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Analysis of Blood Flow Through an Overlapping Mild Arterial Stenosis

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Abstract: The abnormal narrowing of the arteries due to stenosis significantly impacts cardiovascular function by restricting the blood flow. The progression of stenosis, influenced by factors such as blood viscosity, shear stress, and artery curvature, can lead to severe complications, including strokes and heart failure. In this article, we have focused on overlapping stenosis, which poses a unique challenge due to its irregular shapes and greater obstruction of blood flow. This study analytically investigates key hemodynamic parameters such as velocity, volume flow rate, pressure drop, pressure drop ratio, shear stress, and shear stress ratio as blood passes through the overlapping region, treating blood as a Newtonian fluid. The velocity and volume flow rate decrease heavily due to overlapping stenosis, and the pressure drop and shear stress increase in the overlapping stenotic region. With the help of this change, we can predict the real situation of cardiovascular disease, and many lives can be saved using this information.

Keyword: Mild stenosis, Viscosity coefficient, Velocity profile, Volumetric flow rate, Pressure drop, Shear stress, Overlapping model.

1. Introduction

The development and progression of many vascular diseases, which impair cardiovascular function, are closely linked to blood flow characteristics and the deformability of the vascular walls. Stenosis, a hardening of the arterial inner wall due to cholesterol or fatty particle accumulation plays major role for the vascular disease [26]. Stenosis, the abnormal narrowing of arteries, is often caused by atheroma a build up of fats and fibrous tissue within the arterial lumen leading to restriction of the blood flow beyond the narrowed region. This constriction, which can occur at various points in the cardiovascular system, may result in severe consequences [10]. Blood flow is complex, as blood consists of plasma and suspended deformable particles like red and white blood cells and platelets, making its behavior challenging to analyze [13]. Factors such as blood pressure differences and vascular resistance, along with coronary artery motion driven by heart wall motion, are critical in determining blood flow, and the artery's geometry, particularly its curvature, significantly influences hemodynamics [16].

Many researchers have theoretically studied single symmetric and non-symmetric stenoses, but stenoses can also develop in series, have irregular shapes, or overlap. Samundra et al. [6] investigated steady laminar flow through modeled vascular stenosis, while Chakravarty et al. [2] explored mathematical modeling of blood flow through overlapping arterial stenosis. Srivastava and Rastogi [29] analyzed blood flow through a non-symmetrical stenosis in a narrow catheterized artery and recommended selecting a catheter size smaller than half the artery's diameter during catheterization for treatment. Mekheimer and Kot [19] have analyzed the blood flow in the carotid artery under magnetic effects and concluded that for a micropolar fluid, the flow resistance increases with the coupling parameter N, representing the particle size effect, and decreases with the micropolar parameter m, associated with the micropolar spin parameter effect. Gautam et al. [3] have studied the effect of thickness of the stenosis increasing over time on flow parameter taking symmetric shaped stenosis. Ponalagusamy and Manchi [25] have studied various shapes of the stenosis like triangular, bell-shaped, or composite. Stenosis typically occur at low shear rates and high viscosity, often near bifurcations [16]. Key factors like hematocrit levels, wall shear stress, and blood viscosity contribute to stenosis formation [21]. Over time, stenosis can progress to full occlusion, raising blood pressure and altering flow dynamics [1]. Severe stenosis may cause strokes, heart failure, or paralysis, posing a global health threat [4, 8].

Several studies have focused on the impact of stenosis on blood flow, particularly in non-Newtonian fluids. Pokheral et al. have studied the blood flow parameters using blood as a non-Newtonian fluid [7]. Gautam et al. [3] have calculated flow parameters in arteries with mild and progressive stenosis, treating blood as a non-Newtonian fluid. The rate of increment of the aortic stenosis is very high in the case of a cancer patient [1]. rapid stenosis progression in cancer patients with aortic stenosis. [18] a reference for the human coronary artery's diameter, while Pao et al. [23] have explained a detailed method to calculate bending and twisting at bifurcations. Padmanabhan and Jayaraman [22] linked flow impedance to pressure drop in curved stenotic artery, and Santamarina et al. [27] emphasized the effect of arterial curvature on shear stress. Srivastava and Rastogi [29] and Mekheimer and Elkot [19] examined unsteady, viscous, and non-Newtonian flows through overlapping stenoses and permeable arterial walls. More recently Akaber et al. [30] worked on unsteady blood flow in a w-shaped stenosed artery using two-layered model. Sharma et al. [20] have studied the blood flow through a overlapping stenosis taking blood as a non-Newtonian.

