



# Generalized Difference Sequence Spaces with Ideal Convergence in $n$ -Normed Spaces

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**Abstract:** In this paper, we define difference sequence spaces of type  $I$ -convergent,  $I$ -null, bounded  $I$ -convergent, and bounded  $I$ -null in  $n$ -normed space using the Orlicz function. We also study some algebraic and topological properties of these new sequence spaces including some inclusion relations.

**Keywords:** Orlicz function, Sequence space,  $I$ -convergence,  $n$ -Normed space.

## 1. Introduction

Infinite sequence and series play a vital role in mathematics and various scientific discipline. They are often used to represent functions, approximate values and solve complex problems. Understanding the properties and behaviour of infinite sequence and series is fundamental to advance mathematical concept and practical applications in science and engineering.

Numerous mathematicians encountered difficulties when dealing with the limits or sums of infinite sequences and series that have divergent behaviour. As a result, considerable research was started towards determining the limits or sums of such divergent sequences and series through various summability methods. This led to the emergence of a new branch in mathematical analysis, dedicated to assigning limits to divergent sequences and series. This investigation focuses on constructing new sequence spaces by combining difference operators, Orlicz functions, and ideal convergence within the framework of  $n$ -normed spaces. We begin by presenting essential definitions and notations that justify our work.

**Definition 1.1.** An **Orlicz function** [7] is a mapping  $M : [0, \infty) \rightarrow [0, \infty)$  with the following characteristics:

- Continuous, convex, and monotonically increasing;
- Vanishes at zero;
- $M(y) > 0$  for all  $y > 0$ ;
- $\lim_{y \rightarrow \infty} M(y) = \infty$ ;

**Definition 1.2.** An Orlicz function  $M$  satisfies the  $\Delta_2$ -condition [7] if there exists  $L > 0$  such that for all  $y \geq 0$ :

$$M(2y) \leq LM(y).$$

Equivalently, for each  $Q > 1$ , there exists  $L_Q > 0$  satisfying:

$$M(Qy) \leq L_Q M(y), \text{ for all } y \geq 0.$$

Orlicz sequence spaces generalize classical  $l_p$  spaces. They were initially introduced in 1936 by Orlicz. Later, Lindenstrauss and Tzafriri [8] utilized Orlicz functions to construct the Orlicz sequence space  $l_M$ , defined as follows:

$$l_M = \left\{ y = (y_k) \in w : \sum_{k=1}^{\infty} M\left(\frac{|y_k|}{\rho}\right) < \infty \text{ for some } \rho > 0 \right\}$$

of scalars  $(y_k)$ . The first detailed study on Orlicz spaces was given by Krasnosel'skiĭ and Rutickiĭ [7], which is a Banach space with norm

$$\|y\|_M = \inf \left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|y_k|}{\rho}\right) \leq 1 \right\}.$$

Moreover,  $l_M$  looks like the space  $l_p$  with  $M(y) = y^p$ ;  $1 \leq p < \infty$ .

**Definition 1.3.** For a vector space  $Y$  with  $\dim(Y) > 1$ , an  **$n$ -norm** [9] is a function

$$\|\cdot, \dots, \cdot\| : Y^n \rightarrow \mathbb{R}$$

satisfying:

1.  $\|y_1, y_2, \dots, y_n\| = 0$  if and only if  $y_1, y_2, \dots, y_n$  are linearly dependent,
2.  $\|y_1, y_2, \dots, y_n\|$  remains unchanged under any permutation of the vectors,
3.  $\|\alpha y_1, y_2, \dots, y_n\| = |\alpha| \|y_1, y_2, \dots, y_n\|$  for all  $\alpha \in \mathbb{R}$ ,
4. Subadditive in each argument.

The pair  $(Y, \|\cdot, \cdot, \dots, \cdot\|)$  is then referred to as an  **$n$ -normed space**.

Geometrically, the  $n$ -norm measures the volume of the  $n$ -dimensional parallelepiped formed by its vector arguments.

