



Structure of Mathematical Research

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Abstract: *In this paper, we discuss about the structure of various components of mathematical research. We mainly focus on the structure of PhD proposal, PhD thesis and research article. We describe in detail about the standard structure of these components in the context of mathematics.*

Keywords: *References, Citation, Literature review, Plagiarism, Bibliography*

1. Introduction

Mathematics has been the integral part of every scientific research so that high priority is given to mathematical research. Mathematics comes in various areas mostly in natural science, social science and engineering. It is needed in different situations like tracing a rocket in space, designing a shoe, modeling front design of a car, beautifying a painting etc. Broadly the mathematics is the integral part of almost all areas and the research of these areas can be enhanced with the help of mathematics. Mathematical research is done at all academic levels starting from undergraduate to PhD. degree and various independent studies to address the real world problems. The outcomes of mathematical researches are published through various ways such as project, report, thesis, article, monograph etc. These all means of publications in general follow their own standard structure and one has to strictly follow that structure. In this article, we will focus on the structure of three components of mathematical research namely PhD proposal, PhD thesis and research article. This is a complete chain as one needs a proposal to get admission in PhD in an university. After getting a PhD degree, one becomes an independent researcher and then starts to publish his work in the form of articles. We discuss in detail about the standard structure of the proposal, thesis and article. We believe that the article will be beneficial to those who are undertaking or planning to undertake mathematical research. For the basics and structure of project writing in mathematics in undergraduate level, readers are suggested to refer KC and Ghimire [2]. For further reading about the research structure, readers are suggested to refer Alvi [1], Hingham [3], Lane [4], Mason [5], Richland [6] and Saxena [7].

2. Structure of Mathematical Research

In this section, we shall discuss the structure of mathematical research into three categories, namely PhD proposal, PhD thesis and mathematics research article. We discuss these three components in details. We first begin with the structure of PhD proposal.

2.1. PhD Proposal

PhD proposal is normally mandatory in every university. A PhD proposal of mathematics is simply a document outlining the mathematical research we are going to undertake while pursuing the PhD degree. A candidate has to

submit and defend the proposal when applying for a PhD degree. The proposal is very important as it is a great tool that can help us to structure our thinking and outline the path we will be taking during PhD study. Precisely, a proposal must contain three points; what will be done (explanation of proposed research), how it will be done (the methods and techniques to be employed) and why it should be done (novelty and importance of the study).

Regardless of university specific requirements, a standard PhD research proposal in mathematics usually include:

- i. Title and Abstract
- ii. Introduction and literature review
- iii. Research problem and objectives
- iv. Research methodology
- v. Work plan
- vi. Impact and limitations
- vii. References and bibliography

We now discuss each of these components in detail and begin with title and title page.

Title and title page

We are expected to provide a preliminary title or provisional title which will be further elaborated in the process of thesis writing. A title page is expected to have the following things in the stated order:

- i. Provisional title
- ii. Candidate's name
- iii. Name of the supervisors
- iv. Name of the department and University where it is submitted.

Abstract

An abstract is optional in proposal writing. If we want to include the abstract, then it should usually be no longer than a paragraph. Here, we provide a brief summary of what we are going to cover in our research proposal.

Introduction and literature review

This is the main part of the proposal and will probably the lengthiest piece of it. This begins with the basics of the research topic. We state all the definitions, theorems and then we narrow the focus of those details that are especially pertinent to the proposed work. Here, we present what is currently being known and inform all the published work related to chosen title. Moreover, in this section we describe key topics in details which will show the strong understanding and we provide all the sources that are used for understanding these key topics. It is a good idea to mention some notable researchers in the area of study and their contributions and provide some of the key references in the areas of research which requires doing extensive research in our part. The literature review demonstrates applicant's knowledge of the main research achievements in the area of study.

Research problem and objectives

Through the extensive literature review we identify the main gap in the research area on which we are going to focus in our PhD project. Here we state what is known in this field, explain what we expect to see and hope to show through our result. This part should be able to answer some of the following questions:

- i. Does our result strengthen a previous result by giving a more precise characterization of something?
- ii. Have we proved a stronger result of an old theorem by weakening the hypothesis or by strengthening the conclusion?

