The Actuarial Conditions for the Valuation of Pension Liability to Become Zero Under Minimum Funding Standard Architecture

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Abstract: Pension valuation exercises for a defined benefit scheme requires an appraisal of both the schemes assets and its liabilities in different circumstances. The valuations are required to comply with regulatory standards, most notably the minimum funding standard. The objectives of this study are: (i) to compute the estimate of minimum funding standard of pension liability (ii) to establish the actuarial condition under which minimum funding standard liability will be zero. This study used minimum funding standard models for the computations of accruing liabilities for the current and past service liability of employees. Data in respects of different categories of employees were collected over 36 employees from a going concern located in Jos South local government of Plateau State in Nigeria. The data includes the employee’s annual salary and their respective demographic data which includes sex, date of birth, date of employment. This was used to determine the number of years of pensionable service completed and the future years of services to be completed before retirement at the age of 65 years. The study also used life annuity table to compute the service liability of each member of the scheme. From the model used, the result shows that the total service liability of the plan will be vanishingly zero when the newly defined annuity component approaches zero hence this represents the condition for liability of the plan to the members to be zero.

Keywords: Annuity, Liability, Minimum funding standard, Valuation

1. Introduction

Pension liability defines the size of fund that a scheme sponsor has to account for to make future pension payments. In [5] (Brown, 2015) and [10] (Gang & David, 2017), we observe that the pension liability shows the level of variation between the total value due to retirees and the actual value of funds the sponsor has to disburse payments. When the plan sponsor has more funds than it requires to settle future pension, then we simply have pension surplus. In a defined benefit scheme, a pension liability occurs where the sponsor cannot pay all pension benefits to the retired. The expected future pension payments for every member of the scheme is computed by applying the member’s data and scheme provisions. These future benefit payments address every member’s benefits and service profile, the expected time of death, expected disability or the actual retirement date. In [2] (AAA, 2004), the norm is to compute the present value of future benefits (PVFB) by discounting the future payment from the date of payment at the moment under correct actuarial assumptions. The PVFB describes the present value of all benefits expected to be paid by the sponsor to current scheme members. Under correct actuarial assumptions, the sponsor could theoretically improvise the value of fund in the scheme presently such that it would cover...
payments on the scheme and such that this value may take care of both future service that the scheme member is expected to earn and future incremental payments.

However, pension plan sponsor does not seem to recognize the cost of unearned future service. This is seen to be equivalent to recognizing the cost of benefits prior to being paid. In order to distinguish the cost methods, three core areas have been developed. Following [2] (AAA,2004), Actuarial liability $AL$ is the fraction of the $PVFB$ which is assigned to past services benefits. Again in [12] (Moshe,2006), this represents the current value of benefits which has been deferred in previous years and described as the projected benefit obligation ($PBO$). A scheme’s obligation to members prior to retirement can be measured by both accumulated benefit obligation ($ABO$) and project benefit obligation ($PBO$). The ($ABO$) describes the present value of benefits which members of the scheme have earned with their current service based on the current age and salaries for the interval of time they would be collecting pension benefits after they reach the normal retirement age. The ($PBO$) defines the present value of benefits which members of the scheme are likely to earn with their current service and the projected future salaries at the time of retirement.

In [1] (Adeyele & Ogungbenle, 2019); [10] (Gang & David, 2017); [12] (Moshe, 2006); and [15] (Winklevoss, 1993), we observe that the $AL$ may show the expected future payment increments since various pension schemes are developed to ascertain the retirement benefit depend on the payment at retirement. It is interesting that the plan sponsor should recognize the cost of the plan progressively over the member’s working life time, consequently the actuarial assumption treats the fraction of the future benefit due to past service to cover expected future payment increments. The fraction of the $PVFB$ which is assigned to the current year of service represents the Normal Cost $NC$ and defines the present value of benefits which is being deferred this today. This may be equivalent to service cost ($SC$) for the purpose of pensions accounting. Despite other different cost methods are applied to compute the $NC$, it should elicit the current year of services and could also bear the expected future payments increment.

Following [2] (AAA, 2004), the percentage of the $PVFB$ which is assigned to the future years of service is the Present value of future Normal Cost $PVFNC$ and it is expected to include benefits which have not been earned.