Over time, there is always a chance for overlapping stenosis to form due to the obstruction caused by a single stenosis. We recognize that overlapping stenosis poses unique challenges, as it can be more dangerous than other types due to the increased blockage it creates. The distinct geometry of overlapping stenosis significantly impacts blood flow, leading to complications in cardiovascular system. This studies the analytical investigation of hemodynamic parameters, including velocity, pressure, and wall shear stress in the overlapping stenotic region, is crucial for advancing medical science. The advanced analytical approach used here could pave the way for more accurate diagnostic tools and therapeutic strategies for managing cardiovascular conditions.

2. Methods

Taking into consideration steady flow of blood in an axi-symmetric stenosed artery. Let R_0 and R denotes the radius of artery without stenosis and with stenosis respectively. The artery is considered an inelastic circular tube. The flow in radial direction is neglected in light of stenosis.

2.1. Mathematical Model

The geometry of the stenosis which is assumed to be manifested in the arterial segment given by is described in [2] and [12] as :

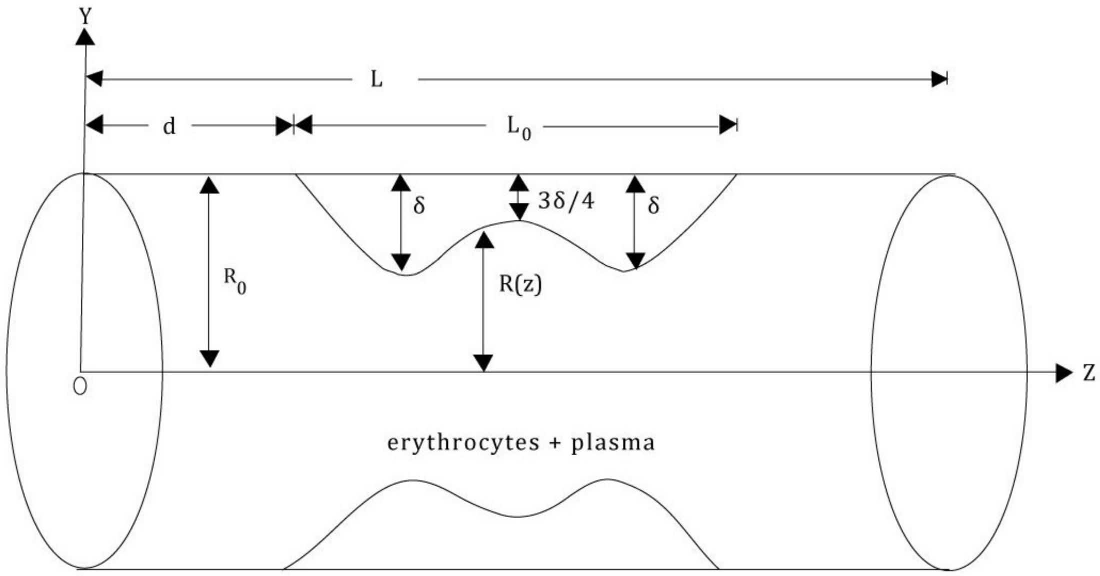

Figure 1: Geometry of the overlapping stenosis

$$\frac{R(z)}{R_0} = \begin{cases} 1 - \frac{3\delta}{2R_0L_0^4}\left[11\left(z-d\right)L_0^3 - 47\left(z-d\right)^2L_0^2 + 72\left(z-d\right)^3L_0 - 36\left(z-d\right)^4\right], d \le z \le d+L_0 \\ = 1, \qquad \text{otherwise} \end{cases} \tag{1}$$

By using Navier-Stokes equations for fluid flow inside a cylinder, blood flow in arteries can be modeled. Let r and P denote the radius and pressure drop of the blood flow in the artery. Component of the velocities along radial, angular and axial directions are v^r, v^θ and v^z respectively. The system can be expressed in cylindrical form as [5]

$$\frac{\partial v^r}{\partial r} + \frac{v^r}{r} + \frac{1}{r}\frac{\partial v^\theta}{\partial \theta} + \frac{\partial v^z}{\partial z} = 0 \tag{2}$$

$$\rho\left(\frac{\partial v^r}{\partial t} + v^r\frac{\partial v^r}{\partial r} + \frac{v^\theta}{r}\frac{\partial v^r}{\partial \theta} - \frac{(u^\theta)^2}{r} + v^z\frac{\partial v^r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left[-\frac{v^r}{r^2} + \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v^r}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 v^r}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v^\theta}{\partial \theta} + \frac{\partial^2 v^r}{\partial z^2}\right] \tag{3}$$