- iii. Have we proven the equivalence of two definitions?
- iv. Is it a classification theorem of structures which were previously defined but not understood?
- v. Does it connect two previously unrelated aspects of mathematics?
- vi. Does it apply a new method to old problems?
- vii. Does it provide a new efficient proof of an old theorem?
- viii. Is it a special case of a larger class?

Research Methodology

In this section, we mainly focus on how to do our research. We describe the methodology and techniques we will use to conduct our research.

Work plan

In order to meet the research objectives, we need to proceed with plan. Normally a PhD degree is expected to complete in three years. Work division for these three years is as follows:

First Year: During this time period, we need to complete the credit courses. The number of credit hours varies with universities. Besides the course credit, we have to spend time for the literature reviews also.

Second Year: We give the continuity of literature reviews. We work on proposed title and start to write paper on the outcomes and then on the publication of the papers. We also attend seminars and conferences.

Third Year: During this time, we focus on research works and paper publications along with thesis writing. We also present our research work in seminars and conferences and defend pre-viva and then get ready for the final viva.

Impact of the research and limitations

In this section we discuss about the impact or contributions we could possibly give by our expected result after the completion of the study. Moreover, we also discuss about any limitations while conducting the research.

References and Bibliography

We mention all the sources (books, article, thesis, report etc.) that was studied to understand the key topics. We must cite all the used sources in the proposal in order to be thankful to the owner of the sources. This is popularly known as references and bibliography. We should follow a uniform way to cite or mention the sources. We discuss in details about references and bibliography at the end.

From the above discussions, we realize that the introduction, literature review, references and bibliography are the major components of the proposal. Thus, one has to spend plenty of time on these parts while preparing the proposal. Next, we discuss about the structure of PhD thesis or dissertation.

2.2 PhD Thesis

All the research findings of the PhD study are documented in a book which we call a thesis or dissertation. A good PhD thesis should be self-contained and must stand on its own as a complete account of the author's work on the subject of investigations. A thesis is formatted like a book, broken into chapters rather than sections. A thesis may include more than one topic in the area of study and should include details presenting original results. As the thesis is self-contained, we should not use the phrases such as "It is easily shown that"

and "We omit the proof" while giving the proof the results. A good PhD thesis is expected to have the following characteristics:

- i. It should make an original and significant contribution to knowledge.
- ii. It should give the evidence of originality of mind and critical judgment in a particular topic of mathematics.
- iii. It contains material worthy of peer-reviewed publications.
- iv. It should be satisfactory in its literary or technical presentation and structure with full bibliography and references.
- v. It demonstrates an understanding of the context of the research.

A mathematical PhD thesis has rigid structure. This rigid structure of a thesis can be divided in to the following components in the stated order:

- i. Title and title page
- ii. Copy right
- iii. Deceleration Page
- iv. Acknowledgments
- v. Dedication
- vi. Table of contents
- vii. List of Figures/symbols/tables
- viii. Abstract
- ix. Chapters 1,2,3,4
- x. Conclusion
- xi. Appendix
- xii. Bibliography

We now describe these components. We first begin with title and title page:

Title and title page

Title of mathematical thesis should be short, descriptive and fairly self-explanatory. The main goal of the title is to give clear understanding of the content. A general reader is expected to guess the content of a thesis after reading the title. The title page of the thesis contains the following items in the stated order:

- i. title of the dissertation
- ii. name of the author
- iii. author's previous degrees
- iv. awarding degree (doctor of philosophy)
- v. official name of the department and university
- vi. year of submission

Copy right

In this section, we give a statement on copyright and ownership of intellectual property rights. In the case of thesis, it normally goes to the student pursuing the degree.

Deceleration Page

Here the author has to declare that the work has not been used in another degree submission.

Acknowledgments

In this page, we acknowledge all the peoples who helped us directly or indirectly. We also thank all the department or offices from where we got help while conducting our research. In practice, the thesis advisor is highly acknowledged. Besides advisor, we also thank all the committee members and faculties of the department. As family and friends always motivate us for the academic work, we also be thankful to them.

