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Research Article

Numerical Approximations Techniques for Transient Analysis in LCR Circuits

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Abstract

There are abundant real world problems in this nature which are governed by differential equations. They are usually non-linear in nature and due to its high complexity, obtaining the analytical solution is very hard or almost impossible. So, we widely use the numerical techniques to approximate the solutions. Likewise, Transient Analysis of an LCR electrical circuit is one of them. LCR circuit encompasses three different elements *i.e.,* Inductor (L), Capacitor (C) and Resistor (R). In this paper, we mainly discuss three major numerical techniques viz. Euler (Forward), fourth-order Runge-Kutta (RK4) and sixth-order Runge-Kutta (RK6) methods to approximate the numerical solutions of second-order differential equation of LCR circuit. Moreover, we compare the numerical solutions with exact solutions with necessary visualization. Concerning the various damping factor values, we discuss the damping conditions and consider the further possibility of discussion and analysis of this numerical methods.

Keywords: LCR Circuit, Transient Analysis, Numerical Methods, Differential Equation, Damping Factor.
AMS(MOS) Subject Classification: Subject classification here.

1 Introduction

Real world encompasses different real-physical phenomena. Many problems in different areas like engineering, physics, medical science, space science etc. needs to be solved mathematically and there comes the actual use of mathematics [1, 6]. Many problems of mathematical physics like planetary motion, motion of simple pendulum clock, circuit analysis,

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motion under gravity, population dynamics, Newton's law of cooling, heat equation, wave equation etc. are governed in the form of differential equation [7]. The development of diverse modeling in science and engineering heavily relies on the solution of differential equations [6]. While encountering differential equations of these types in modeling, nearly all seems to be non-linear. Ordinary differential equations can be solved analytically in a variety of ways. However, in most real life situations, the differential equation that models the problem is too complicated or almost impossible to solve the analytically [1, 7]. Consequently, in order to obtain some approximation to such challenging problems, a pragmatic and rigorous method is required. One such way is the numerical techniques [5, 6, 8].

According to Otoo and Islam, transient energy bursts in an electrical circuit have the potential to cause harm to specific circuit components [5, 7]. Typically, transient causes an electrical circuit's component states to change. It usually occurs during switching period. It is frequently used for circuit analysis, energy efficiency, power quality, control system design etc. There are many millions of components like inductors, resistors, capacitors etc. embedded inside the integrated circuit and are most dangerous to this small increment of current [16]. In an electrical circuit, it is exceedingly difficult for the inductor voltage and the capacitor voltage to take on a new steady state value. Consequently, the evolution of the capacitor and inductor voltage with time can be ascertained by transient analysis [11]. The analysis of a system in an unstable condition is known as transient analysis. Steady (equilibrium) state of system is referred when the variables that define its state do not vary over time. It is in an unstable state if not. Because it may be used to analyze the performance of any electrical circuit, transient analysis is extremely important [13]. Thus, there can be several forms of voltage and current for an electrical current or voltage passing through an electrical circuit. For example, when looking at circuits that combine resistive circuits and time-varying signals, the resulting Kirchhoff's Voltage law (KVL) & Kirchhoff's Current law (KCL) resemble differential equations more than algebraic equations [5, 15]. But due to its high order and complexity, the analytical solution is not easy to solve. Also, for an LCR circuit which is an electrical circuit consisting of inductor, capacitor and resistor which are connected in either series or parallel combinations, the circuit equations are second-order differential equations [5, 15]. By differentiating with regard to time, we may transform these equations into ordinary differential equations. As a result, one can approach the transient analysis of an LCR circuit numerically [11, 15]. The Runge-Kutta method's effectiveness in resolving second-order differential equations was emphasized by the author.

Suhag (2013) [15], carried out a transient response of a second order LCR circuit and recorded the system's reaction when the conditions were changed from one steady state value to next state. His conclusion was that the Runge-Kutta method seems to be very efficient method in solving the differential equations which are of second order [15]. Kee and Ranom (2018) [11], used the RK4 approach with different time step sizes h to study the transient response of a LCR circuit of series connection for under damped, critically damped, & over damped situations. Henry et al. (2018) [5], used two iteratives approximations *i.e.* Heun's and Runge-Kutta methods to solve the transient analysis of LCR circuit. They proved that the RK4 method converged exact solution faster than Heuns method. Ogbuka (2008) [12], simulated a sample of complex electrical circuit in mathematical tools with strong mathematical background and basic laws of circuit analysis to study transient response. Despande (2014) [11], found that the DC voltage applied in any LCR circuit gives raise to transient response and estimation of voltage was of crucial for control purpose. In their study, Hossain et al. (2017) [6], examined a numerical example of solving a second-order IVP for an ODE utilizing the fourth and fifth-order RK methods. The IVP for the fourth order Runge-Kutta method was solved by Ahamad and Charan (2019) using the Runge-Kutta method of fifth order [1]. The modified Euler and Runge-Kutta approach was employed by Kamruzzaman and Nath (2018) to compare the analytical and numerical solution of an ODE using IVP [10]. Kafle et al. (2020) [8], experimented the BRK5 method to analyze the different damping conditions of LCR circuit for both series and parallel. For universal second-order differential equations, Fehlberg (1974) produced classical formulations for the seventh, sixth, and fifth order Runge-Kutta-Nystróm with step-size control [4].