$$\rho\left(\frac{\partial v^\theta}{\partial t} + v^r\frac{\partial v^\theta}{\partial r} + \frac{v^\theta}{r}\frac{\partial v^\theta}{\partial \theta} + \frac{v^r v^\theta}{r} + v^z\frac{\partial v^\theta}{\partial z}\right) = -\frac{\partial p}{\partial \theta} + \mu\left[-\frac{v^\theta}{r^2} + \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v^\theta}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 v^\theta}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v^\theta}{\partial \theta} + \frac{\partial^2 v^\theta}{\partial z^2}\right] \tag{4}$$

$$\rho\left(\frac{\partial v^z}{\partial t} + v^r\frac{\partial v^{z\text{‘}}}{\partial r} + \frac{v^\theta}{r}\frac{\partial v^z}{\partial \theta} + v^z\frac{\partial v^z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v^z}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 v^z}{\partial \theta^2} + \frac{\partial^2 v^z}{\partial z^2}\right] \tag{5}$$

In axisymmetric flow, we assume $v^\theta = 0$, and v^r, v^z and p are independent of θ. Viscosity μ and density ρ are assumed constant for the steady flow of blood. Velocity component parallel to z- axis is v. We have considered axially symmetric flow along z-axis only, so $v^r = 0$, $v = 0$, and $v^z = v$, then equation (5) becomes

$$\frac{\partial v}{\partial z} = 0, -\frac{\partial p}{\partial r} = 0, 0 = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 v}{\partial r^2} + \frac{\partial^2 v}{\partial z^2} + \frac{1}{r}\frac{\partial v}{\partial r}\right) \tag{6}$$

Suppose pressure term as P = $-\frac{\partial p}{\partial z}$, equation (6) reduces to

$$-\frac{pr}{\mu}=\frac{\partial(r\frac{\partial v}{\partial r})}{\partial r} \tag{7}$$

2.1.1. Volocity

On integration with respect to r,

$$r\frac{\partial v}{\partial r}=-\frac{pr^2}{2\mu}+C(z) \tag{8}$$

Using $\frac{\partial v}{\partial r}=0$ at r = 0 gives C(z) = 0

$$r\frac{\partial v}{\partial r}=-\frac{pr^2}{2\mu} \tag{9}$$

$$\frac{\partial v}{\partial r}=-\frac{pr}{2\mu} \tag{10}$$

Integrating with respect to r, we get

$$v=-\frac{pr^2}{4\mu}+D(z) \tag{11}$$

Using v = 0 at r = R gives D(z) = $\frac{pr^2}{4\mu}$
The velocity is,

$$v=\frac{pR^2}{4\mu}-\frac{pr^2}{4\mu}\frac{r^2}{\mu}=\frac{p}{4\mu}(R^2-r^2) \tag{12}$$

$$v=\frac{p}{4\mu}\left\{R_0^2\left[1-\frac{3\delta}{2R_0L_0^4}\left(11\,(z-d)\,L_0^3-47\,(z-d)^2L_0^2+72\,(z-d)^3L_0-36\,(z-d)^4\right)\right]^2-r^2\right\} \tag{13}$$

Neglecting the higher order of δ, we get,

$$v=\frac{p}{4\mu}\left\{R_0^2-r_0^2-\frac{3\delta R_0}{L_0^4}\left[11\,(z-d)\,L_0^3-47\,(z-d)^2L_0^2+72\,(z-d)^3L_0-36\,(z-d)^4\right]+0(\delta)\right\} \tag{14}$$

which is the required velocity of the overlapping stenosis artery.

2.1.2. Volumteric Flow Rate

The volumetric flow rate (Q) through the cylindrical tube can be obtained by (Kapur 1985)

$$Q=\int_0^R 2\pi rvdr=\frac{n\pi}{3n+1}\left(\frac{p}{2\mu}\right)^{\frac{1}{n}}R^{\frac{1}{n}+3} \tag{15}$$

Put n = 1 for Newtonian fluid, we get,

$$Q=\frac{\pi pR_0^4}{8\mu} \tag{16}$$

$$Q=\frac{\pi pR_0^4}{8\mu}\left[1-\frac{3\delta}{2R_0L_0^4}\left\{11\left(z-d\right)L_0^3-47\left(z-d\right)^2L_0^2+72\left(z-d\right)^3L_0-36\left(z-d\right)^4\right\}\right]^4 \tag{17}$$

Using Binomial expansion and neglecting the higher power of δ, we get

$$Q=\frac{\pi pR_0^4}{8\mu}\left[1-\frac{6\delta}{R_0L_0^4}\left\{11\left(z-d\right)L_0^3-47\left(z-d\right)^2L_0^2+72\left(z-d\right)^3L_0-36\left(z-d\right)^4\right\}\right] \tag{18}$$