In pension valuation, the projected unit credit cost method ($PUC$), the traditional unit credit ($UC$) cost methods, Entry age normal (EAN) are commonly employed. The $PUC$ cost method addresses the expected future payment increments in the actuarial computations of liability and normal cost by distributing benefits to member’s service period. It also seems to generate progressively increasing normal costs while the $UC$ cost method does not adopt expected future payment increments but the liability. Following [10] (Gang and David, 2017), the cost prorated technique $EAN$ which distributes costs over the members service period seems to have high normal cost at the initial years of members service. The unit credit addresses the expected growth in payment in the normal cost for a year.

Following [3] (Blake & Orszag,1997); [5] (Brown, 2015); [6] (Brynjar, 2019) and [7] (Collinson, 2001); there is a particular distinction between the actuarial measures of pensions funding status and the appropriate measure of its market value. Following [3] (Blake, 1999); [10] (Gang & David, 2017); and [15] (Winklevoss, 1993), it is argued that in pensions fund valuation, the long term investment institution has
liabilities of the most extended duration and although these liabilities are equivalent, there tends to be computational differences. Life insurance schemes make provisions for such futures as policy loan and early surrender options in such a manner that the pension fund would not consider. It is also apparent that defined benefits schemes have integrated options structures on the invested asset in such a way that life policies do not have such structures. The valuation of pension liabilities is necessary in the event that sponsors are unable to meet the pension overheads further.

The Purpose of an Actuarial Valuation

An actuarial valuation represents an appraisal of a pension scheme financial profile serving as regular check to ensure that the trustees funding program is not adversely altered. In [4] (Brien, 2020), [8] (Dufresne, 1988a); [9] Dufresne, 1988b); and [10] (Gang & David, 2017) valuation represents the actuaries’ estimate of the scheme’s solvency where the liabilities are computed as the cost of purchasing annuities from a life office. We observe that the valuation exercise is carried out at a specified date (valuation date) and it is conducted on behalf of the trustees by the scheme’s actuary. It is a legal requirement that actuarial valuation be conducted on all defined benefit pension schemes at least once in three years. A defined benefits scheme pay a set of retirement income benefits to members depending on their promotional salaries and years of membership. This pension promise extends to many years into the future and payments continue irrespective of how long members would survive and when a member dies. Following [13] (Owadally & Haberman; 1999); [14](Steven & Igbal, 2001); and [15] (Winklevoss, 1993) defined benefits schemes are usually administered by the board of trustees who are legally responsible to ensure that scheme members receive their pensions at the appropriate time.

In carrying out an actuarial valuation, assumptions are made about a range of future events such as how long scheme members will survive, probable investment return, salary and price inflation.

The Main Elements of Defined Pension Model

\( s \): planning horizon or date of the end of the pension plan, with \( 0 < s < \infty \)

\( f(s) \): value of the fund asset at time \( s \)

\( P(s) \): Benefit advanced to the member at time \( s \)

\( C(s) \): Contribution rate made by the sponsor at time \( s \)

\( l(s) \): Actuarial liabilities at time \( s \) that is, total liabilities of the sponsor

\( n(s) \): Normal cost at time \( s \) of the fund assets

\( ul(s) \): Unfunded actuarial liability at time \( s \) equal to \( l(s) - f(s) \)

\( S(s) \): Supplementary cost at time \( t \), equal to \( C(s) - n(s) \)

\( P(x) \): Percentage of the value of the future benefit accumulated until age \( x \in [R, m] \) where \( R \) defines retirement age and \( m > R \)

\( \delta(s) \): variable rate of valuation of the liabilities which is defined by the regulatory authorities.