In this paper, series and parallel LCR circuit are taken with DC source and this can be formulated by differential equation of second order. Due to complexity, at first we have converted the second order ODE with IVP into system of first order differential equation and solved numerically. All three numerical methods *i.e.,* Euler (Forward), RK4 and RK6 methods are compared for solving the IVP of ODE. We note that the analytical solution of the Euler technique converges to an abnormally tiny step size. Therefore, a lot of computation is required. The Runge-Kutta technique, on the other hand, produces better results, converges to the analytical solution more quickly, and requires less iterations to provide an accurate solution. It is found that RK6 method is the best among these methods as it converges very faster toward exact solution with comparatively less error than compared to other two methods. Due to its high accuracy, RK6 approximation is selected to study the different damping characteristics of LCR circuit for both series and parallel circuit under different conditions of resistors.

2 Mathematical Theory

A LCR circuit constitutes three elements *i.e.,* Inductor (L), Capacitor (C) and Resistance (R). The LCR circuit equation is governed by second order differential equation [12]. The LCR circuit has several uses, but two of the most significant ones are as oscillators and radio or audio receiver transformers [8]. A transient response complete solution is created by applying the circuit equations to analyze the circuit and develop analytical solutions for transient analysis [11]. However, analytical solution of such LCR ciruits in complex network is almost impossible to find or very difficult to solve. Therefore, numerical methods offer the best solution technique to the system [5]. Here, we solve the circuit equation in an LCR circuit using all feasible iterative strategies while comparing the various approaches. Figure 1 is for series combination and Fig. 2 is for parallel combination of LCR circuit. We use the Kirchhoff's Voltage Law for $t > 0$ in Fig. 1.

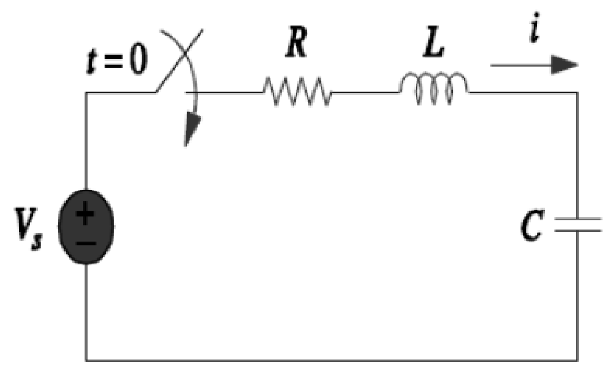

Figure 1: Series LCR Circuit [18].

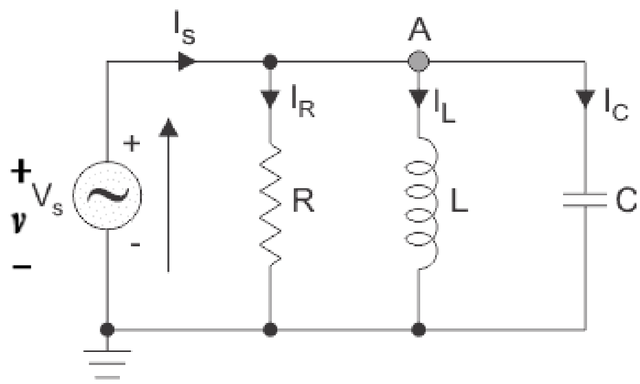

Figure 2: Parallel LCR Circuit [19].