2.1.3. Pressure Drop

The pressure drop (Δp) in the stenosed part is obtaibed as,

$$\Delta p=\int_{-z}^{z}pdz=\frac{8\mu Q}{\pi}\int_{-z}^{z}R^{-4}dz \tag{19}$$

$$R^{-4}=R_0^{-4}\left[1-\frac{3\delta}{2R_0L_0^4}\left\{11\left(z-d\right)L_0^3-47\left(z-d\right)^2L_0^2+72\left(z-d\right)^3L_0-36\left(z-d\right)^4\right\}\right]^{-4} \tag{20}$$

Using Binomial expansion and neglecting the higher order of δ, we get

$$R^{-4}=R_0^{-4}\left[1+\frac{6\delta}{R_0L_0^4}\left\{11\left(z-d\right)L_0^3-47\left(z-d\right)^2L_0^2+72\left(z-d\right)^3L_0-36\left(z-d\right)^4\right\}\right] \tag{21}$$

$$\Delta p=\frac{8\mu Q}{\pi R_0^4}\int_{-z}^{z}dz+\frac{48\mu Q\delta}{\pi R_0^5L_0^4}[11L_0^3\int_{-z}^{z}(z-d)dz-47L_0^2\int_{-z}^{z}(z-d)^2dz+72L_0\int_{-z}^{z}(z-d)^3dz-36\int_{-z}^{z}(z-d)^4dz] \tag{22}$$

After calculation, we get

$$\Delta p=\frac{16\mu Qz}{\pi R_0^4}-\frac{48\mu Q\delta}{\pi R_0^5L_0^3}\left[22zdL_0^2+\frac{94}{3}L_0(4z^3-3z^4-3z^2d^2)-144(z^2+d^2)\right] \tag{23}$$

When there is no satenosis, δ=0, then the pressure drop without stenosis becomes,

$$(\Delta p)_0=\frac{4\mu z_0Q^n}{R_0^{3n+1}}\left(\frac{n\pi}{3n+1}\right)^n \tag{24}$$

Then the pressure drop ratio is

$$\frac{\Delta p}{(\Delta p)_0} = \frac{4zR_0^{3n-3}}{\pi z_0 Q^{n-1}}\left(\frac{n\pi}{3n+1}\right)^n - \frac{12\delta R_)^{3n-4}}{\pi z_0^3 Q^{n-1}}\left(\frac{n\pi}{3n+1}\right)^n \begin{bmatrix} 22zdL_0^2 + \frac{47}{2}(8z^3 - 6z^4 - 6z^2d^2) \\ -144zd(z^2+d^2) \end{bmatrix} \tag{25}$$

$$\frac{\Delta p}{(\Delta p)_0} = \frac{4zR_0^{3n-3}}{\pi z_0 Q^{n-1}}\left(\frac{n\pi}{3n+1}\right)^n - \frac{12\delta R_)^{3n-4}}{\pi z_0^3 Q^{n-1}}\left(\frac{n\pi}{3n+1}\right)^n \left[22zdL_0^2 + \frac{47}{2}(8z^3 - 6z^4 - 6z^2d^2) - 144zd(z^2+d^2)\right] \tag{26}$$

2.1.4. Shear Stress

The shear stress (τ) on the stenosis surface is obtained from (Kapur 1985)

$$\tau = \frac{PR}{2} = \mu Q^n \left(\frac{3n+1}{n\pi}\right)^n \frac{1}{R^{3n}} \tag{27}$$

$$\tau = \mu Q^n \frac{3n+1}{n\pi} \frac{1}{R_0^{3n}\left[1 - \frac{3\delta}{2R_0L_0^4}\left\{11(z-d)L_0^3 - 47(z-d)^2L_0^2 + 72(z-d)^3L_0 - 36(z-d)^4\right\}\right]^{3n}} \tag{28}$$

When δ=0, there is no stenosis, so that $f(\frac{\delta}{R_0})$=1, the shear stress in absence of stenosis is,

$$\tau_P = \mu Q^n \left(\frac{3n+1}{n\pi}\right)^n \frac{1}{R_0^{3n}} \tag{29}$$

2.1.5. Shear Stress Ratio

The ratio of shear stress $\left(\frac{\tau}{\tau_0}\right)$ with and without stenosis is (Kapur 1985)

$$\frac{\tau}{\tau_0} = \frac{R_0^{3n}}{R_0^{3n}} = \frac{1}{\left[1 - \frac{3\delta}{2R_0L_0^4}\left\{11(z-d)L_0^3 - 47(z-d)^2L_0^2 + 72(z-d)^3L_0 - 36(z-d)^4\right\}\right]^{3n}} \tag{30}$$

