The Nepali Math. Sc. Report, Vol. 40, No.1 and 2, 2023, 43-54
DOI: https://doi.org/10.3126/nmsr.v40i1-2.61495

# GENERALIZED STATUS HARARY BASED INDICES

N. HARISH [1] AND B. CHALUVARAJU [1]

[1] *Department of Mathematics, Bangalore University,*
*Jnanabharathi Campus, Bengaluru - 560 056, INDIA*
*E-mail: harish.jaga16@gmail.com*
*E-mail: bcrbub@gmail.com*

**Abstract:** In this article, we initiate the study of generalized status Harary based indices such as $k$-sum status Harary index, $k$-product status Harary index and $k$-difference status Harary index of a non-trivial, undirected, simple connected graph, where $k$ is a positive integer. Here many bounds of the generalized status Harary based indices are obtained and its exact values for some specific families of graphs are found. Also, its relationship with other graphical indices are investigated. In addition that, we explore the comparative analysis of the molecular graphs of paraffin hydrocarbons.

**Key Words**: Molecular graph; Graphical indices; status index; Harary index; status Harary index.
**AMS (MOS) Subject Classification.** 05C07, 05C09, 05C12.

## 1. Introduction

Let $G = (V, E)$ be a simple, finite connected graph with vertex set $V(G) = V$ and $E(G) = E$ edge set. The cordinality of vertices and edges are denoted by $|V(G)| = n$ and $|E(G)| = m$. The number of vertices are adjacent to $u$ called its degree of the vertex and is denoted by $d_G(u)$. The minimum and maximum degree of the vertex are $\delta(G) = \delta$ and $\Delta(G) = \Delta$. The length of the shortest path between any two vertices $u$ and $v$ called its distance and is denoted by $d(u, v)$. The maximum distance between any pair of vertices in $G$ is called its diameter of a graph $G$ and is denoted by $diam(G) = D$. The minimum among all the distance between a vertex to all other vertices called its radius and is denoted by $rad(G)$. The sum of its distance from every other vertex of a graph $G$ is called its status [7] and is represented by

$$\sigma(u) = \sum_{u \in V(G)} d(u, v). \tag{1.1}$$

For more information on graph theoretic notion and terminology, we refer to [5, 8, 27].

Received: May 31, 2023 Accepted: 20 December, 2023 Published Online: 29 December, 2023 .

| **Graphical indices** | **Mathematical Representation** |
|---|---|
| First status connectivity index, [20] | $S_1(G) = \sum_{\{u,v\}\subseteq V(G)} (\sigma(u) + \sigma(v)).$ |
| Second status connectivity index, [20] | $S_2(G) = \sum_{\{u,v\}\subseteq V(G)} \sigma(u)\ \sigma(v).$ |
| Wiener index, [26] | $W(G) = \sum_{\{u,v\}\subseteq V(G)} d(u,v) = \frac{1}{2} \sum_{u\in V(G)} \sigma(u).$ |
| First Zagreb index, [6] | $M_1(G) = \sum_{uv\in E(G)} d_G(u) + d_G(v)$ |
| Second Zagreb index, [6] | $M_2(G) = \sum_{uv\in E(G)} d_G(u)\ d_G(v)$ |
| Generalized Harary index, [4] | $H_k(G) = \sum_{\{u,v\}\subseteq V(G)} \frac{1}{d(u,v)+k}.$ |
| Irregular index or Albertson index, [2] | $irr(G) = \sum_{uv\in E(G)} \lvert d_G(u) - d_G(v)\rvert.$ |

TABLE 1. Graphical indices and its representaion.

Now, we initiate the generalized status Harary based indices as follows:

Let $G$ be a non-trivial connected graph. Then

(i) The $k$-sum Status Harary index of a graph $G$ is defined as

$$SSH_k(G) = \sum_{\{u,v\}\subseteq V(G)} \frac{\left[\sigma(u) + \sigma(v)\right]}{d(u,v)+k}. \tag{1.2}$$

(ii) The $k$-product Status Harary index of a graph $G$ is defined as

$$PSH_k(G) = \sum_{\{u,v\}\subseteq V(G)} \frac{\sigma(u).\sigma(v)}{d(u,v)+k}. \tag{1.3}$$

(iii) The $k$-difference Status Harary index of a graph $G$ is defined as

$$DSH_k(G) = \sum_{u,v\subseteq V(G)} \frac{\lvert\sigma(u) - \sigma(v)\rvert}{d(u,v)+k}. \tag{1.4}$$

For more details on status of a vertex and their related graphical indices, we refer to [1], [9]-[14], [16]-[19],[21]-[24].