Here, the total voltage dropped along the series LCR circuit is given as [15]:

$$V_S = V_R + V_L + V_C \tag{2.1}$$

where, V_S is the DC source voltage and V_R, V_L and V_S are voltage across resistor, inductor and capacitor respectively. The LCR circuit's second order differential equation with constant coefficients is given as [14]:

$$L\frac{d^2V}{dt^2} + R\frac{dV}{dt} + \frac{1}{C}V = \frac{V_s}{C} \tag{2.2}$$

Once we close the switch, we analyze the transient characteristic in this series LCR circuit. Various iterative techniques can be employed to solve equation (2.2). The amount by which a system's oscillation gradually reduces with time (t) is known as the damping factor (μ). The damping factor's (μ) value determines the transient response. The damping factor in an LCR circuit is provided by [5].

$$\mu = \frac{\alpha}{\omega_0} \tag{2.3}$$

where, $\alpha = \frac{R}{2L}$ (damping coefficient) & $\omega_o = \frac{1}{\sqrt{LC}}$ (natural oscillation). Then, the equation (2.3) becomes

$$\mu = \frac{R}{2}\sqrt{\frac{C}{L}} \tag{2.4}$$

Different values of μ describes the nature of oscillation of the system. The given system is (i) critically damped when $\mu = 1$, (ii) under damped when $\mu < 1$, & (iii) over damped in all other cases [15].

Again, in Fig. 2 for the loop for $t > 0$, we apply the Kirchhoff's Current Law [11]. The total current flowing through this circuit is given as [15]:

$$I_S = I_L + I_C + I_R \tag{2.5}$$

where, source current is denoted by I_S, and the currents via the resistor, inductor, and capacitor are denoted by I_R, I_L, and I_C, respectively. Based on the loop current approach, differential equation of the LCR circuit in Fig. 2 is as follows:

$$\frac{1}{R}V + \frac{1}{L}\int V\,dt + C\frac{dV}{dt} = I_S \tag{2.6}$$

Differentiating equation (2.6) on both sides, we obtain differential equation of second order for LCR circuit with constant coefficients and is given as [14].

$$\frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dV}{dt} + \frac{1}{LC}V = 0 \tag{2.7}$$

with $V(0) = 6$, $i(0) = 0$, and $\frac{dV(0)}{dt} = -12$.

After the switch is closed, the transient characteristics of the parallel LCR circuit are examined. Iterative techniques can be employed to solve equation (2.7). The amount by which a system's oscillation gradually reduces with time (t) is known as the damping factor (ξ). The damping factor (ξ) [5] determines the transient response. The damping factor in an LCR circuit is provided by:

$$\xi = \frac{\alpha}{\omega_0} \tag{2.8}$$

where, $\alpha = \frac{1}{2RC}$ (damping coefficient) & $\omega_o = \frac{1}{\sqrt{LC}}$ (natural oscillation). Then, the equation (2.8) becomes

$$\xi = \frac{1}{2R}\sqrt{\frac{L}{C}} \tag{2.9}$$

According to Suhag, the system is referred to as (i) critically damped when $\xi = 1$, (ii) under damped when $\xi < 1$, & (iii) excessively damped when $\xi > 1$ [15].

3 Numerical Methodologies

This section discusses various numerical techniques for solving ordinary differential equations (ODEs) of an initial value problem (IVP). First, we provide the explicit Euler technique. Next, we present the fourth order Runge-Kutta method (RK4) and the sixth order Runge-Kutta method (RK6) respectively [3, 11].

3.1 Euler's (Forward) Method

The most basic one-step method is Euler's method and in 1768, he approached his method for initial value problems (IVP). The solutions to explicit first-order equations can be approximated using this numerical method [2]. It is predicated on progressively approximating the solution through linear processes. Examine the following starting value issue for a smooth function:

$$y'(t) = f(t, y(t)), \ \ y(t_0) = y_0 \tag{3.1}$$

With a step size of h, let y_n represent the position at time t_n. According to the meaning of derivative, we get

$$\frac{dy}{dt} = \lim_{h \to 0} \frac{y(t+h) - y(t)}{h}$$
$$\frac{dy}{dt} \approx \frac{y(t+h) - y(t)}{h}$$

Let (t_n, y_n) be the slope, then

$$\frac{y_{n+1} - y_n}{h} \approx \frac{dy}{dt} = f(t_n, y_n).$$

which implies

$$y_{n+1} = y_n + hf(t_n, y_n) \tag{3.2}$$

Equation (3.2) is called Euler's (Forward) or Explicit method [10].

3.2 Runge-Kutta Methods

The Runge-Kutta family of explicit and implicit iterative techniques includes the widely recognized Euler method, which is applied in temporal discretization to approximate ODE solutions. Carl Runge and Martin Kutta, two German mathematicians, created these techniques in 1900. The Runge-Kutta method is the most often used because it is simple to program and has high accuracy and is more stable [7]. This method can be identified by its order, which agrees with Taylor's series solution up to terms of h^r, where r is the method's order. Here, we will discuss two extensively used Runge-Kutta methods of order fourth and sixth *i.e.,* RK4 & RK6 respectively.