Dedication

In this page, we dedicate the work to our closest one. This section is optional.

Table of Contents

In this page, we list all headings and subheadings of the thesis with page numbers. We note that the table of content does not include table of contents and abstract.

Abstract

Abstract is a brief summary of the thesis which presents a brief introduction to the issue of the thesis. It is one of the major components of the thesis which should be written very carefully. Abstract should be stated in simple declarative sentences. Here we give a brief summary of how we want to address the issue and also include a possible implication of the work. The length of the abstract of a mathematical thesis varies, ranging from a paragraph to a page.

List of Figures/tables/symbols

In this page, all the figures and tables used in the thesis are listed with their respective page numbers. As we use many sets and symbols of mathematics, we should list all the symbols that are used with their description.

Chapters 1-2

The first two chapters of the thesis are considered as the preliminary chapters. In these chapters, the mathematical problems being addressed must be clearly described and put into context. The related information collected from literature review demonstrating a sound knowledge and understanding of existing work on the proposed topic is appropriate to put in these chapters. Moreover, all basic results such as definitions, theorems and examples which are used in the main body are stated or proved. We also describe the methodology of the work if there are.

Chapters 3-4

In these chapters, we put main work of the thesis. The main work of the thesis includes new or original results. Thus, these chapters are the main part of the thesis and we spend plenty of time to write.

Conclusion

Conclusion is written at the end of the thesis. Conclusion of the thesis must be carefully drawn with overall contributions of the thesis assessed. Moreover, it is also a good idea to identify open problems and future directions for research in the conclusion. Being able to identify open problems and future directions of the research is one of the attributes required to be a good researcher.

Appendix

Appendix contains information that is essential to the thesis but does not fit comfortably into the body of the text. The most common use of appendix is to present detailed analysis that would distract the reader if it were given at the point where the results of the analysis are needed. So, appendix may or may not be the part of a thesis depending upon the context.

Bibliography

In this section, we need to list all the sources that are used to justify mathematical statements or theorems we are using in the thesis. Moreover, we list all the sources that are used in the background study of the area of research. Thus, we list all the used and unused sources in the thesis. We will be describing it in detail at the end.

Finally, we discuss the structure of mathematical research article. We will give more attention to this structure of mathematical research.

2.3 Mathematics Research Article

In general, mathematics research papers are read by specialists in a given domain. Thus, we need to think about the specific reader who will be interested in our article. Our goal is to make those readers interested in the article we write and thus we can leave out the proof of trivial results. A good mathematics research article should tell a story where organization is very important. If the organization of the article is good, then the revision lies in detailing the content. We need to spend plenty amount of time in editing and polishing. Most important thing while writing the article is that we do not plagiarize. We will discuss about the plagiarism at the end. Before we start to write an article, we must have a result worth of publishing,

A mathematics research article normally has the following components:

- i. Title
- ii. List of authors
- iii. Abstract
- iv. Mathematics Subject Classification
- v. Key words and phrases
- vi. Introduction
- vii. Body of the work (sections)
- viii. Appendix/Conclusion/Future work
- ix. Acknowledgement
- x. References

Title and title page

The title of the article should be brief and should specifically describe the content of the paper. The title should not be too general. We do not use abbreviations or complicated symbols. The title page contains the following:

- i. a concise and informative title
- ii. name(s) of the author(s)
- iii. affiliation(s) of the author(s), i.e., institution, department, city, country
- iv. an active e-mail address of the author(s) with indication of the corresponding author.

Abstract

The abstract of an article should have the following features:

- i. It should present the main results of the paper.
- ii. It should be short and crisp.
- iii. It should not have complicated formula.
- iv. It should be as independent from the article as possible. The reader should not have to refer to the paper to understand the abstract.
- v. Many mathematics journals state a maximum size for the abstract, usually between 200 to 300 words.
- vi. It should not make claims that are not justified in the paper.
- vii. It should also be catchy enough to attract the attention of a reader.
- viii. It should give indications of the conclusions of the paper.