**Definition 1.4.** Kizmaz introduced difference sequence spaces [5]:

$$\begin{aligned} c_0(\Delta) &= \{y : \Delta y \in c_0\}, \\ c(\Delta) &= \{y : \Delta y \in c\}, \\ \ell_{\infty}(\Delta) &= \{y : \Delta y \in \ell_{\infty}\}, \end{aligned}$$

where  $y = (y_k)$  and the difference operator is given by  $\Delta y = (\Delta y_k) = (y_k - y_{k+1})$ . He also demonstrated that such spaces are Banach spaces having norm given by:

$$\|y\| = |y_1| + \|\Delta y\|_{\infty}.$$

The concept of ideal convergence was first introduced by Kostyrko et al. [6] as a generalization of statistical convergence. For more detailed discussions on these types of sequence spaces, one

may refer to the works of Mursaleen et al. [14], Ghimire and Pahari [2], Hazarika et al. [4], Mursaleen and Alotaibi [11], Mursaleen and Mohiuddine [12], Mursaleen and Sharma [13], Raj et al. [16], Salat et al. [17], Mohiuddine et al. [10], Savas [18, 19], Tripathy and Hazarika [20], and many others.

**Definition 1.5.** A family  $I \subseteq 2^Y$  is called an **ideal** [6] on  $Y$  if

1.  $\emptyset \in I$ ,
2.  $C \in I$  and  $D \subseteq C \Rightarrow D \in I$  (hereditary),
3.  $C, D \in I \Rightarrow C \cup D \in I$  (finite additivity).

**Definition 1.6.** In an  $n$ -normed space  $(Y, \|\cdot, \dots, \cdot\|)$ , a sequence  $(y_k)$  is **I-convergent** [17] to  $l \in Y$  if for all  $v_1, \dots, v_{n-1} \in Y$  and  $\varepsilon > 0$ :

$$\{k \in \mathbb{N} : \|y_k - l, v_1, \dots, v_{n-1}\| \geq \varepsilon\} \in I.$$

This is denoted by

$$I - \lim_k \|y_k - l, v_1, \dots, v_{n-1}\| = 0.$$

**Definition 1.7.** A sequence space  $Y$  is a **sequence algebra** [20] if it is closed under termwise multiplication.

**Definition 1.8.** A sequence space  $Y$  is **solid** [15] if it is closed under multiplication by bounded scalar sequences.

## 2. Main Results

The work done in [3] is extended by introducing and investigating the following classes of ideal convergent difference sequence spaces defined by Orlicz function in  $n$ -normed space [1].

Let  $(Y, \|\cdot, \dots, \cdot\|)$  be an  $n$ -normed space and  $\omega$  the space of all vector-valued sequences. We now proceed to define the following sequence spaces:

Let  $(Y, \|\cdot, \dots, \cdot\|)$  be an  $n$ -normed space and let  $\omega$  denote the set of  $Y$ -valued sequences.

$$s^I(\Delta, M, \|\cdot\|_n) = \left\{ y = (y_k) \in \omega : \exists \rho > 0, \text{ and } l \in Y \text{ such that for all } v_1, \dots, v_{n-1} \in Y, \right. \\ \left. I - \lim_k M\left(\frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho}\right) = 0 \right\},$$

$$s_0^I(\Delta, M, \|\cdot\|_n) = \left\{ y = (y_k) \in \omega : \exists \rho > 0 \text{ such that for all } v_1, \dots, v_{n-1} \in Y, \right. \\ \left. I - \lim_k M\left(\frac{\|\Delta y_k, v_1, \dots, v_{n-1}\|}{\rho}\right) = 0 \right\},$$

$$\ell_\infty(\Delta, M, \|\cdot\|_n) = \left\{ y = (y_k) \in \omega : \exists \rho > 0 \text{ such that for all } v_1, \dots, v_{n-1} \in Y, \right. \\ \left. \sup_k M\left(\frac{\|\Delta y_k, v_1, \dots, v_{n-1}\|}{\rho}\right) < \infty \right\}.$$

Additionally, we define:

$$\begin{aligned} u^I(\Delta, M, \|\cdot\|_n) &= s^I(\Delta, M, \|\cdot\|_n) \cap \ell_\infty(\Delta, M, \|\cdot\|_n), \\ u_0^I(\Delta, M, \|\cdot\|_n) &= s_0^I(\Delta, M, \|\cdot\|_n) \cap \ell_\infty(\Delta, M, \|\cdot\|_n). \end{aligned}$$

The sequence spaces  $s^I(\Delta, M, \|\cdot\|_n)$ ,  $s_0^I(\Delta, M, \|\cdot\|_n)$ ,  $u^I(\Delta, M, \|\cdot\|_n)$ , and  $u_0^I(\Delta, M, \|\cdot\|_n)$  represent I-convergent, I-null, bounded I-convergent, and bounded I-null sequences respectively.