3.2.1 Fourth Order Runge-Kutta Method

The fourth order Runge-Kutta method [3] is given as follows:

$$y_{n+1} = y_n + h[\frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6}] \tag{3.3}$$

where,

$k_1 = f(t_n, y_n)$, $k_2 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1)$, $k_3 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2)$, $k_4 = f(t_n + h, y_n + hk_3)$

3.2.2 Sixth Order Runge-Kutta Method

The sixth order Runge-Kutta method formula [3] is given as:

$$y_{n+1} = y_n + \frac{h}{90}[7k_1 + 32k_3 + 12k_4 + 32k_5 + 7k_6] \tag{3.4}$$

where,

$$k_1 = hf(t_n, y_n),\ k_2 = hf(t_n + \tfrac{h}{3}, y_n + \tfrac{k_1}{3}),\ k_3 = hf(t_n + \tfrac{h}{3}, y_n + \tfrac{k_1+k_2}{6}),$$
$$k_4 = hf(t_n + \tfrac{h}{2}, y_n + \tfrac{k_1}{8} + \tfrac{3k_3}{8}),\ k_5 = hf(t_n + \tfrac{2h}{3}, y_n + \tfrac{k_1}{2} - \tfrac{3k_3}{2} + 2k_4),$$
$$k_6 = hf(t_n + \tfrac{5h}{6}, y_n - \tfrac{k_1}{2} + \tfrac{5k_3}{2} + \tfrac{k_5}{2}),\ k_7 = hf(t_n + h, y_n + \tfrac{k_1}{7} + \tfrac{2k_3}{7} + \tfrac{12k_4}{7} - \tfrac{12k_5}{7} + k_6),$$

For the solution of higher order problems for ordinary differential equations, we can expand on the previously stated iterative [6].

4 Numerical Formulation of LCR Circuit

Here, we will convert the second order differential equation of LCR circuit into the system of differential equation of first order for both series and parallel circuit and formulate numerically.

4.1 For Series circuit

$$\text{Let us suppose, } V = x \text{ and } \frac{dV}{dt} = \frac{y}{C} \tag{4.1}$$

then, we can write equation (2.2) as

$$\frac{dy}{dt} = \frac{V_s - x - R * y}{L} \tag{4.2}$$

Thus, equations (4.1) and (4.2) give the system of differential equation which is of first order [5]. And, we let

$$f = \frac{dV}{dt} = \frac{y}{C} \ \ and \ \ g = \frac{dy}{dt} = \frac{V_s - x - R * y}{L}.$$

4.2 For Parallel Circuit

$$\text{We let, } V = x \text{ and } \frac{dV}{dt} = \frac{y}{C} \tag{4.3}$$

then, we can write equation (2.7) as

$$\frac{dy}{dt} = \frac{V_s}{LC} - \frac{y}{RC} - \frac{x}{L} \tag{4.4}$$

Hence, equations (4.3) and (4.4) give the system of differential equation which is of first order [5].

Let,

$$f = \frac{dV}{dt} = \frac{y}{C} \ \ and \ \ g = \frac{dV}{dt} = \frac{Vs}{LC} - \frac{y}{RC} - \frac{x}{L}.$$

4.3 Numerical Methods

Here, we apply three major numerical techniques *i.e.,* Euler's, RK4 and RK6 methods to compute the approximate solution for both LCR circuits.

4.3.1 Euler's (Forward) Method for LCR Circuit

$$x_{i+1} = x_i + hf(t_i, x_i, y_i), \;\; y_{i+1} = y_i + hg(t_i, x_i, y_i)$$

where $i = 0, 1, 2, 3, ..., n$ and h is step-size.

4.3.2 RK4 method for LCR Circuit

$$x_{i+1} = x_i + h[\frac{f_1}{6} + \frac{f_2}{3} + \frac{f_3}{3} + \frac{f_4}{6}], \;\; y_{i+1} = y_i + h[\frac{g_1}{6} + \frac{g_2}{3} + \frac{g_3}{3} + \frac{g_4}{6}]$$

where,

$$f_1 = f(t_i, x_i, y_i), \;\; g_1 = g(t_i, x_i, y_i),$$

$$f_2 = f(t_i + \frac{h}{2}, (x_i + \frac{h}{2}f_1), (y_i + \frac{h}{2}g_1)), \;\; g_2 = g(t_i + \frac{h}{2}, (x_i + \frac{h}{2}f_1), (y_i + \frac{h}{2}g_1)),$$

$$f_3 = f(t_i + \frac{h}{2}, (x_i + \frac{h}{2}f_2), (y_i + \frac{h}{2}g_2)), \;\; g_3 = g(t_i + \frac{h}{2}, (x_i + \frac{h}{2}f_2), (y_i + \frac{h}{2}g_2)),$$

$$f_4 = f(t_i + h, (x_i + hf_3), (y_i + hg_3)), \;\; g_4 = g(t_i + h, (x_i + hf_3), (y_i + hg_3)).$$