Using binomial expansion and neglecting the higher order of δ, we get,

$$\frac{\tau}{\tau_0} = \frac{R_0^{3n}}{R_0^{3n}} = \frac{1}{1 - \frac{9n\delta}{2R_0L_0^4}\left\{11(z-d)L_0^3 - 47(z-d)^2L_0^2 + 72(z-d)^3L_0 - 36(z-d)^4\right\}} \tag{31}$$

For Newtonian fluid, i.e., $n = 1$, so

$$\frac{\tau}{\tau_0} = \frac{R_0^{3n}}{R_0^{3n}} = \frac{1}{1 - \frac{9\delta}{2R_0L_0^4}\left\{11(z-d)L_0^3 - 47(z-d)^2L_0^2 + 72(z-d)^3L_0 - 36(z-d)^4\right\}} \tag{32}$$

3. Results and Discussion

The present model has been developed to analyze blood flow parameters in an overlapping artery with mild stenosis along axial direction. Analytical solutions of the velocity of the blood flow, pressure drop, shear stress, and effective viscosity of the blood following in an overlapping artery with mild stenosis are obtained and analyzed. The following simulated result for different blood flow parameters are obtained by using computational software.

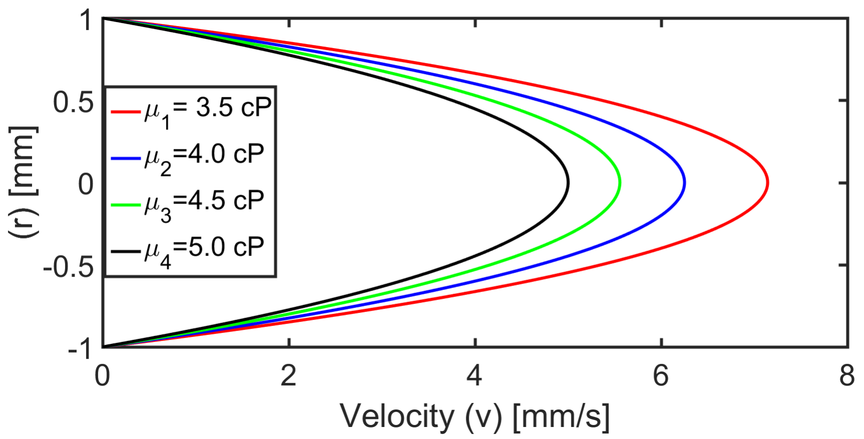

Figure 2: Velocity profile of blood flow through an overlapping stenotic artery for different viscosity μ.

3.1. Velocity profile of blood flow through an overlapping stenotic artery when viscosity changes

Figure (2) shows that the viscosity effects the velocity of blood flow for the artery having radius 1 mm. Velocity decreases rapidly when viscosity increases by a small quantity, which can be seen easily from the figure. For the viscosities 3.5 cP, 4.0 cP, 4.5 cP and 5.0 cP, the approximate values of velocities are 7.14 mm/s, 6.25 mm/s, 5.56 mm/s, 5.00 mm/s respectively. This simulation result shows that the velocity decreases as the viscocity increases and velocity increasese as the viscosity decreases. Near the inner wall of the artery the velocity is approximately zero, which increases gradually when we move towards center. When viscosity increases from 3.5 cP to 5.0 cP, velocity decreases by 30 percent.

3.2. Volumetric flow rate of an stenosis of blood flow through an overlapping artery

Figure (3)**A** describes the volumetric flow rate of blood in an artery across stenosis with various values of z (0,-0.4) mm. Flow rate is least at value μ_4=3.5 cP and maximum at value μ_1=2 cP with value z=0.4 mm. For the viscosities μ are 2 cP, 2.5 cP, 3 cP and 3.5 cP, the approximate volume of blood flow are 6000 mm^3/s, 4773 mm^3/s, 4000 mm^3/s, 3399 mm^3/s respectively. This simulation result shows that the volume decreases as the viscosity increases. At the end of the stenosis z=0.4 mm of the artery the volume is approximately zero. When viscosity increases from 2 cP to 3.5 cP, volume decreases by 43 percent. This figure shows that the volumetric flow rate decreases with the increase in viscosity.

Figure (3)**B** displays the relation between volumetric flow rate and length of the stenosis. When the length of stenosis is 1 mm the volumetric flow rate decreases rapidly and when the length of stenosis is 4 mm the volumetric flow rate decreases slowly. When the length of stenosis is 1 mm the volumetric flow rate decreases rapidly from 4876 mm^3/s to 180 mm^3/s and when the length of stenosis is 4 mm the volumetric flow rate decreases slowly from 622 mm^3/s to 180 mm^3/s. The volumetric flow rate of blood decreases with increasing viscosity and stenosis length. Higher viscosities result in significantly reduced flow rates, while longer stenoses cause a gradual decline. These trends highlight the sensitivity of blood flow to viscosity and arterial narrowing.