**Theorem: 2.1** The spaces  $s^I(\Delta, M, \|\cdot\|_n)$ ,  $s_0^I(\Delta, M, \|\cdot\|_n)$ ,  $u^I(\Delta, M, \|\cdot\|_n)$ , and  $u_0^I(\Delta, M, \|\cdot\|_n)$  are linear.

*Proof:-* We demonstrate linearity for  $s_0^I(\Delta, M, \|\cdot\|_n)$ ; other cases follow similarly.

Suppose  $x = (x_k)$  and  $y = (y_k)$  belong to  $s_0^I(\Delta, M, \|\cdot\|_n)$ . Then we can find constants  $\rho_1, \rho_2 > 0$ :

$$\begin{aligned} I - \lim_k M \left( \frac{\|\Delta x_k, v_1, \dots, v_{n-1}\|}{\rho_1} \right) &= 0, \\ I - \lim_k M \left( \frac{\|\Delta y_k, v_1, \dots, v_{n-1}\|}{\rho_2} \right) &= 0. \end{aligned}$$

for all  $v_1, \dots, v_{n-1} \in Y$  For any  $\varepsilon > 0$ , define

$$\begin{aligned} A_1 &= \left\{ k \in \mathbb{N} : M \left( \frac{\|\Delta x_k, v_1, \dots, v_{n-1}\|}{\rho_1} \right) > \frac{\varepsilon}{2} \right\}, \\ A_2 &= \left\{ k \in \mathbb{N} : M \left( \frac{\|\Delta y_k, v_1, \dots, v_{n-1}\|}{\rho_2} \right) > \frac{\varepsilon}{2} \right\}. \end{aligned}$$

Then,  $A_1, A_2 \in I$

If we choose

$$\rho = \max\{2|\beta|\rho_1, 2|\gamma|\rho_2\},$$

where  $\beta, \gamma$  are scalars, then by convexity and monotonicity of Orlicz function  $M$ ,

$$\begin{aligned} M \left( \frac{\|\beta \Delta x_k + \gamma \Delta y_k, v_1, \dots, v_{n-1}\|}{\rho} \right) &\leq M \left( \frac{|\beta| \|\Delta x_k, v_1, \dots, v_{n-1}\|}{\rho} + \frac{|\gamma| \|\Delta y_k, v_1, \dots, v_{n-1}\|}{\rho} \right) \\ &\leq M \left( \frac{\|\Delta x_k, v_1, \dots, v_{n-1}\|}{\rho_1} \right) + M \left( \frac{\|\Delta y_k, v_1, \dots, v_{n-1}\|}{\rho_2} \right). \end{aligned}$$

Therefore,

$$\left\{ k \in \mathbb{N} : M \left( \frac{\|\beta \Delta x_k + \gamma \Delta y_k, v_1, \dots, v_{n-1}\|}{\rho} \right) > \varepsilon \right\} \subseteq A_1 \cup A_2.$$

Since  $A_1, A_2 \in I$  and  $I$  is an ideal, the left set also belongs to  $I$ . Hence,  $s_0^I(\Delta, M, \|\cdot\|_n)$  is linear.

**Theorem: 2.2** If the Orlicz functions  $M_1, M_2$  satisfy the  $\Delta_2$ -condition, then

$$(a) \quad T(\Delta, M_1, \|\cdot\|_n) \subseteq T(\Delta, M_2 \circ M_1, \|\cdot\|_n)$$

$$(b) \quad T(\Delta, M_1, \|\cdot\|_n) \cap T(\Delta, M_2, \|\cdot\|_n) \subseteq T(\Delta, M_1 + M_2, \|\cdot\|_n) \text{ for } T = s^I, s_0^I, u^I, u_0^I.$$

*Proof:-* Here the results (a), (b) are proved for the space  $T = s^I$ . For remaining spaces, results can be proved in a similar manner.