4.3.3 RK6 method for LCR Circuit

$$x_{i+1} = x_i + \frac{h}{90}[7f_1 + 32f_3 + 12f_4 + 32f_5 + 7f_6], \;\; y_{i+1} = y_i + \frac{h}{90}[7g_1 + 32g_3 + 12g_4 + 32g_5 + 7g_6]$$

where,

$$f_1 = hf(t_i, x_i, y_i), \quad g_1 = hg(t_i, x_i, y_i),$$

$$f_2 = hf(t_i + \frac{h}{3}, (x_i + \frac{f_1}{3}), (y_i + \frac{g_1}{3})), \quad g_2 = hg(t_i + \frac{h}{3}, (x_i + \frac{f_1}{3}), (y_i + \frac{g_1}{3})),$$

$$f_3 = hf(t_i + \frac{h}{3}, (x_i + \frac{f_1 + f_2}{6}), (y_i + \frac{g_1 + g_2}{6})), \quad g_3 = hg(t_i + \frac{h}{3}, (x_i + \frac{f_1 + f_2}{6}), (y_i + \frac{g_1 + g_2}{6})),$$

$$f_4 = hf(t_i + \frac{h}{2}, (x_i + \frac{f_1}{8} + \frac{3f_3}{8}), (y_i + \frac{g_1}{8} + \frac{3g_3}{8})), \quad g_4 = hg(t_i + \frac{h}{2}, (x_i + \frac{f_1}{8} + \frac{3f_3}{8}), (y_i + \frac{g_1}{8} + \frac{3g_3}{8})),$$

$$f_5 = hf(t_i + \frac{2h}{3}, (x_i + \frac{f_1}{2} - \frac{3f_3}{2} + 2f_4), (y_i + \frac{g_1}{2} - \frac{3g_3}{2} + 2g_4)),$$

$$g_5 = hg(t_i + \frac{2h}{3}, (x_i + \frac{f_1}{2} - \frac{3f_3}{2} + 2f_4), (y_i + \frac{g_1}{2} - \frac{3g_3}{2} + 2g_4)),$$

$$f_6 = hf(t_i + \frac{5h}{6}, (x_i - \frac{f_1}{2} + \frac{5f_3}{2} + \frac{f_5}{2}), (y_i - \frac{g_1}{2} + \frac{5g_3}{2} + \frac{g_5}{2})),$$

$$g_6 = hg(t_i + \frac{5h}{6}, (x_i - \frac{f_1}{2} + \frac{5f_3}{2} + \frac{f_5}{2}), (y_i - \frac{g_1}{2} + \frac{5g_3}{2} + \frac{g_5}{2})),$$

$$f_7 = hf(t_i + h, (x_i + \frac{f_1}{7} + \frac{2f_3}{7} + \frac{12f_4}{7} - \frac{12f_5}{7} + f_6), (y_i + \frac{g_1}{7} + \frac{2g_3}{7} + \frac{12g_4}{7} - \frac{12g_5}{7} + g_6)),$$

$$g_7 = hg(t_i + h, (x_i + \frac{f_1}{7} + \frac{2f_3}{7} + \frac{12f_4}{7} - \frac{12f_5}{7} + f_6), (y_i + \frac{g_1}{7} + \frac{2g_3}{7} + \frac{12g_4}{7} - \frac{12g_5}{7} + g_6)).$$

5 Results and Discussion

The numerical solution of LCR circuits using the various iterative techniques discussed above is compared to the analytical solution in the ensuing subsection. Also, we analyze the different characteristics of damping factor for the both series and parallel LCR circuit by using the best efficient numerical method which is sixth order Runge Kutta method (RK6).

5.1 Comparison of numerical and analytical solutions

In the given subsection, we simulate the numerical solution for series and parallel circuits respectively for under damped conditions with their analytical solution and comparison is done by calculating the absolute error at specific time period.

5.1.1 For Series LCR Circuit

The simulation time in this experiment is from 0 s to 20 s for all situations taking a step size of $h = 0.1s$. Likewise, let direct source voltage $(Vs) = 6V$, Resistance $(R) = 1\ ohm$, Inductane $(I) = 1H$ & Capacitance $(C) = 0.25F$ to obtain the simulation result of LCR circuit [5].