- ix. Mathematical formulas and bibliographic references should be kept to a minimum in it.
- x. Any bibliographic references must be written out in full (not given by number) in it.
- xi. We avoid using the phrases like "In this article, we prove among other results that". Simply write "We prove that."

Keywords

The purpose of giving keywords is to give a good indication of paper's content so that a reader can think about the content of the article. Then the number of keywords needed in the article is usually 10 or less. Keywords are normally used in computer searches of an article.

Mathematics Subject Classification

Mathematics Subject Classification, abbreviated MSC, is alphanumeric classification scheme and is mandatory for most of journals these days. The current version of Mathematics Subject Classification is MSC2020. MSC is a hierarchical scheme, with three levels of structure. In this scheme, a classification can be two, three or five digits long, depending on how many levels of the classification scheme are used. Here the first level is represented by a two-digit number, the second by a letter, and the third by another two-digit number. This scheme divides the mathematics into four categories with numbers between 00-97 sections. These sections are further divided into many subsections. For example, section 65 covers for numerical analysis and has 106 subsections. In this section, the subsection 65F05 is reserved for direct method of solving linear systems whereas 65B10 is reserved for the summation of series. Readers can find the all the details about the MSC online in [8].

Introduction

In any mathematics research articles introduction along with abstract are considered as the main selling points of the paper. Most of the reader only read abstract and introduction and then discard the paper. So, we need to work very carefully on introduction. Introduction of an article can drive a reader to the rest of the paper. Introduction part should be fairly short, say a few hundred words. Depending upon the type of articles, we give the motivation of the topic. While writing a very technical paper that builds upon earlier work in a difficult area and addressing ourselves as an expert, then it not necessary to motivate the topic. But while writing a survey paper, we need to motivate the topic, relate it to other areas, and explain and unify the topic we are surveying. We need to start the introduction in such a way that we make the reader interested in the first paragraph. In introduction part we state the problems, give the history of work, and related problems or pointers to relevant work. Moreover, we also state main results/theorems.

Sections, Main sections

This is the main body of the paper where we present the main results or analysis. In other words, we present what is new or original in the paper. This section contains theorems and proofs or algorithms or numerical approaches or experiments or computations or comparisons or interpretations or all or some at a time. This section is considered as the heart of the article.

Conclusion/Future direction

It is good to discuss the conclusion of the research work so that a reader can get the entire information from the conclusion. So, most of the researcher prefer to conclude their work in the article. Moreover, they also discuss their future work in the form of some open questions. This will show a direction of further research for the reader and opens a path. Being able to specify future direction of research is an important characteristic of a good researcher.

Appendix

An appendix contains information that is essential to the article but does not fit comfortably into the body of the text. Most common use of appendix is to present detailed analysis that would distract the reader if it

were given at the point where the results of the analysis are needed. Depending upon the content of the article, appendix may or may not be needed.

Acknowledgement

All the acknowledgments should be included at the very end of the article before the references. Anyone who makes a contribution to the research (or) article, but not a listed author, should be acknowledged for their contribution. In general, we do not forget to acknowledge the grant provider.

Finally, we discuss about the references and bibliography which are needed in all the above discussed structures.

References and Bibliography:

In order to support and justify an idea in a research work, authors generally take the help of books, article, thesis etc. In return, authors have to thank authors of the used sources. This is called referencing. Thus, referencing is simply a method of acknowledging and recognizing the authors for their innovative and original work. In some of the academic writings, bibliography is used in the substitution of reference list. In the bibliography we list all the sources that are referred in the main text and include(s) all the sources consulted even if they are not cited in the work for the future use. For example, general background reading to familiarize with the topic. Bibliography is used in thesis, report and in proposal. References only includes the sources that are cited or referred in the work. In articles, we use references.