Figure (3)**C** shows the effect of viscosity on volumetric flow rate at different height of the stenosis. This simulation result shows that the viscosity and volumetric flow rate hold inverse relation. When viscosity

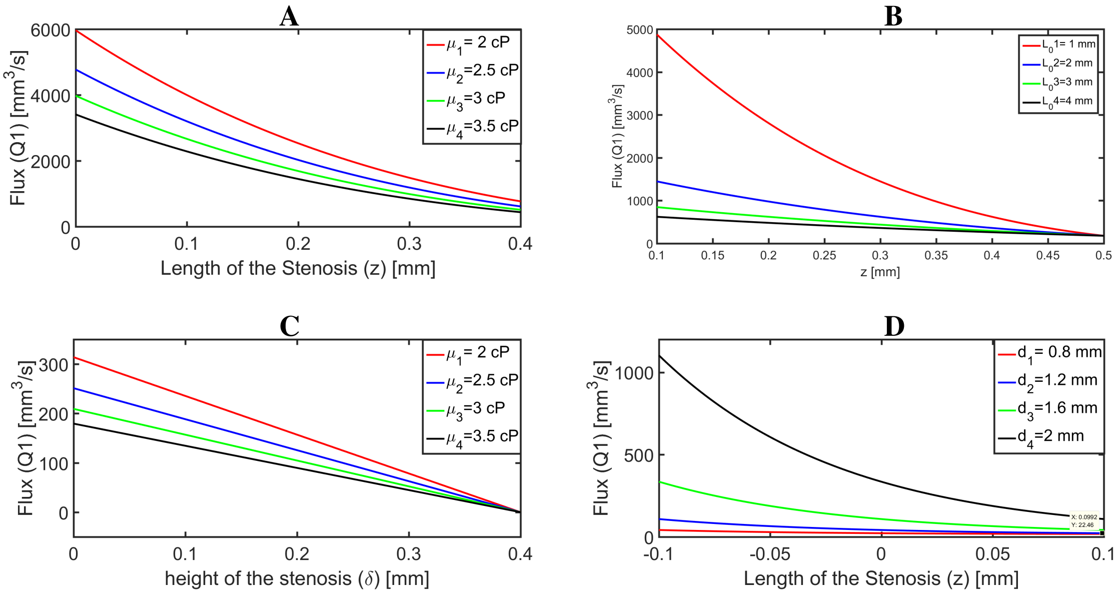

Figure 3: Volumetric flow rate of blood flow through an overlapping stenotic artery **A:** for different locations (d), **B:** for different length of stenosis (L), **C:** for different height of stenosis (δ), **C:** for different locations of stenosis (d).

values are μ_1=2 cP, μ_2=2.5 cP, μ_3=3 cP, μ_4=3.5 cP, approximate values of volumetric flow are 313 mm^3/s, 250 mm^3/s, 209 mm^3/s, 179 mm^3/s respectively. When viscosity increases from 2 cP to 3.5 cP, volumetric flow rate decreases by 43 percent. This figure shows that the volumetric flow rate decreases with the increase in viscosity.

Figure (3) **D** depicts volumetric flow rate in an artery for different locations of stenosis. This simulation result shows that the locations of stenosis and volumetric flow rate hold inverse relation. When d=0.8 mm the volumetric flow rate decreases from 1103 mm ^{3}/s to 109 mm^3/s approximately. When d=1.2 mm the volumetric flow rate decreases from 334 mm ^{3}/s to 42 mm^3/s approximately.At d=1.6 mm the volumetric flow rate decreases from 108 mm ^{3}/s to 22 mm^3/s approximately.When d=2 mm the volumetric flow rate decreases from 42 mm ^{3}/s to 21 mm^3/s approximately.When d increases from 0.8 mm to 2 mm, volumetric flow rate decreases rapidly. From this figure we conclude that the volumteric flow rate de creases more quickly when we move far from the origine. The volumetric flow rate of blood is inversely affected by both viscosity and the location of stenosis. Higher viscosity results in a noticeable reduction in flow rate, indicating that increased resistance impedes blood flow. Additionally, as the stenosis is located farther from the origin, the flow rate decreases more rapidly, showing the compounded impact of stenosis position on flow dynamics. These observations underscore the intricate relationship between flow rate, viscosity, and stenosis location.