(a) Suppose  $y = (y_k)$  belong to  $s^I(\Delta, M_1, \|\cdot\|_n)$ . Then, there exists a positive constant  $\rho$  :

$$I - \lim_k M_1 \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho} \right) = 0, \text{ for } v_1, \dots, v_{n-1} \in Y.$$

Let  $\varepsilon > 0$  be given. Then, there exists  $0 < \delta < 1$  with  $M_1(u) < \varepsilon$ , for all  $0 \leq u \leq \delta$ , since  $M_1$  is an Orlicz function. Let us define the following sets:

$$B_1 = \left\{ k \in \mathbb{N} : M_1 \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho} \right) \leq \delta \right\},$$

$$B_2 = \left\{ k \in \mathbb{N} : M_1 \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho} \right) > \delta \right\}.$$

If  $k \in B_2$ , then

$$M_1 \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho} \right) < \frac{1}{\delta} M_1 \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho} \right) < 1 + \left\{ \frac{1}{\delta} M_1 \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho} \right) \right\}$$

Then by convexity and monotonicity of  $M_2$ , we have

$$M_2 \left\{ M_1 \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho} \right) \right\} < M_2 \left\{ 1 + \left\{ \frac{1}{\delta} M_1 \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho} \right) \right\} \right\}$$

$$< \frac{1}{2} M_2(2) + \frac{1}{2} M_2 \left\{ 2 \cdot \frac{1}{\delta} M_1 \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho} \right) \right\}.$$

Since  $M_2$  satisfies the  $\Delta_2$ -condition, we have

$$M_2 \left\{ M_1 \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho} \right) \right\} < \frac{1}{2} L \left\{ \frac{1}{\delta} M_1 \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho} \right) \right\} M_2(2)$$

$$+ \frac{1}{2} L \left\{ \frac{1}{\delta} M_1 \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho} \right) \right\} M_2(2)$$

$$= \frac{L}{\delta} M_2(2) M_1 \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho} \right).$$

For  $k \in B_1$ , we have

$$M_1 \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho} \right) < \delta \implies M_2 \left\{ M_1 \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho} \right) \right\} < \varepsilon.$$

Hence, we conclude that

$$s^I(\Delta, M_1, \|\cdot\|_n) \subseteq s^I(\Delta, M_2 \circ M_1, \|\cdot\|_n).$$

(b) Suppose  $y = (y_k)$  lies in  $s_0^I(\Delta, M_1, \|\cdot\|_n) \cap s_0^I(\Delta, M_2, \|\cdot\|_n)$ .

Then we can find positive constants  $\rho_1$  and  $\rho_2$ , such that

$$I - \lim_k M_1 \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho_1} \right) = 0, \text{ for } v_1, \dots, v_{n-1} \in Y,$$

and

$$I - \lim_k M_2 \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho_2} \right) = 0, \text{ for all } v_1, \dots, v_{n-1} \in Y.$$

Choose  $\rho = \max\{\rho_1, \rho_2\}$ . Then,

$$\begin{aligned} (M_1 + M_2) \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho} \right) &= M_1 \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho} \right) + M_2 \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho} \right) \\ &\leq M_1 \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho_1} \right) + M_2 \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho_2} \right) \end{aligned}$$

This implies that

$$s^I(\Delta, M_1, \|\cdot\|_n) \cap s^I(\Delta, M_2, \|\cdot\|_n) \subseteq s^I(\Delta, M_1 + M_2, \|\cdot\|_n)$$

**Theorem: 2.3.** The following inclusion relations always hold:

$$s_0^I(\Delta, M, \|\cdot\|_n) \subseteq s^I(\Delta, M, \|\cdot\|_n) \subseteq l_\infty^I(\Delta, M, \|\cdot\|_n).$$

*Proof:-* The relation  $s_0^I(\Delta, M, \|\cdot\|_n) \subseteq s^I(\Delta, M, \|\cdot\|_n)$  is trivial.

Suppose  $y = (y_k) \in s^I(\Delta, M, \|\cdot\|_n)$ . Then we can find  $\rho > 0$ , and  $l \in Y$  with

$$I - \lim_k M \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho} \right) = 0$$

for all  $v_1, \dots, v_{n-1} \in Y$ .