\( r(s) \): variable risk-free market interest rate

Actuarial liabilities which are estimated every year for each pension fund can be categorized into three. (i) total accrued liability, (ii) total future liability and future liability and total liability. Total accrued liability is the liability at hand when the estimation takes place and it is dependent on the accrued benefits of members. The total future liability represents the estimated future liability provided that all current members continuously pay premiums into the fund and receive benefits in accordance with these premium payments. Total liability is the addition of both total accrued liability and total future liability.
In actuarial valuation, we define the process of comparing pension assets with liabilities so as to confirm if variations occur between the two components. The actuarial valuation exercise is conducted on both the accrued position of the fund and with the future premiums and benefits. The actuarial valuation on the accrued position considers the current asset value, \( A(s) \) and the accrued liabilities \( l(s) \) due to benefits that members have currently received. The total actuarial valuation considers the present value of the current member’s future premiums and the liabilities that are due as a result of these future premiums. It is often assumed that all current members will continuously pay premiums into the fund up to retirement and that no new members will enter the fund in the future. Again, the consideration of death is not accounted for so that the members are all assumed to survive to retirement. The accrued position would be defined as follows.

\[
AP_s = \frac{A(s)}{l(s)} - 1
\]

(1)

where \( AP_s \) represent the accrued actuarial position of the fund.

The total actuarial position would then be

\[
\bar{AP}_s = \frac{A(s) + PV(\Pi)}{l(s)} - 1 - \frac{\bar{l}}{l(s)} = \frac{A(s) + PV(\Pi) - l(s) - \bar{l}}{l(s) + \bar{l}}
\]

(2a)

\[
\bar{AP}_s = \frac{A(s) + PV(\Pi)}{l(s)} - 1 - \frac{\bar{l}}{l(s)} = \frac{(AP_s + 1) + PV(\Pi)}{l(s)} - 1 - \frac{\bar{l}}{l(s)}
\]

(2b)

where \( AP_s \) is the accrued actuarial position of the fund, \( PV(\Pi) \) defines the present value of all future premium payments from the current membership and \( \bar{l} \) is the liability that arises from the future premium payments \( \Pi \). \( A(s) \) is the current asset value and \( l(s) \) is the accrued liability of the pension fund.

The present value of an immediate yearly annuity of 1 per annum to \( (x) \) is denoted by \( a_x \) and can be considered as the sum of series of pure endowment of \( 1 \), i.e.

\[
a_x = A_{x\cdot} + A_{x\cdot} + A_{x\cdot} + \ldots + A_{x+\omega-x-1} = \frac{D_{x+1}}{D_x} + \frac{D_{x+2}}{D_x} + \frac{D_{x+1}}{D_x} + \ldots + \frac{D_{\omega-1}}{D_x}
\]

(3)

\[
a_x = \frac{D_{x+1}}{D_x} + \frac{D_{x+2}}{D_x} + \frac{D_{x+1}}{D_x} + \ldots + \frac{D_{\omega-1}}{D_x}
\]

(4)

\[
a_x = \frac{1}{D_x} \sum_{t=1}^{\omega-x-1} D_{x+1}
\]

(5)

\[
a_x = 1 + a_x
\]

(6)
Material and Methods:

The Minimum Funding Standard Valuation Model

In [11] (Mce Nally & Connor, 2018), the Minimum funding standard valuation is required to ascertain whether the fund satisfies the standard set by the regulatory authority. The minimum funding standard valuation is essentially designed to confirm if the scheme holds adequate assets to meet the benefits which have accrued to members at the date of the valuation. Following [11] (Mce Nally & Connor, 2018), the MFS is designed to spell out the minimum assets that a defined benefits scheme must hold and what course of action should be taken where the asset of the scheme falls below this minimum. The funding standard is satisfied if in the actuary’s sense of judgment, the schemes assets on the date of valuation exceed the addition of the transfer values at that date to which the members would be entitled to and the estimated expenses of winding up the scheme. The transfer values is computed by projecting the benefit payments to which the members will earn based on their date of employment, a correct margin for mortality improvement and assuming a prescribed investment rate of return as a discount factor.