## 2. SOME SPECIFIC FAMILIES OF GRAPHS

The following easily computed values of generalized status Harary based indices of some specific families of graphs are stated without proof.

**Proposition 2.1.** *Let $G$ be a $r$-regular graph with $n \geq 3$. Then*

(i) $SSH_k(G) = \dfrac{n(n-1)[2(n-1)-r]}{1+k}$.

(ii) $PSH_k(G) = \dfrac{n(n-1)(2(n-1)-r)^2}{2(1+k)}$.

(iii) $DSH_k(G) = 0$.

*Proof.* Since for each vertex $u$ of a graph $G$ and $\sigma(u) = (2n - 2 - d_G(u))$, we have

$$SSH_k(G) = \sum_{\{u,v\}\subseteq V(G)} \frac{\left[\sigma(u) + \sigma(v)\right]}{d(u,v) + k}.$$
$$= \sum_{\{u,v\}\subseteq V(G)} \frac{(2n - 2 - d_G(u)) + (2n - 2 - d_G(v))}{1 + k}$$

By simplification, we have

$$SSH_k(G) = \frac{n(n-1)[2(n-1) - r]}{1 + k}.$$

Similarly, we have to (ii) and (iii). □

**Proposition 2.2.** *For any Complet graph $K_n$ with $n \geq 2$,*

(i) $SSH_k(K_n) = \dfrac{n(n-1)^2}{1+k}$.
(ii) $PSH_k(K_n) = \dfrac{n(n-1)^3}{2(1+k)}$.
(iii) $DSH_k(K_n) = 0$.

**Proposition 2.3.** *For any Cycle $C_n$ with $n \geq 3$,*

(i) $SSH_k(C_n) = \begin{cases} \dfrac{n^3}{2(1+k)}, & \textit{if} \quad n \quad \textit{is} \quad \textit{even} \\ \dfrac{n(n^2-1)}{2(1+k)}, & \textit{if} \quad n \quad \textit{is} \quad \textit{odd} \end{cases}$
(ii) $PSH_k(C_n) = \begin{cases} \dfrac{n^5}{16(1+k)}, & \textit{if} \quad n \quad \textit{is} \quad \textit{even} \\ \dfrac{n(n^2-1)^2}{16(1+k)}, & \textit{if} \quad n \quad \textit{is} \quad \textit{odd} \end{cases}$
(iii) $DSH_k(C_n) = 0$.

**Proposition 2.4.** *For any Complete regular bipartite graph $K_{s,s}$ with $s \geq 1$,*

(i) $SSH_k(K_{s,s}) = \dfrac{2s(6s^2 - 7s + 2)]}{1+k}$.
(ii) $PSH_k(K_{s,s}) = \dfrac{2s(18s^3 - 33s^2 + 20s - 4)}{1+k}$.
(iii) $DSH_k(K_{s,s}) = 0$.

## 3. BOUNDS ON STATUS HARARY BASED INDICES

**Theorem 3.1.** *Let $G$ be a non-trivial connected graph with diameter $D$. Then*

(i) $\dfrac{n(n-1)[2(n-1) - \delta]}{(n-1) + k} \leq SSH_k(G) \leq \dfrac{n(n-1)\Delta[D(2n-3) + 1]}{1+k}$
(ii) $\dfrac{n(n-1)(2(n-1) - \delta)^2}{2((n-1) + k)} \leq PSH_k(G) \leq \dfrac{n(n-1)\left[D(n-1) + \Delta(D-1)\right]^2}{2(1+k)}$.
(iii) $\dfrac{|d_G(u) - d_G(v)|}{(n-1) + k} \leq DSH_k(G) \leq \dfrac{(D-1)|d_G(u) - d_G(v)|}{1+k}$.

*Equality holds if and only if $D \leq 2$.*

*Proof.* Let $G$ be a non-trivial connected graph with $D \leq 2$. Then for any vertex $u \in V(G)$, there are $d_G(u)$ vertices which are at distance 1 from $u$ and the remaining $(n-1-d_G(u))$ vertices are at distance atmost $D$.
Therfore,

$$d_G(u) + 2(n-1-d_G(u)) \leq \sigma(u) \leq d_G(u) + D(n-1-d_G(u)).$$

This implies that

$$2(n-1) - d_G(u)) \leq \sigma(u) \leq D(n-1) - (D-1)d_G(u)). \tag{3.1}$$

Similarly, for any vertex $v \in V(G)$, we have

$$2(n-1) - d_G(v)) \leq \sigma(v) \leq D(n-1) - (D-1)d_G(v)). \tag{3.2}$$

Adding Equation (3.1) and (3.2), we have

$$4(n-1) - [d_G(u) + d_G(v)] \leq \sigma(u) + \sigma(v) \leq 2D(n-1) - (D-1)[d_G(u) + d_G(v)]. \tag{3.3}$$