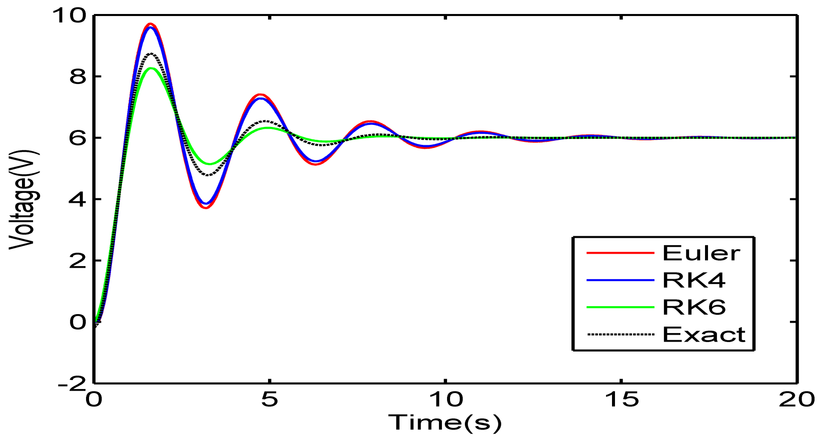

Figure 3: Comparison of Euler, RK4, RK6 and Exact solution for series circuit.

Table 1: Results from simulations using numerical techniques & analytical solution with an absolute error.

Time (sec)	Euler		RK4		RK6		Exact
	Voltage	Error	Voltage	Error	Voltage	Error	Voltage
0.0 sec	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.1 sec	0.000000	0.114378	0.011960	0.102418	0.168186	0.053808	0.114378
0.2 sec	0.240000	0.203654	0.261484	0.182170	0.533736	0.090082	0.443654
0.3 sec	0.696000	0.256189	0.723512	0.228677	1.061245	0.109056	0.952189
0.4 sec	1.336800	0.275764	1.366120	0.246444	1.713234	0.100670	1.612564
0.5 sec	1.336800	1.029671	2.152233	0.214238	2.451763	0.085292	2.366471
0.6 sec	2.125680	1.064150	3.041423	0.148407	3.239892	0.050062	3.189830
0.7 sec	3.022200	0.982692	3.991735	0.013157	4.042936	0.038044	4.004892
0.8 sec	4.968810	0.084868	4.961454	0.077512	4.829500	0.054442	4.883942
0.9 sec	5.935740	0.243419	5.910749	0.218428	5.572277	0.120044	5.692321
1.0 sec	6.847225	0.422573	6.803138	0.378486	6.248608	0.176044	6.424652

The simulation results of the iterative solution of the LCR circuit with the exact solution under the previously stated conditions is shown in Fig. 3. The computed points of three numerical methods *i.e.,* Euler, RK4 & RK6 are taken at specific point for the comparison of accuracy of the numerical methods and is given by Tab. 3. Likewise, the oscillation is more for Euler method than RK4, RK6 and exact methods. We see that error between oscillation of Euler and RK4 is comparatively less than each other but is more than RK6 and exact solutions. The oscillation decreases slowly after the increment of time and at infinite time all solution converges uniformly.

5.1.2 For Parallel LCR Circuit

The simulation time in this experiment is from 0 s to 20 s for all situations taking a step size of $h = 0.1s$. Likewise, let direct source voltage $(Vs) = 6V$, Resistance $(R) = 2\ ohm$, Inductane $(I) = 1H$ & Capacitance $(C) = 0.25F$ to obtain the simulation result of LCR circuit [5].

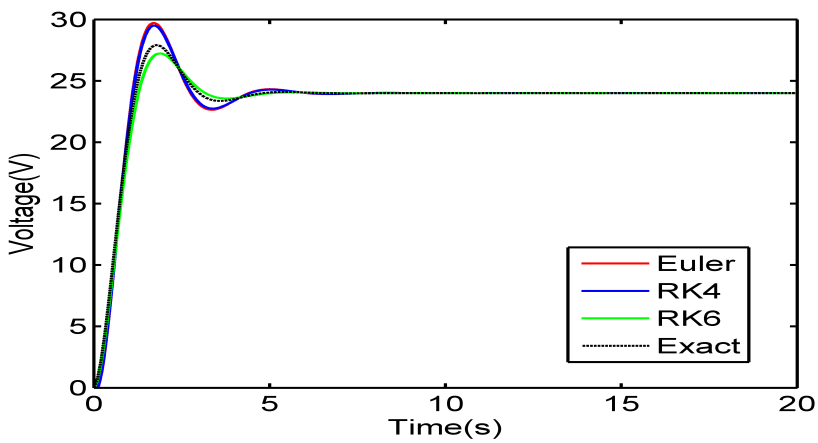

Figure 4: Comparison of Euler, RK4, RK6 and Exact solution for parallel circuit.

Table 2: Results from simulations using numerical techniques & analytical solution with an absolute error.