The precise models used to calculate the pension fund liability under minimum funding standard valuation are setout in the models below:

**MODEL 1**

\[
L_{MFS} = \left(\frac{S}{60}\right) \times SAL \times (1 + EI)^n \times \left(1 - \left(\frac{1}{1 + AF}\right)^n\right) \times MVA
\]

\[
L_{MFS} = f(\alpha, AF)
\]

where \(\alpha = a_s\) and \(a_s = a_s + 1\)

\[
f(\alpha, AF) = \begin{cases} 
\ g(\alpha) & \text{for } \alpha = a_s \\
\ g(AF) & \text{for } \alpha = AF 
\end{cases}
\]

**MODEL 2**

\[
1 + AR = 1 + \frac{d_r - p_g}{1 + p_g}
\]

\[
1 + AR = 1 + \frac{p_g + d_r - p_g}{1 + p_g}
\]

\[
1 + AR = \frac{1 + d_r}{1 + p_g}
\]

\[
(1 + AR)(1 + p_g) = 1 + d_r
\]

\[
d_r = (1 + AR)(1 + p_g) - 1
\]

\[
\frac{1}{1 + AR} = \left(\frac{1 + p_g}{1 + d_r}\right)
\]

\[
AF = \left[1 - \left(\frac{1 + p_g}{1 + d_r}\right)^\frac{n}{AR}\right]
\]

**MODEL 3**

\[
AR = \left(\frac{1 + d_r}{1 + p_g}\right) - 1
\]
\[
AR = \frac{1 + d_x}{1 + p_g} - 1
\]
(18)
\[
AR = \frac{1 + d_x - 1 - p_g}{1 + p_g}
\]
(19)
\[
AR = \frac{d_x - p_g}{1 + p_g}
\]
(20)

Theorem 1

Under the condition that
\[\begin{align*}
(i). & \quad AR = \frac{d_x - p_g}{1 + p_g} \\
(ii). & \quad AF = \left[ 1 - \left( \frac{1 + p_g}{1 + d_x} \right) \right]^N_{AR}
\end{align*}\]
(21)

where \( N \) is expectation of life at retirement. Then \( L_{MFSL} \rightarrow 0 \)

To prove this, it is sufficient to prove that
\[
AF = \left[ 1 - \left( \frac{1 + p_g}{1 + d_x} \right) \right]^N_{AR} = 0
\]
(23)

Proof

Note that \( \lim_{n \to \infty} a^n = 0 \) if \( |a| < 1 \)
(24)

We assume
\[
\frac{1 + p_g}{1 + d_x} < 1
\]
(25)

Again since \( N \in \mathbb{Z}^+ \), \( AR \in \mathbb{R} \) and specifically \( AR < 1 \), then \( \frac{N}{AR} \to \infty \)

Consequently
\[
1 - \left( \frac{1 + p_g}{1 + d_x} \right) < 1 \text{ and }
\]
\[
1 - \left( \frac{1 + p_g}{1 + d_x} \right)^N_{AR} \to 0 \text{ hence } L_{MFSL} = 0
\]
(26)

Q.E.D

Variable Description

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_{MFSL} )</td>
<td>Total service liability</td>
</tr>
<tr>
<td>( S_c )</td>
<td>Numbers of years of pensionable service completed to date.</td>
</tr>
<tr>
<td>( SAL )</td>
<td>Current salary.</td>
</tr>
<tr>
<td>( EI )</td>
<td>Expected rate of inflation.</td>
</tr>
<tr>
<td>( X )</td>
<td>Numbers of years to retirement</td>
</tr>
<tr>
<td>( r_{pre} )</td>
<td>Pre-retirement discount rate</td>
</tr>
</tbody>
</table>
Based on the current events in our local financial setting, we assume the following values of the parameter are shown below for the purpose of this work

- $N$ Life expectancy after retirement 10 years
- $P_g$ Pension increase from 1% to 4.5%
- $EI$ Expected rate of inflation 17.93%
- $d_r$, Pre-retirement discount rate 7.75%
- $d_p$, Post-retirement discount rate 4.5%
- $MVA$, Market value adjustment 1.42

The market value adjustment ($MVA$) in equation (7) is an instrument which life offices and annuity issuer’s use to manage risk. A market value adjustment could be favourable when life annuity is surrendered at the time when interest rates are low. Before an annuity is surrendered, it is necessary to ensure the terms of the contract is well understood or if an $MVA$ or other penalties fees will come in. The interest rate trajectories should be well monitored since the timing matters for market value adjustments. The $MVA$ of fixed annuities are more common and preferred by consumers since they offer bigger rates than the book value fixed annuities and many fixed annuity consumers buy with the aim that they would not surrender their contracts. In a continuously increasing interest rate regime, the $MVA$ provides interest rate shield to the life office by charging the insured supplementary fees to surrender which reimburses them for having to sell assets at a loss. The supplementary charge would mitigate any potential benefit of higher rates a scheme holder would find somewhere else. Consequently, the protection against downside interest rates which the $MVA$ fixed annuity offers permits the life office to experience higher rates because they are protected against an influx of liquidation requests in the event interest rates rise.