Since, $1 \leq d(u,v) \leq D$ and

$$\frac{1}{D+k} \leq \frac{1}{d(u,v)+k} \leq \frac{1}{1+k}. \tag{3.4}$$

By equation (3.3) and (3.4), we have

$$\frac{4(n-1) - \left[d_G(u) + d_G(v)\right]}{D+k} \leq \frac{\sigma(u)+\sigma(v)}{d(u,v)+k}$$
$$\leq \frac{2D(n-1) - (D-1)\left[d_G(u) + d_G(v)\right]}{1+k}.$$

The above inequality, which satisfies for each $\{u, v\} \subseteq V(G)$ left and right side of the inequalities, we have

$$\sum_{\{u,v\} \subseteq V(G)} \frac{4(n-1) - \left[d_G(u) + d_G(v)\right]}{D+k} \leq SSH_k(G) \leq$$
$$\sum_{\{u,v\} \subseteq V(G)} \frac{2D(n-1) - (D-1)\left[d_G(u) + d_G(v)\right]}{1+k}. \tag{3.5}$$

Since $\delta \leq \{d_G(u), d_G(v)\} \leq \Delta$ implies $2\delta \leq \{d_G(u)+d_G(v)\} \leq 2\Delta$, and $1 \leq d(u,v) \leq (n-1)$ implies

$$\frac{1}{(n-1)+k} \leq \frac{1}{d(u,v)+k} \leq \frac{1}{1+k}.$$

Hence equation (3.5) becomes the desired result (i). □

By equation (3.5) and the definition of $M_1(G)$, we have

**Corollary 3.1.** Let $G$ be a non-trivial connected graph with diameter $D$. Then

(i) $\dfrac{2n(n-1)^2 - M_1(G)}{(n-1)+k} \leq SSH_k(G) \leq \dfrac{Dn(n-1)^2 - (D-1)M_1(G)}{1+k}$.

(ii) $\dfrac{2n(n-1)^3 - 2(n-1)M_1(G) + M_2(G)}{(n-1)+k} \leq PSH_k(G)$
$\dfrac{n(n-1)^3D^2 - 2D(D-1)(n-1)M_1(G) + 2(D-1)^2M_2(G)}{2(1+k)}$

(iii) $\frac{irr(G)}{(n-1)+k} \leq DSH_k \leq \frac{(D-1)irr(G)}{1+k}$.

Equality holds if and only if $D \leq 2$.

By equation (3.5) with $d_G(u) = d_G(v) = r$, if $G$ is a connected graph, then we have

**Corollary 3.2.** Let $G$ be a $r$-regular graph with diameter $D$. Then

(i) $\frac{n(n-1)[2(n-1)-r]}{(n-1)+k} \leq SSH_k(G) \leq \frac{nr(n-1)[D(2n-3)+1]}{1+k}$.

(ii) $\frac{n(n-1)(2(n-1)-r^2)}{2((n-1)+k)} \leq PSH_k(G) \leq \frac{n(n-1)\left[D(n-1)+r(D-1)\right]^2}{2(1+k)}$.

(iii) $\frac{irr(G)}{(n-1)+k} \leq DSH_k \leq \frac{(D-1)irr(G)}{1+k}$.

Equality holds if and only if $D \leq 2$.

By equation (3.5) with $rad(G) \leq d(u,v) \leq 2rad(G)$, we have

**Corollary 3.3.** Let $G$ be a non-trivial connected graph with diameter $D$. Then

(i) $\frac{n(n-1)[2(n-1)-\delta]}{2rad(G)+k} \leq SSH_k(G) \leq \frac{n(n-1)\Delta[D(2n-3)+1]}{rad(G)+k}$.

(ii) $\frac{n(n-1)(2(n-1)-\delta)^2}{2(2rad(G)+k)} \leq PSH_k(G) \leq$
$\frac{n(n-1)\left[D^2(n-1)^2 - 2\Delta D(D-1)(n-1) + \Delta^2(D-1)^2\right]}{2(rad(G)+k)}$.

(iii) $\frac{irr(G)}{2rad(G)+k} \leq DSH_k \leq \frac{(D-1)irr(G)}{rag(G)+k}$.

Equality holds if and only if $D \leq 2$.