Thus, list of references and bibliography indicate that we have read widely and deeply in the chosen topic of research. It also helps us to show anyone who reads your work that we understand the topic and can demonstrate our own thoughts on this. It is very important to refer a source that has been used in the work. If someone's work is not referred, then it is called plagiarism. As the references and bibliography is written at the end, we have to keep track of all the sources from the beginning. In every academic writing, referencing is done at two levels. At the first level we give a short reference in the body of the text and this is called "in-text citation". At the second level a detailed reference is provided at the end of the document in the form of a list and this list is popularly known as references or bibliography. Journal articles and most books are peer reviewed and can be referred whereas one has to be careful in taking information from web sites. For example, matters taken cannot be referred from the source Wikipedia which is an open source and anyone can make corrections on the content. We now discuss about various styles of referencing.

Referencing Styles

At the first submission, normally any referencing styles can be used. But later on, when the paper gets accepted, they want a specific standard style or their own style of referencing. In general, the institutions or publishing houses decide their referencing style so that the referencing styles vary with publishing houses. The commonly and widely used referencing styles are listed below:

- i. AMS (American Mathematical Society)
- ii. Harvard
- iii. Vancouver
- iv. APA (American Psychological Association)
- v. MLA (Modern Language Association)
- vi. Chicago
- vii. ACS (American Chemical Society)
- viii. AGLC (Australian Guide to Legal Citation)
- ix. AMA (American Medical Association)
- x. CSE (Council of Science Editors)
- xi. IEEE (Institute of Electrical and Electronics Engineers)

These referencing styles differ in the order of information, use of punctuation, in formatting and these differences occur in both the level of referencing. AMS style refers to the citation format established by the American Mathematical Society. With AMS, the number sign in brackets represents the order that the citation is mentioned in the text of the paper. For example, [5] would indicate that this is the fifth citation found in the text. For more about the AMS style of referencing, please refer webpage AMS. As an example, we now discuss in detail about the Harvard style of referencing.

Harvard Style of Referencing

This style of referencing was introduced by Harvard University. Harvard Author-Date style of referencing is commonly used in natural sciences. In this style, all the used sources (books, articles etc) are listed in alphabetical order by author's last name and given at the end. In-text citation is simply done by (Last Name, Year of Publication) or Last Name (Year of Publication).

In the case of books with multiple authors, all the names should be included in the order they appear in the book.

We follow the following sequence in listing a book:

- i. Authors (Last Name)
- ii. Initials (First Name)
- iii. Year
- iv. Title of Book
- v. Edition (if not the first)
- vi. Place of Publication (Town or City)
- vii. Publisher.

For journal articles we follow the following sequence in listing:

- i. Authors (Last Name)
- ii. Initials (First Name)
- iii. Year
- iv. Title of article
- v. Full title of journal
- vi. Volume Number (issue/part number)
- vii. Page numbers.

Articles from web-based journal:

- i. Authors (Last Name)
- ii. Initials (First Name)
- iii. Year
- iv. Title of article
- v. Full title of journal
- vi. [online] Available at: web address (quote the exact URL for the article) [Accessed date].

Articles with DOIs

We can choose to use the DOI (Digital Object Identifier) instead of the format/location/access date. The DOI is a permanent identifier and replaces a permanent web address for online articles. They are often found at the start/end of an article. Not all articles are assigned a DOI. We follow the following sequence in listing the articles with DOIs.

- i. Authors (Last Name)
- ii. Initials (First Name)

- iii. Year
- iv. Title of article
- v. Full title of journal
- vi. [e-journal] Volume number (Issue/Part number), Page numbers if available. DOI.

Thesis

For the thesis, we follow the following sequence:

- i. Authors (Last Name)
- ii. Initials (First Name)
- iii. Year of Publication
- iv. *Title of Dissertation*
- v. Level
- vi. Official Name of the University.

If there are several works by one author published in different years, then these should be arranged in chronological order, with the earliest date first. If there are several authors with same last name, we order them by alphabetical order of their first name. If there are several works by one author and published in the same year, then they should be differentiated by adding a lower-case letter after the date.

3. Conclusion

In this article, we discussed in detail about the structures of various mathematical documents, namely, PhD proposal, PhD thesis and mathematical research article.

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