**Method of Data Presentation and Analysis**

The analysis evaluates the salary data of 36 employees from the human resource department of a manufacturing firm in Jos south local government area of plateau state. The salary data was then validated against error for each staff. Along the salary data, the demographic data of each employee were also collected to enable us carry out full computation of the service liability of the employees. The data was analyzed using minimum funding standard model in R-language. The findings were depicted in graphs and tabular form for easy access of the numerical values.

**Table 1: Table of Liability $L_{nf}$**

<table>
<thead>
<tr>
<th>PCN</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
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<tbody>
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<td>001</td>
<td>0.183333</td>
<td>19609024.45</td>
<td>0.143599</td>
<td>0.032231</td>
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</table>
In table 1 above we have computed the liabilities of the plan for each member. The total service liability of the plan was computed using minimum funding standard model. In column \( E \), the total liability of the plan is 1303777.19 based on the parameters we have set below. where PCN is the permanent code number

\[
A = \frac{S}{60} \quad \text{(27)}
\]

\[
B = SAL \times (1 + EI)^X \quad \text{(28)}
\]

\[
C = \frac{1}{(1 + r_{pre})^X} = (1 + r_{pre})^{-X} \quad \text{(29)}
\]

\[
D = \left[ \frac{1}{AF} \left( 1 + \frac{1}{1 + AF} \right)^X \right] MVA \quad \text{(30)}
\]
where PCN is the permanent code number of the plan was computed using minimum funding standard model. In column plan is

\[
C = \begin{pmatrix}
0.07 & 0.36 & 0.35 & 0.34 & 0.33 & 0.32 & 0.31 & 0.30 & 0.29 & 0.28 & 0.27 & 0.26 & 0.25 & 0.24 & 0.23 & 0.22 & 0.21 & 0.20 & 0.19 & 0.18 & 0.17 & 0.16 & 0.15 & 0.14 & 0.13 & 0.12 & 0.11 & 0.10 & 0.09 & 0.08 & 0.07 & 0.06 & 0.05 & 0.04 & 0.03 & 0.02 & 0.01
\end{pmatrix}
\]

Above we have computed the liabilities of the plan for each member. The total service liability

\[
T \left( \frac{1}{1 + r_{pre}} \right)^X \left[ \frac{1}{1 + AF} \right]^N MVA
\]

(31)

Table 2: Table of Service Years Completed.

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<th>G</th>
<th>X</th>
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<td>036</td>
<td>509664</td>
<td>6</td>
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<td>29</td>
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</tbody>
</table>

Total 13998690.54

Source: Author’s computation
Table 2 above shows the computations of years of service completed the current age and the number of future service years. The annual salary of each member was also computed and the total annual salary of the plan is $139,869,054$ based on the parameter set below.

Where; $PCN =$ permanent code number.

- $F =$ (monthly salary) $\times$ 12
- $G = S_c$ = current age – entry age
- $x =$ current year – year of birth
- $I =$ retirement age – current age

Table 3: Table of Ordinary Annuity

<table>
<thead>
<tr>
<th>PCN</th>
<th>J</th>
<th>K</th>
<th>L</th>
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</thead>
<tbody>
<tr>
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<td>0</td>
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<td><strong>0.616883</strong></td>
<td><strong>0</strong></td>
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</table>

Source: Author’s computation
In table 3 a sensitivity analysis was carried out between annuity rate and annuity factor, it is apparent that annuity factor is zero. This was proved in theorem 1 above and the total value of annuity rate of the plan is $0.616883$ based on the parameter set below.