## 4. BOUNDS IN TERMS OF OTHER GRAPHICAL INDICES

To prove next couple of bounds, we make use of the following lemma.

**Lemma 4.1.** [3] *Let $G$ be a non-trivial connected graph. Then*

(i) $1 \leq d(u,v) \leq (n-1)$.
(ii) $1 \leq d(u,v) \leq diam(G)$.
(iii) $rad(G) \leq d(u,v) \leq 2rad(G)$.

**Theorem 4.2.** *Let $G$ be a non-trivial connected graph. Then*

(i) $\frac{S_1(G)}{RH_k(G)} \leq SSH_k(G) \leq S_1(G).H_k(G)$.

(ii) $\frac{S_2(G)}{RH_k(G)} \leq PSH_k(G) \leq S_2(G).H_k(G)$.

(iii) $\frac{irr(G)(G)}{RH_k(G)} \leq DSH_k(G) \leq irr(G).H_k(G)$.

*Where $RH_k(G)$ denotes the reciprocal of generalized Harary index of a graph $G$.*

*Proof.* By Cauchy-Scharz inequality, we have

$$\sum_{\{u,v\}\subseteq V(G)} \frac{\sigma(u)+\sigma(v)}{d(u,v)+k} \geq \frac{\sum\limits_{\{u,v\}\subseteq V(G)} \sigma(u)+\sigma(v)}{\sum\limits_{\{u,v\}\subseteq V(G)} d(u,v)+k}$$
$$SSH_k(G) \geq \frac{S_1(G)}{RH_k(G)}.$$

Similarly, we have

$$\sum_{\{u,v\}\subseteq V(G)} \frac{\sigma(u)+\sigma(v)}{d(u,v)+k} \leq \sum_{\{u,v\}\subseteq V(G)} [\sigma(u)+\sigma(v)] \sum_{\{u,v\}\subseteq V(G)} \frac{1}{d(u,v)+k}.$$

Therefore,

$$\frac{S_1(G)}{RH_k(G)} \leq SSH_k(G) \leq S_1(G).H_k(G).$$

Similarly, we have (ii) and (iii). □

**Theorem 4.3.** *Let $G$ be a non-trivial connected graph with $D \leq 2$. Then*

(i) $SSH_k(G) \leq [2n(n-1)^2 - M_1(G)]H_k(G)$.
(ii) $PSH_k(G) \leq \big[2n(n-1)^3 - 2(n-1)M_1(G) + M_2(G)\big]H_k(G)$.
(iii) $DSH_k(G) \leq irr(G)\, H_k(G)$.

*Proof.* Let $G$ be a $(n,m)$ - connected graph with diameter $D \leq 2$. Then the $d_G(u)$ vertices at distance 1 from the vertex $u$ and remaining $(n-1-d_G(u))$ vertices at distance 2 from $u$ in $G$. Thus for each vertex $u$ in $G$, we have

$$\sigma(u) = d_G(u) + 2(n-1-d_G(u)) = 2n-2-d_G(u).$$

(i) Consider

$$\begin{aligned}
SSH_k(G) &= \sum_{\{u,v\}\subseteq V(G)} \frac{\big[\sigma(u)+\sigma(v)\big]}{d(u,v)+k}. \\
&\leq \sum_{\{u,v\}\subseteq V(G)} [\sigma(u)+\sigma(v)] \sum_{\{u,v\}\subseteq V(G)} \frac{1}{d(u,v)+k} \\
&\leq \sum_{\{u,v\}\subseteq V(G)} \big[2n-2-d_G(u)+(2n-2-d_G(v))\big] H_k(G) \\
&\leq \sum_{\{u,v\}\subseteq V(G)} \big[(4n-4)-[d_G(u)+d_G(v)]\big]\; H_k(G) \\
&\leq [2n(n-1)^2 - M_1(G)]H_k(G).
\end{aligned}$$

(ii) Consider

$$\begin{aligned}
PSH_k(G) &= \sum_{\{u,v\}\subseteq V(G)} \frac{\sigma(u)\ \sigma(v)}{d(u,v)+k}.\\
&\leq \sum_{\{u,v\}\subseteq V(G)} [\sigma(u).\sigma(v)] \sum_{\{u,v\}\subseteq V(G)} \frac{1}{d(u,v)+k}\\
&\leq \sum_{\{u,v\}\subseteq V(G)} (2n-2-d_G(u))(2n-2-d_G(v))H_k(G)\\
&\leq \sum_{\{u,v\}\subseteq V(G)} \Big[4n^2-8n+4-2n\big[d_G(u)+d_G(v)\big]+2\big[d_G(u)+d_G(v)\big]\\
&\quad + (d_G(u)+d_G(v))\Big]\ H_k(G)\\
&\leq \big[2n(n-1)^3-2(n-1)M_1(G)+M_2(G)\big]H_k(G).
\end{aligned}$$