3.3. Pressure drop ratio of blood flow through an overlapping stenotic artery

Figure(4)**A** shows the ratio of pressure drop against length of stenosis at different height of the stenosis. For stenosis height δ = 0.8 mm, 0.6 mm, 0.4 mm, 0.2 mm the ratio of pressure drop are 109 mm of Hg, 81 mm of Hg, 53 mm of Hg, 26 mm of Hg respectively. This figure shows that the ratio of pressure drop increases with the increase in height of the stenosis. Figure (4)**B** shows the ratio of pressure drop against

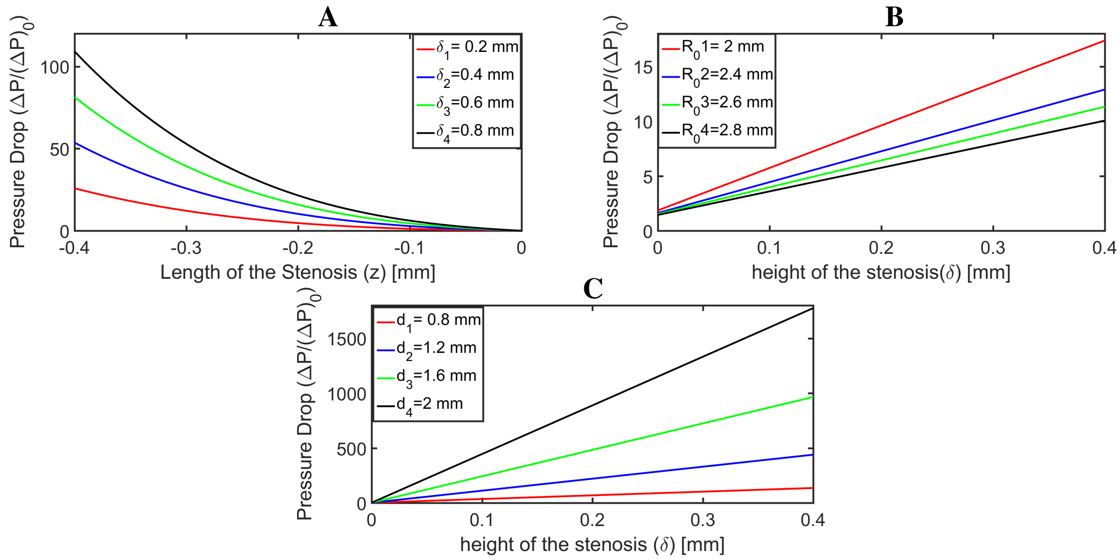

Figure 4: Pressure drop ratio of blood flow through overlapping stenotic artery**A:** for different height height of stenosis (δ), **B:** across the variation of the radius of the artery (R_0), **C:** at different locations of the stenosis(d).

height of the stenosis for different radii. For radii R_0 = 2 mm, 2.4 mm, 2.6 mm, 2.8 mm the pressure drop are 17 mm of Hg, 13 mm of Hg, 11 mm of Hg, 10 mm of Hg respectively. As the radii increases from 2 mm to 2.8 mm, the pressure drop decreases from 17 mm of Hg to 10 mm of Hg.

Figure(4) **C** Shows the ratio of pressure drop against height of the stenosis at different positions of stenosis. For d = 0.8 mm, 1.2 mm, 1.6 mm, 2.0 mm the pressure drop ratios are 135 mm of Hg, 438 mm of Hg, 965 mm of Hg, 1775 mm of Hg respectively. As the position increases from 0.8 mm to 2.0 mm, the pressure drop ratio also increases from 135 mm of Hg to 1775 mm of Hg, $1.856*10^6$ mm of Hg, $3.4163*10^6$ mm of Hg to $3.4163*10^6$ mm of Hg. From this figure we conclude that the pressure drop ratio increases more quickly when we move far from the origine. The pressure drop ratio increases with stenosis height and position due to increased obstruction and greater resistance to blood flow. Higher stenosis height narrows the arterial lumen, increasing velocity and pressure loss to maintain flow. Similarly, stenoses located farther from the origin experience cumulative effects of upstream resistance, amplifying the pressure drop. Conversely, larger artery radii reduce resistance, leading to a lower pressure drop ratio as wider lumens facilitate easier flow. These relationships align with principles of fluid dynamics, where flow resistance is governed by geometry and flow path characteristics.