PCN is the permanent code number

$$J = \frac{P_g}{100}$$

$$K \Rightarrow AR = \frac{d_r - p_g}{1 + p_g}$$

$$L \Rightarrow AF = \left[ 1 - \left( \frac{1 + p_g}{1 + d_r} \right)^{\frac{N}{AR}} \right]$$

Table 4: Table of Life Annuity.

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<th>N</th>
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<td>19.849</td>
<td>18.849</td>
</tr>
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</table>

Source: Neill A. (1977)
Table 4 above shows the computation of life annuity due and annuity immediate and for each member of the scheme based on their respective age according to the parameters below.

where:

- $PCN$ is the Permanent code number.
- $x$ is the current year – year of birth

\[
M = a_x + 1 = a_{x+1} \quad \text{(35)}
\]

\[
N = a_x = a_{x-1} \quad \text{(36)}
\]

**GRAPHS**

![Graph 1: Graph of $L_{mfs}$ against Age](image1)

Figure 1: Graph of $L_{mfs}$ against Age

Figure 1 above shows the graph of total service liability ($L_{mfs}$) against their respective ages ($x$) for each of the member in the scheme.

![Graph 2: Graph of $L_{mfs}$ against $S_c$](image2)

Figure 2: Graph of $L_{mfs}$ against $S_c$

Figure 2 above shows the graph of total service liability $L_{mfs}$ against the number of pensionable service completed ($S_c$) to date by each member of the scheme.
Table 4 above shows the computation of life annuity due and annuity immediate and for each member of the scheme based on their respective age according to the parameters below.

where;

PCN is the Permanent code number.

\[ x = \text{current year} - \text{year of birth} \]

\[ P_{x} a_x \]

\[ L_{x} a_x \]

GRAPHS

Figure 1: Graph of \( mfsL \) against Age

Figure 1 above shows the graph of total service liability \( L_{mfs} \) against their respective ages \( x \) for each member of the scheme.

Figure 2: Graph of \( mfsL \) against \( cS \)

Figure 2 above shows the graph of total service liability \( L_{mfs} \) against the number of pensionable service completed \( cS \) to date by each member of the scheme.

Figure 3: Graph of \( mfsL \) against Salary

Figure 3 above shows the graph of total service liability \( L_{mfs} \) against their respective salary (sal) of each member of the scheme.

Figure 4: Graph of \( L_{mfs} \) against Future Service

Figure 4 shows the graph of total service liability \( L_{mfs} \) against the future years of service \( X \) to be completed.
Figure 5: Graph of $L_{mf}$ against Annuity $a_x$

Figure 5 shows the graph of total service liability ($L_{mf}$) against life annuity $a_x$.

**Discussion of Results**

**Theorem 2. Main Result**

Given that

$$f(\alpha, \zeta) = \begin{cases} L_{MFS}(\alpha) \neq 0 & \text{for } \alpha = a_x \\ L_{MFS}(\zeta) = 0 & \text{for } \zeta = AF \end{cases}$$  \hspace{1cm} (37a)$$

$$L_{MFS} = \left(\frac{S_p}{60}\right) \times [SAL \times (1 + EI)^x] \times \left(1 - \frac{1}{1 + \alpha}\right)^N \times \frac{\alpha}{\zeta} \times MVA$$  \hspace{1cm} (37)$$

$$L_{MFS} = \left(\frac{S_p}{60}\right) \times [SAL \times (1 + EI)^x] \times \left(1 - \frac{1}{1 + \zeta}\right)^N \times \frac{\alpha}{\zeta} \times MVA$$  \hspace{1cm} (38)$$

In particular, if we define

$$\zeta = AF = \left[1 - \left(\frac{1 + p_g}{1 + d_r}\right)\right]^{\frac{N}{AR}}$$ \hspace{1cm} (39)$$

Then

$AF = 0$ and $L_{MFS} = 0$

**Proof:**

Under the condition that

(i) $AR = \frac{d_r - p_g}{1 + p_g}$  \hspace{1cm} (40)$$

(ii) $AF = \left[1 - \left(\frac{1 + p_g}{1 + d_r}\right)\right]^{\frac{N}{AR}}$  \hspace{1cm} (41)$$