(iii) Consider

$$\begin{aligned}
DSH_k(G) &= \sum_{\{u,v\}\subseteq V(G)} \frac{|\sigma(u)-\sigma(v)|}{d(u,v)+k}.\\
&\leq \sum_{\{u,v\}\subseteq V(G)} |\sigma(u)-\sigma(v)| \sum_{u,v\subseteq V(G)} \frac{1}{d(u,v)+k}\\
&\leq \sum_{\{u,v\}\subseteq V(G)} |2n-2-d_G(u)-[2n-2-d_G(v)]|H_k(G)\\
&\leq \sum_{\{u,v\}\subseteq V(G)} |d_G(u)-d_G(v)|\ H_k(G)\\
DSH_k(G) &\leq irr(G)\ H_k(G).
\end{aligned}$$

Thus the result follows. □

To prove our next result, we make use of the following Cauch-Schwarz inequality.

**Lemma 4.4.** *Let $a_1, a_2, ..., a_n$ and $b_1, b_2, ..., b_n > 0$ be two sequence of real numbers. Then*

$$\sum_{i=1}^{n} \frac{a_i^2}{b_i} \geq \frac{\left(\sum_{i=1}^{n} a_i\right)^2}{\sum_{i=1}^{n} b_i}.$$

**Theorem 4.5.** *Let $G$ be a non-trivial connected graph. Then*

(i) $SSH_k(G) \geq \dfrac{S_1^2(G)}{RH_k(G)}$.

(ii) $PSH_k(G) \geq \dfrac{S_2^2(G)}{RH_k(G)}$.

(iii) $DSH_k(G) \geq \dfrac{irr^2(G)}{RH_k(G)}$.

*Proof.* (i) Let $G$ be a non-trivial connected graph. By Lemma 4.4, we have $a_i = \sigma(u_i)+\sigma(v_i)$ and $b_i = \dfrac{1}{d(u_i,v_i)+k}$ for all $1 \leq i \leq n$. Then

$$\sum_{i=1}^{n} \frac{\big(\sigma(u_i)+\sigma(v_i)\big)^2}{d(u_i,v_i)+k} \geq \frac{\left(\sum_{i=1}^{n} \sigma(u_i)+\sigma(v_i)\right)^2}{\sum_{i=1}^{n} \dfrac{1}{d(u_i,v_i)+k}}$$

Therefore, $SSH_k(G) \geq \dfrac{S_1^2(G)}{RH_k(G)}$.
Similarly, we have the results (ii) and (iii). $\square$

## 5. Comparative analysis of Molecular graphs

For chemical applicability of generating elementary reactions of complex systems of Paraffinic hydrocarbons. This group of hydrocarbons consisting of linear molecules with the formula $C_kH_{2k+2}$. The following molecular graph of paraffin hydrocarbons as shown in Figure 1, which is used for producing petrochemicals range from the simplest hydrocarbon methane, to heavier hydrocarbon gases and liquid mixtures present in crude oil fractions and residues. For more details on molecular graphs and its related concepts, we refer to [15, 25, 26].