Now,

$$M \left( \frac{\|\Delta y_k, v_1, \dots, v_{n-1}\|}{2\rho} \right) \leq \frac{1}{2} M \left( \frac{\|\Delta y_k - l, v_1, \dots, v_{n-1}\|}{\rho} \right) + \frac{1}{2} M \left( \frac{\|l, v_1, \dots, v_{n-1}\|}{\rho} \right).$$

Taking the supremum over  $k$  on both sides, we can prove that

$y \in l_\infty^I(\Delta, M, \|\cdot\|_n)$ .

Hence,

$$s_0^I(\Delta, M, \|\cdot\|_n) \subseteq s^I(\Delta, M, \|\cdot\|_n) \subseteq l_\infty^I(\Delta, M, \|\cdot\|_n).$$

**Theorem: 2.4.** The classes  $s_0^I(\Delta, M, \|\cdot\|_n)$ ,  $u_0^I(\Delta, M, \|\cdot\|_n)$  are solid.

*Proof:-* We start by proving that  $s_0^I(\Delta, M, \|\cdot\|_n)$  is solid. Second one can be proved similarly.

Suppose  $y = (y_k) \in s_0^I(\Delta, M, \|\cdot\|_n)$ . Then we can find a positive constant  $\rho$  such that

$$I - \lim_k M \left( \frac{\|\Delta y_k, v_1, \dots, v_{n-1}\|}{\rho} \right) = 0 \quad \text{for all } v_1, \dots, v_{n-1} \in Y.$$

Suppose a sequence of scalars  $(\beta_k)$  satisfying  $|\beta_k| \leq 1$  for each  $k \in \mathbb{N}$ . Then

$$M\left(\frac{\|\beta_k \Delta y_k, v_1, \dots, v_{n-1}\|}{\rho}\right) \leq |\beta_k| M\left(\frac{\|\Delta y_k, v_1, \dots, v_{n-1}\|}{\rho}\right) \leq M\left(\frac{\|\Delta y_k, v_1, \dots, v_{n-1}\|}{\rho}\right).$$

This implies that

$$I - \lim_k M\left(\frac{\|\beta_k \Delta y_k, v_1, \dots, v_{n-1}\|}{\rho}\right) = 0.$$

This proves that  $\alpha_k y_k \in s_0^I(\Delta, M, \|\cdot\|_n)$ , and so  $s_0^I(\Delta, M, \|\cdot\|_n)$  is solid.

**Theorem: 2.5.** The classes  $s^I(\Delta, M, \|\cdot\|_n)$ ,  $s_0^I(\Delta, M, \|\cdot\|_n)$  are sequence algebras.

*Proof:-* We start by proving that  $s_0^I(\Delta, M, \|\cdot\|_n)$  is a sequence algebra. Second one can be proved similarly

Suppose  $(x_k), (y_k)$  belong to  $s_0^I(\Delta, M, \|\cdot\|_n)$ . Then we can find positive constants  $\rho_1$  and  $\rho_2$  such that, for all  $v_1, \dots, v_{n-1} \in Y$ ,

$$I - \lim_k M\left(\frac{\|\Delta x_k, v_1, \dots, v_{n-1}\|}{\rho_1}\right) = 0$$

and

$$I - \lim_k M\left(\frac{\|\Delta y_k, v_1, \dots, v_{n-1}\|}{\rho_2}\right) = 0.$$

Choose  $\rho = \rho_1 \rho_2$ . Then one can easily show that

$$I - \lim_k M\left(\frac{\|\Delta(x_k y_k), v_1, \dots, v_{n-1}\|}{\rho}\right) = 0.$$

This implies that  $(x_k).(y_k) = (x_k y_k) \in s_0^I(\Delta, M, \|\cdot\|_n)$ , which means that  $s_0^I(\Delta, M, \|\cdot\|_n)$  is a sequence algebra.

### 3. Conclusion

This research establishes fundamental properties of difference sequence spaces in n-normed spaces with ideal convergence and Orlicz functions. These results also provide a foundation for constructing new sequence spaces of similar types with enhanced algebraic and geometric properties.

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