$N$ is expectation of life at retirement then $L_{MFS} \to 0$

To prove this, it is sufficient to prove that

$$AF = \left[1 - \left(\frac{1 + p_g}{1 + d_r}\right)\right]^{\frac{N}{AR}} = 0$$  \hspace{1cm} (42)$$
Note that \( \lim_{n \to \infty} a^n = 0 \) if \( |a| < 1 \) \( (43) \)

We assume \( \frac{1 + p_g}{1 + d_r} < 1 \) \( (44) \)

Again since \( N \in \mathbb{Z}^+ \), \( AR \in \mathbb{R} \) and specifically \( AR < 1 \), then \( \frac{N}{AR} \to \infty \) \( (45) \)

Consequently \( 1 - \left( \frac{1 + p_g}{1 + d_r} \right)^{N/AR} \to 0 \) \( (47) \)

and \( \left( 1 - \left( \frac{1}{1 + AF} \right) \right)^N = 0 \) \( (48) \)

hence \( L_{MFS} = 0 \) \( (49) \)

If we substitute \( AF = 0 \) into \( L_{MFS} \) then

\[
L_{MFS} = \left( \frac{S_r}{60} \right) \times MVA \times \left[ SAL \times \left( 1 + EI \right)^x \right] \times \left( \frac{1}{1 + r_{pre}}^x \right) \times \left( \frac{1 - \left( \frac{1}{1 + AF} \right)^N}{AF} \right) = 0
\] \( (50) \)

If \( \alpha = AF \) and given the size of the fund, we obtain

\[
u l(s) = l(s) - f(s)
\] \( (51) \)

\[
u l(s) = -f(s) \implies ul(s) + f(s) = 0
\] \( (52) \)

If \( \alpha = a_x \)

\[
L_{MFS} = \left( \frac{S_r}{60} \right) \times MVA \times \left[ SAL \times \left( 1 + EI \right)^x \right] \times \left( \frac{1}{1 + r_{pre}}^x \right) \times \left( \frac{1 - \left( \frac{1}{1 + a_x} \right)^N}{a_x} \right)
\] \( (54) \)

\[
L_{MFS} = \left( \frac{S_r}{60} \right) \times MVA \times \left[ SAL \times \left( 1 + EI \right)^x \right] \times \left( \frac{1}{1 + r_{pre}}^x \right) \times \left( \frac{1 - \frac{1}{a_x}}{a_x} \right)^N
\] \( (55) \)
must be defined as years. This also includes the market value years while those that are above the age of years, the total service liability was . The pension accrual of is also higher than the discount rate pre-retirement. Column are not evenly distributed. In figure years and Column years while the older member shows the total service liability of each . Column shows the projection of the annual salary figure of each member as compared to the expected number of years that each member is expected to survive. The number of years that each member is shows the computation of annuity factor based on the respective age of each member based on the pensionable years of service completed divided by . This is confirmed in column E of table 1 and consequently we obtain

\[
ul (s) + f (s) = \left( \frac{S_c}{60} \right) \times MVA \times \left[ SAL \times (1 + EI) \right] \times \left( \frac{1}{1 + r_{pre}} \right)^x \times \left( \frac{a_s - 1}{a_s} \right)^N \left( \frac{a_s}{a_s} \right)^{N-1} \neq 0 \] (57)

\[
L_{MFS} = \left( \frac{S_c}{60} \right) \times MVA \times \left[ SAL \times (1 + EI) \right] \times \left( \frac{1}{1 + r_{pre}} \right)^x \times \left( \frac{a_s}{a_s} \right)^N \] (56)

\[
L_{MFS} = \left( \frac{S_c}{60} \right) \times MVA \times \left[ SAL \times (1 + EI) \right] \times \left( \frac{1}{1 + r_{pre}} \right)^x \times \left( \frac{a_s}{a_s} \right)^N \] (58)

This is confirmed in column E of table 1 and consequently we obtain

\[
ul (s) + f (s) = \left( \frac{S_c}{60} \right) \times MVA \times \left[ SAL \times (1 + EI) \right] \times \left( \frac{1}{1 + r_{pre}} \right)^x \times \left( \frac{1 - \frac{1}{1 + AF}}{AF} \right)^N = 1303777.19 \] (59)