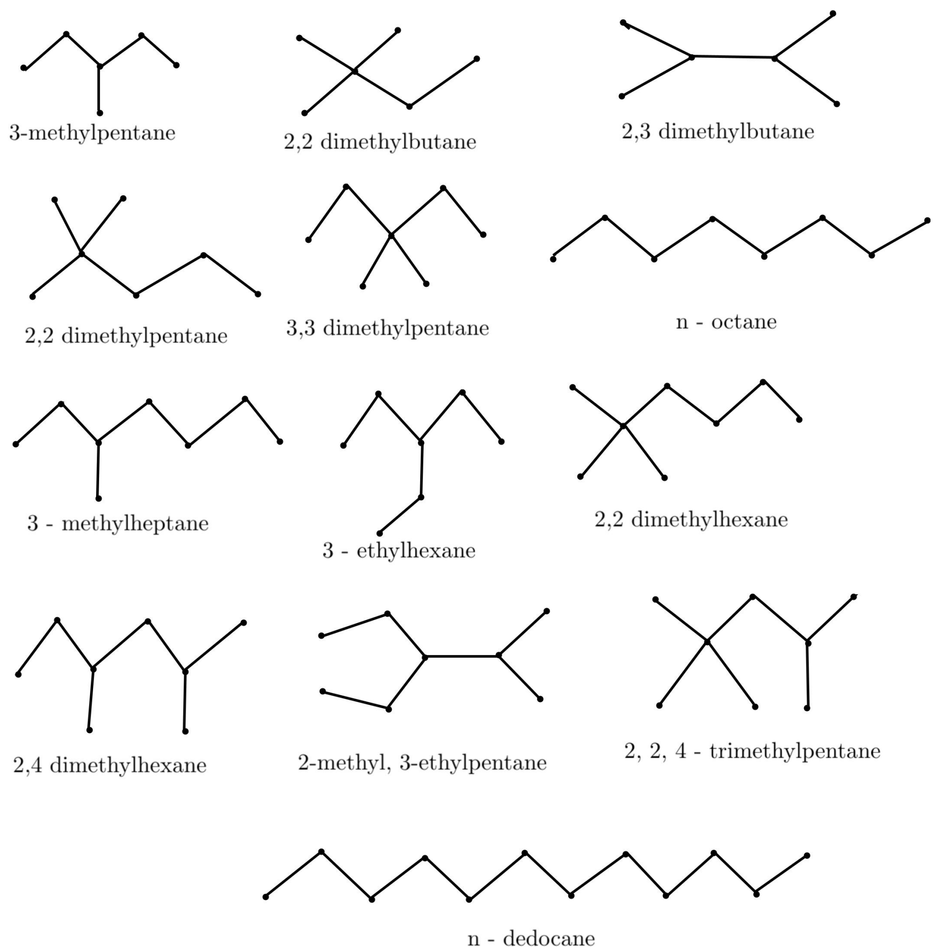


Figure 1. Paraffin Hydrocorbons

| **Molecular graphs** | $SSH_k$ | | | $PSH_k$ | | | $DSH_k$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | k=0 | k=1 | k=2 | k=0 | k=1 | k=2 | k=0 | k=1 | k=2 |
| 3-methylpentane | 360.3 | 215.7 | 157.1 | 1748.8 | 1058.9 | 775.1 | 52 | 32.6 | 23.4 |
| 2,2-dimethylbutane | 340 | 203 | 147 | 1500 | 912 | 666 | 52 | 29 | 20.3 |
| 2,3-dimethylbutane | 344 | 205.333 | 148.999 | 1557.331 | 943.666 | 689 | 50.666 | 28.666 | 20.266 |
| 2,2-dimethylpentane | 610 | 368.733 | 270.866 | 3819 | 2358.4 | 1748.366 | 86 | 50.8 | 36.8 |
| 3,3-dimethylpentane | 589.333 | 359.8 | 264.799 | 3507.333 | 2175.733 | 1608.599 | 90.666 | 53 | 38.133 |
| n-octane | 1117.066 | 692.437 | 516.967 | 11358.132 | 7063.866 | 5288.373 | 128 | 82.894 | 62.98 |
| 3-methylheptane | 1043.932 | 648.484 | 483.184 | 9523.866 | 5960.818 | 4461.96 | 140.199 | 89.037 | 66.839 |
| 3-ethylhexane | 617 | 377.8 | 278.6 | 4012.5 | 2488.8 | 1848 | 92 | 55 | 40 |
| 2,2-diethylhexane | 933.466 | 564.666 | 413.514 | 8596.333 | 5377.366 | 4011.105 | 129.2 | 79.8 | 59.147 |
| 2,4-dimethylhexane | 912.933 | 587.666 | 443.370 | 7937 | 5181 | 3929.799 | 127.1 | 82.799 | 62.542 |
| 2-methyl,3-ethylpentane | 957 | 596.133 | 443 | 7612.5 | 4807.133 | 3598 | 145.333 | 87.533 | 63.932 |
| 2,2,4-trimethylpentane | 964.666 | 597.866 | 442.666 | 7612 | 4780.666 | 356.733 | 124.666 | 74.8 | 54.533 |
| n-dodecane | 5565.478 | 3033.679 | 2331.968 | 108725.782 | 70651.18 | 54357.934 | 500 | 354.115 | 283.295 |

