the applicability of the above methods, particularly the S-curve approach, can be extended to analyze more complex real-world scenarios. Expanding its application to incorporate a broader range of factors and constraints can enhance its utility in addressing the intricacies inherent in real-world financial decision-making. Future scope of study lies in expanding the data set by more investment options including real life applications. The problem can be explored with more different types of nonlinear membership functions.

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