Q.E.D

In table 1, column A shows the pension accrual of each member of the scheme which is the number of pensionable years of service completed divided by . The total pension accrual of the plan is . Column B shows the projection of the annual salary figure of each member as compared to the expected rate of inflation in the country. This also depends on the number of years to retirement of each member of the scheme, the total value of the salary projection is . Column C shows the pre-retirement discount rate of each member of the scheme and the total value is . Column D shows the computation of annuity factor based on the respective age of each member based on the number of years that each member is expected to survive. The number of years that each member is expected to survive after retirement is assumed to be years. This also includes the market value adjustment \( MVA \). The total value is . Column E shows the total service liability of each member of the scheme. The computation was based on minimum funding standard model and the total value is .

Therefore, the projected annual salary of is by far higher than the total service liability of . The pension accrual of is also higher than the discount rate pre-retirement.
In table 2 column \( F \) shows the annual salary of each member of the scheme, the total annual salary is 13998690.54. Column \( G \) shows the number of pensionable service completed to date by each member of the scheme. The minimum is 2 years while the maximum is 33 years. Column \( X \) shows the current age of the members of the scheme, the younger member has the age of 25 years while the older member has the age of 57 years. Column \( I \) shows the number of years that each member still have to serve. The lowest age a member still has to serve is 8 years while the highest age is 40 years.

In table 3 column \( J \) shows the pension growth rate from 1.0\% to 4.5\% with the total pension growth rate of 0.99. Column \( K \) shows the computation of annuity rate which is based on post retirement discount rate \( d_r \) and the pension growth rate \( P_g \). The total value is 0.616883. Column \( L \) shows the computation of the annuity factor and the total is 0. The Column \( L \) confirms our theorem 2 forming the basis of this paper.

In table 4 column \( X \) shows the respective age of the members. Column \( M \) shows the life annuity due of each member according to their respective age. Column \( N \) shows the life annuity immediate of each member according to their respective age.

The distribution of liabilities in the figures 1–5 are not evenly distributed. In figure 1, the total service liability of the plan was plotted against current age of the members. The graph shows that majority of the plan are clustered around ages 30 and 45 years while those that are above the age of 45 are dispersed. In figure 2, the total service liability of the plan was plotted against the number of pensionable years completed to date. The graph shows that high number of pensionable years of service completed are clustered between 2 years and 11 years while those that have completed more 11 years of service are dispersed. In figure 3, the total service liability of the plan was plotted against salary of each member of the scheme. The graph shows that the annual salary of the scheme members are clustered between 120000 to 257000 and those above 257000 are dispersed. In figure 4, the total service liability was plotted against future years of service. The graph shows that the years of future service of the members are dispersed below 20 years while those above 25 and 35 years of future service are clustered. In figure 5, the total service liability was plotted against life annuity. The graph shows that the liability of each member is unevenly distributed over the life annuity from 1 to 35 units.

**Conclusion**

Retirement plan funding anticipates that over long term, both contribution rate of administrative expenses and investment earnings less investment fees will be needed to cover benefit payments. Retirement plan asset changes as a result of the net impact of these income and expenses component. It is necessary to lower level and predictable plan cost from year to year. For this reason, the valuation method considered market value adjustment. The objective of this study is to determine the total service liability of employees in the plan using \( MFS \) valuation model. The \( MFS \) valuation computes the lump sum required to satisfy the future pension benefits of the scheme’s member based on the completed years of pensionable service to date and their current salary. Future pension benefits is computed by taking the expected annual pension benefits in the year of retirement and multiplying it by an annuity factor to reflect the expected lifespan of the member after retirement. The lump sum computed is then discounted back to the present by considering the pre and post retirement investment growth rates.

In this study, we have computed the total service liability of the plan based on the following key parameters; pension accrual, annual salary, expected rate of inflation, number of years to retirement, discount rate before retirement and the market value adjustment. In order that we establish the condition that \( L_{MFS} \to 0 \), the function \( AF \) must be defined as

\[
AF = \left[ 1 - \frac{(1 + P_g)}{(1 + d_r)} \right]^{\frac{N}{d_r}}
\]  
(60)
References