TABLE 2. Graphical indices of paraffin hydrocarbons

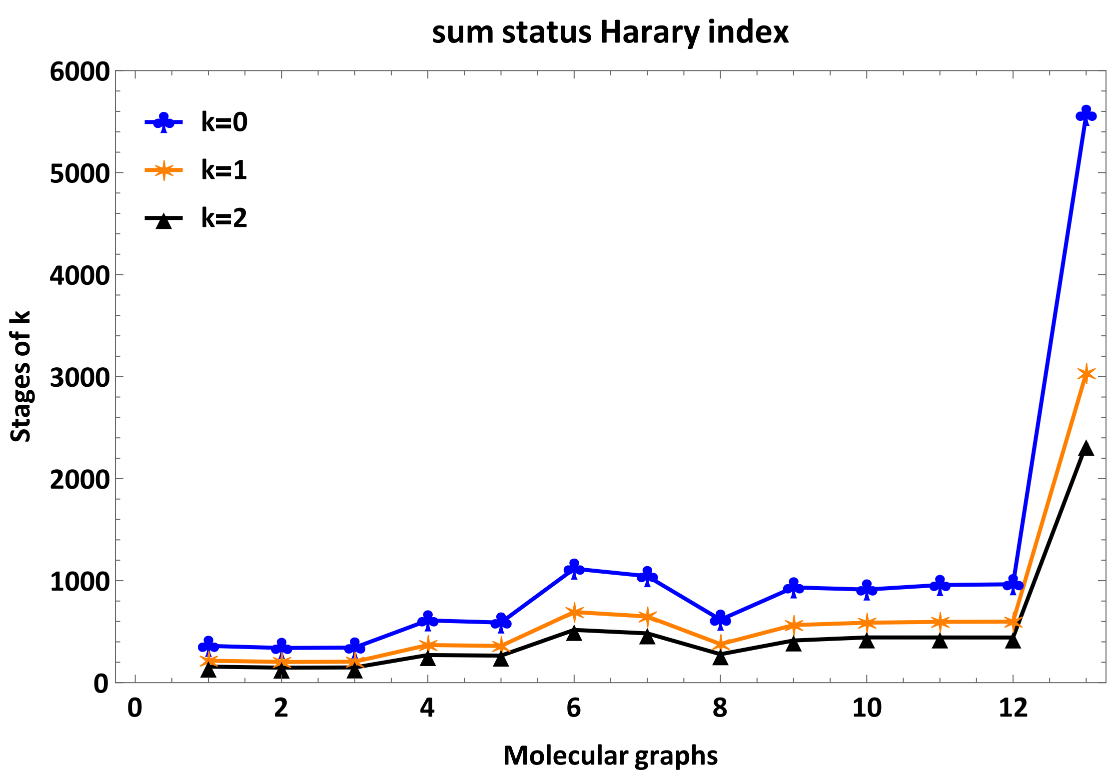


FIGURE 2. Sum status Harary index of Molecular graphs

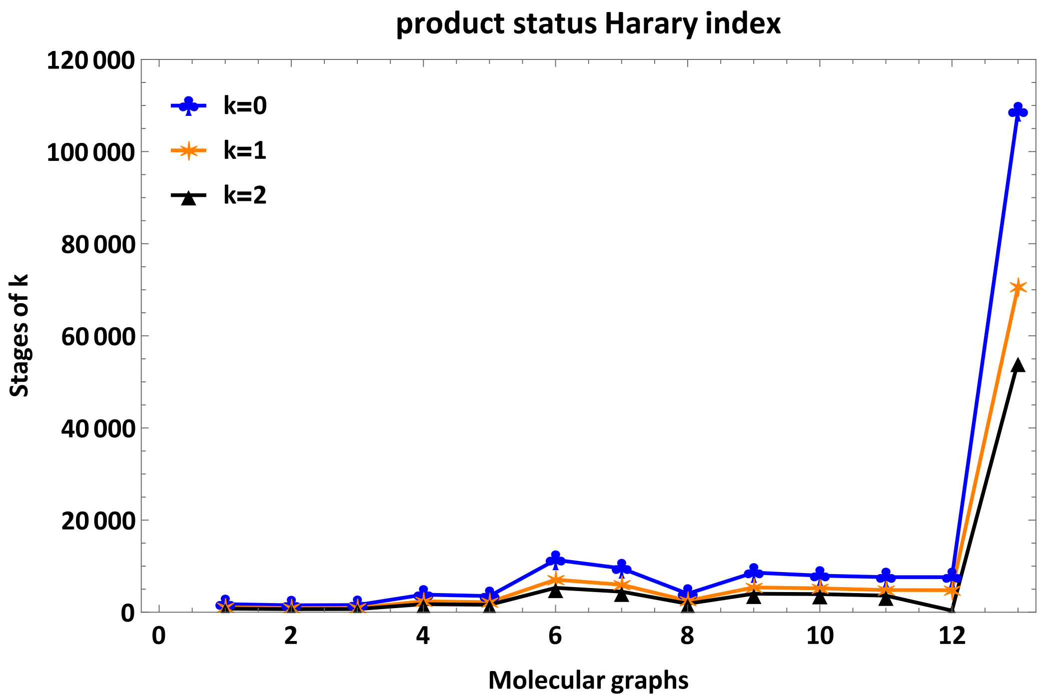


FIGURE 3. Product status Harary index of Molecular graphs

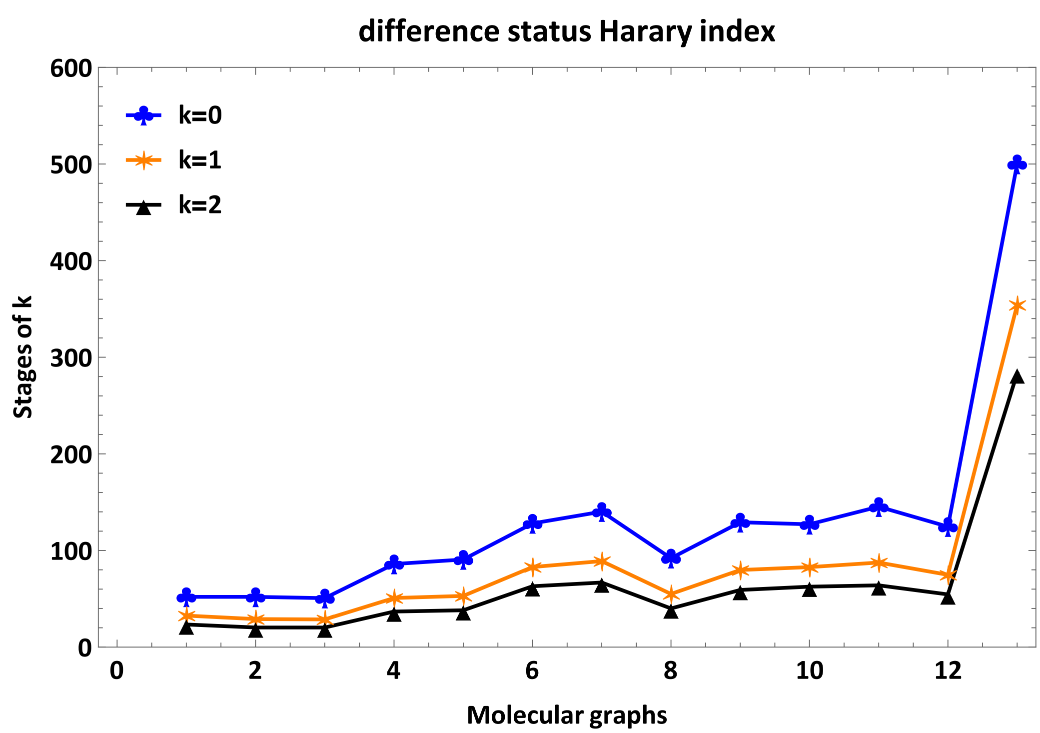


FIGURE 4. Difference status Harary index of Molecular graphs

The graphical representation shows the comparative analysis of generalized status Harary indices such as sum status Harary index, product status Harary index and difference status Harary index as shown in Figures 2, 3, and 4. From this graphical representation, we concluded that the product status Harary index gives the highest values compared to the sum and difference status Harary index. When $0 \leq k \leq 2$ in this condition, the molecular graph of $n$-dodecane values varies from 108.7K to 54.3K. Otherwise, the remaining molecular graphs are changing their values vary in consistency as shown in Figure 3 and 4. We can represents mathematicaly as $DSH_k(G) < SSH_k(G) < PSH_k(G)$.

## 6. Conclusion and Open Problems

In this paper, we calculated the exact values for some specific families of graphs and many bounds of the generalized status Harary based indices are obtained. And also, we shows the relationship between the Harary based indices and molecular graph of paraffin hydrocorbons. For the comparative advantages, applications, and mathematical point of view, many questions are suggested by this research, among them are the following.

1. Find the extremal values and extremal graphs of the generalized status Harary based indices.
2. Characterize among the graphical indices of $DSH_k(G)$, $SSH_k(G)$ and $PSH_k(G)$ for appropriate value of $k$.
3. Find QSPR/QSAR/QSTR related study on the generalized status Harary based indices.

## 7. DECLARATIONS

**Conflict of Interest:** The authors declare that there is no conflict of interest regarding the publication of this article.

**Acknowledgements:** Authors would like to thank the reviewers for their valuable comments and suggestions to improve the quality of the paper.

**Author Contributions:** All authors have contributed equally to this manuscript.

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