Time (sec)	Euler		RK4		RK6		Exact
	Voltage	Error	Voltage	Error	Voltage	Error	Voltage
0.0 sec	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.1 sec	0.000000	0.737550	0.047680	0.689870	0.644902	0.092468	0.737550
0.2 sec	0.960000	1.221952	1.036162	1.145790	1.972012	0.2098860	2.181952
0.3 sec	2.68800	1.464886	2.773533	1.379353	3.806136	0.34675	4.152886
0.4 sec	4.992000	1.491691	5.069390	1.414301	5.987573	0.496118	6.483691
0.5 sec	7.687680	1.337233	7.742125	1.282788	8.375207	0.649706	9.024913
0.6 sec	10.304544	1.042045	10.624699	1.021890	10.847579	0.799010	11.646589
0.7 sec	13.590528	0.648876	13.568884	0.670510	13.303391	0.936013	14.239404
0.8 sec	16.515133	0.199770	16.448082	0.266821	16.261162	0.513741	16.714903
0.9 sec	19.271197	0.266292	19.158826	0.153921	19.411536	0.406631	19.004905
1.0 sec	21.775442	0.715153	21..621120	0.560831	21.466798	0.406569	21.060289

Likewise, we see that the oscillation is comparatively less in parallel circuit than the series circuit and this means it is damped quickly. In this circuit, we found that Euler method gives high oscillation than other methods but the error is more than RK4, RK6 and exact solutions. Damping increases with increment of time and as time increases infinitely all oscillations are damped and all solution converge uniformly.

Hence, when compared to the other numerical techniques *i.e.,* Euler and RK4, the numerical approximation solution curve computed by the Runge-Kutta method of sixth order (RK6) is approaching quicker towards the analytical solution. Errors have been calculated individually at each time second and this shows that the relative error between the analytical solution and RK6 solution is comparatively less than other two methods. Thus, we deduce that the RK6 approach is the most effective way to estimate the solution of the LCR circuit.

5.2 Damping Conditions

In an LCR circuit, damping is the term used to describe how a resistive component gradually reduces oscillations or vibrations in the circuit. It is the process that eventually results in a steady state by gradually reducing the oscillation's amplitude.

5.2.1 In Series Circuit

RK6 approach is used in this circuit to visualize all three damping scenarios (*i.e.,* under damped, critically damped and over damped). Table 3 gives the values of electrical elements *i.e.,* source voltage, resistance, inductor and capacitor for different damping condition. Additionally, results are simulated under the previously described conditions with various time-specific points in Fig. 5. Table 4 gives the values of three damping conditions by using RK6 method for series circuit.

Element	Damping Value		
	Cond. 1	Cond. 2	Cond. 3
Resistor	Under-Damped	Critically-Damped	Over-Damped
Voltage	6 Volt	6 Volt	6 Volt
Resistance	$< 4\Omega$	$4\ \Omega$	$> 4\Omega$
Inductor	1 Henry	1 Henry	1 Henry
Capacitor	0.25 Farad	0.25 Farad	0.25 Farad

Table 3: Electrical elements values for three distinct damping circumstances in a series [11].

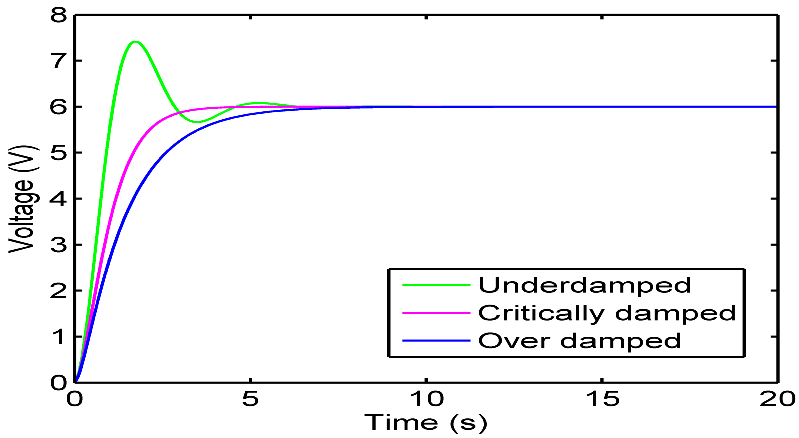

Figure 5: Comparison of all three different damping conditions by applying RK6 series.

Table 4: All damping features of RK6 and their respective values for series.