3.4. Shear Stress ratio of blood flow through an overlapping stenotic artery

Figure (5)**A** shows the ratio of Shear Stress against z of the stenosis at different positions of stenosis. For d = 0.8 mm, 1.2 mm, 1.6 mm, 2.0 mm the Shear Stress ratio are 80 mm, 118 mm, 337 mm, 594 mm respectively. As the position increases from 0.8 mm to 2.0 mm, the Shear Stress increases from 80 mm to 594 mm. Shear Stress ratio increases by 87 percent when we move 60 percent far from the origine. From this figure we conclude that the Shear Stress ratio increases more quickly when we move far from the origine. Figure (5)**B** Shows the ratio of Shear Stress ratio against height of the stenosis for different radii. For radii R_0 = 2 mm, 2.4 mm, 2.6 mm, 2.8 mm the Shear Stress ratio are 0.4 mm, 0.4 mm, 0.4 mm, 0.4

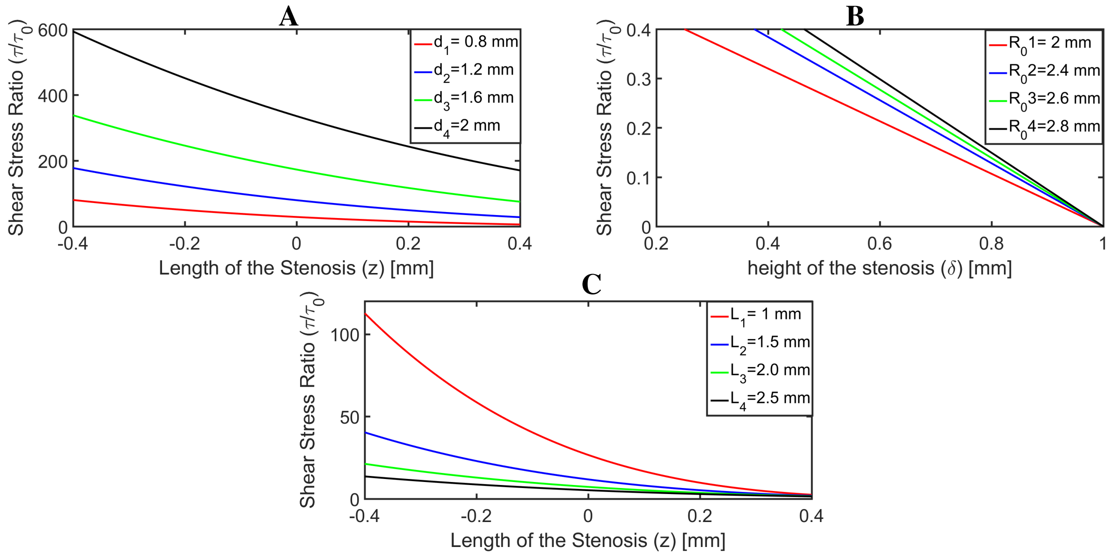

Figure 5: Shear Stress ratio of blood flow through an overlapping stenotic artery **A:** at different locations(d), **B:** with the variation of the stenotic height of the artery (δ), **C:** with the variation of the stenotic length of the artery (L_0).

mm respectively. As the radii increases from 2 mm to 2.8 mm, the Shear Stress ratio remains same.

Figure (5)**C** Shows the shear stress ratio against z of the stenosis for different length of the artery. For L_0 = 1 mm, 1.5 mm, 2.0 mm, 2.5 mm the Shear Stress ratio are 112 mm, 40 mm, 21 mm, 14 mm respectively. As the length of the artery increases from 1 mm to 2.5 mm, the Shear Stress ratio decreases from 112 mm of Hg to 14 mm of Hg. From this figure we conclude that the shear stress ratio decreases when length of the artery increases. The shear stress ratio increases with the position of the stenosis, as resistance intensifies farther from the origin due to cumulative effects. Conversely, it decreases with artery length, as longer arteries distribute the flow resistance over a greater distance, reducing localized shear stress. The shear stress ratio remains constant with varying artery radii, indicating that radius changes do not significantly impact shear stress under these conditions. These trends align with fluid dynamics principles, where shear stress depends on flow resistance and arterial geometry.

4. Conclusions

This study demonstrates that blood flow is significantly impacted by overlapping stenosis, with key parameters such as flow velocity, volume, pressure drop, and shear stress being influenced by factors like viscosity, stenosis length, and position. As viscosity increases, the flow velocity and volumetric flow rate decrease, highlighting the strong resistance effect of higher viscosity on blood flow. Similarly, as stenosis height and position move farther from the origin, both pressure drop and shear stress increase, reflecting the increased resistance and obstruction. Longer stenoses cause a more gradual decrease in flow, while larger artery radii reduce resistance, leading to a lower pressure drop. The shear stress ratio decreases with longer arteries, as the resistance is spread out over a greater length. These findings underscore the complex relationship between viscosity, stenosis geometry, and blood flow dynamics, with practical implications for understanding the impact of arterial blockages on cardiovascular health.

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