Time (sec)	Different Value of RK6 Method		
	Under-Damped	Critically-Damped	Over-Damped
0.0 sec	0.000000	0.000000	0.000000
0.1 sec	0.164656	0.148423	0.136930
0.2 sec	0.512743	0.425057	0.371173
0.3 sec	1.063963	0.781100	0.650941
0.4 sec	1.599025	1.180798	0.965764
0.5 sec	2.260901	1.598392	1.280176
0.6 sec	2.955797	2.015728	1.589688
0.7 sec	3.653875	2.420398	1.888212
0.8 sec	4.329716	2.804294	2.172432
0.9 sec	4.962558	3.162494	2.440755
1.0 sec	5.536324	3.492399	2.692653

5.2.2 In Parallel Circuit

Again, all three damping scenarios (*i.e.,* Under damped, critically damped & over damped) visualized by RK6 method. Table 5 gives the values of all electrical elements like resistor, source voltage, inductor and capacitor for different damping condition. Also, results are simulated under the previously described conditions with various time-specific points in Fig. 6. Table 6 gives the values of three damping conditions by using RK6 method for parallel circuit.

Element	Damping Values		
	Cond. 1	Cond. 2	Cond. 3
Resistor	Under-Damped	Critically-Damped	Over-Damped
Voltage	6 Volt	6 Volt	6 Volt
Resistance	$> 1\Omega$	$1\ \Omega$	$< 1\Omega$
Inductor	1 Henry	1 Henry	1 Henry
Capacitor	0.25 Farad	0.25 Farad	0.25 Farad

Table 5: Electrical elements values for three distinct damping circumstances in a parallel [11].

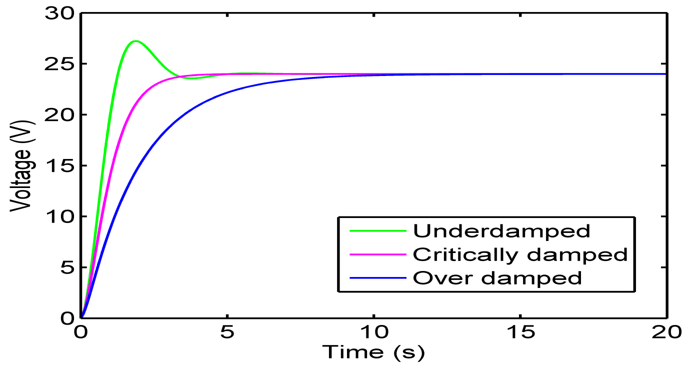

Figure 6: Comparison of all three different damping conditions by applying RK6 parallel.

Table 6: All damping features of RK6 and their respective values for parallel.

Time (sec)	Different Value of RK6 Method		
	Under-Damped	Critically-Damped	Over-Damped
0.0 sec	0.000000	0.000000	0.000000
0.1 sec	0.644902	0.593693	0.506146
0.2 sec	1.972027	1.700228	1.311301
0.3 sec	3.806136	3.124398	2.255384
0.4 sec	5.987573	4.723191	3.250947
0.5 sec	8.375207	6.393567	4.250781
0.6 sec	10.847579	8.062911	5.229934
0.7 sec	13.303391	9.681590	6.175750
0.8 sec	15.661162	11.217177	7.082329
0.9 sec	17.858274	12.649976	7.947467
1.0 sec	19.849543	13.969596	8.770952

6 Conclusions

In this study, we analyzed the transient analysis of both series and parallel LCR circuits by applying the analytical, Euler (Forward), RK4 and RK6 methods. Computational software is dominantly used for obtaining the transient analysis systematically and conveniently. Due to circuit complexity, obtaining analytical solution is very difficult so our experiment suggests and shows that the numerical approximation is suitable for transient analysis. At first, we modeled the second order differential equations into system of first order differential equation and solved them using different numerical approximations. It has been found that the for under damped condition for series and parallel LCR circuits both, RK6 method is very efficient method due to less error than other two methods *i.e.,* Euler (Forward) and RK4. We also found that the oscillation behaves differently for series and parallel circuit. If we see Fig. 3 and Fig. 4, the damping is faster in parallel circuit for all numerical and exact solution than series circuit. Likewise, we also experimented the three damping characteristics for both series and parallel circuits by using RK6 method by taking different values resistance and found that oscillation gradually decreases with increase of time in different nature. The oscillation is high for under damped condition and reduces gradually for critically and over damped respectively in both circuits. It also shows that the oscillation for series circuit for under damped condition is more than parallel circuit. Difference between critically and over damped conditions is comparatively less in series circuit than parallel circuit. As time increases infinitely, all three damping conditions converges uniformly in both series and parallel LCR circuit. Our experiment clearly shows that the under damped decay for both circuits are oscillatory and exponential in nature. This response aids us to build a system that satisfies our needs, and we can additionally improve the time domain settings of the system. Thus, we deduce that by varying the conditions from one steady value to next while conducting a system's transient analysis, we can determine the system's response accurately. Hence, to obtain the more accuracy, we prefer using high order numerical methods.

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