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Research Article

On Banach Space Valued Function Space of Bounded Type Defined by Orlicz Function

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Abstract

The present work introduces and investigates a new class of Banach space-valued functions of bounded type, constructed using an Orlicz function. It serves as a natural extension of the classical space of bounded complex sequences commonly studied in functional analysis. In addition, we examine the linear and topological properties of this space under a suitable natural norm. The role of the Δ_2-condition in ensuring the equality of associated classes of Banach space-valued functions of bounded type, is highlighted through illustrative examples.

Keywords: Orlicz function, Orlicz space, Orlicz sequence space, Normal space.
AMS (MOS) Subject Classification: 46A45, 46B45.

1 Introduction

Function spaces play a significant role across various branches of mathematics, for instance, functional analysis, differential equations, probability theory, and numerical analysis. In functional analysis, the space ℓ_p serves as a natural extension of the p-norm defined on finite-dimensional linear spaces and possesses a rich topological structure. The concept of Orlicz sequence spaces was introduced by W. Orlicz in 1931 as a generalization of the classical ℓ_p spaces.

A large number of research works have explored on different types of topological structures of Orlicz sequence and function spaces. Numerous studies have introduced, explored,

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and analyzed various topological and algebraic properties of these spaces using Orlicz functions, serving as generalizations of classical sequence and function spaces in several contexts: for instance, we refer to a few: Wilansky[1], Tripathy and Mahanta [2], Ghosh and Srivastava[3], Kolk[4], Savas and Patterson[5], Maddox [6], Srivastava and Pahari[8], Ghimire and Pahari([10], [11]), Rao and Subremanina[13], Basarir and Altundag[15], Pahari ([17],[18]), Kamthan and Gupta[19], Parashar and Choudhary[20], Khan[21], Karakaya[22], Bhardwaj and Bala[23], Altun and Bilgin[24]. Before delving into the main results, we give a review of some basic definitions and notations that will support the developments in this study.

Definition 1.1. A function $\Phi : [0, \infty) \longrightarrow [0, \infty)$ is said to be an Orlicz function if it is continuous, non-decreasing and convex with $\Phi(0) = 0$, $\Phi(x) > 0$ for $x > 0$, and $\Phi(x) \longrightarrow \infty$ as $x \longrightarrow \infty$ [14].

An Orlicz function Φ can be represented in the following integral form

$$\Phi(x) = \int_0^x q(t)dt$$

where q, known as the kernel of Orlicz function Φ, is right- differentiable for $t \geq 0$, $q(0) = 0, q(t) > 0$ for $t > 0$, q is non-decreasing, and $q(t) \longrightarrow \infty$ as $t \longrightarrow \infty$[14].

Definition 1.2. An Orlicz function Φ is said to satisfy the Δ_2 condition for all values of x, if there exists a constant $K > 0$ such that $\Phi(2x) \leq K\Phi(x)$, for all $x \geq 0$[14] . The Δ_2 condition is equivalent to the satisfaction of the inequality

$\Phi(Qx) \leq KQ\Phi(x)$ for all values of x for which $Q > 1$[14].

Definition 1.3. Lindenstrauss and Tzafriri [12] used the concept of Orlicz function to define the following sequence space:

$$\ell_\Phi = \left\{ x = (x_k) \in \omega : \sum_{k=1}^{\infty} \Phi\left(\frac{|x_k|}{r}\right) < \infty \text{ for some } r > 0 \right\},$$

known as the Orlicz sequence space, where ω denotes the space of all sequences $x = (x_k)$.

The space ℓ_Φ becomes a Banach space when equipped with the Luxemburg norm, defined by

$$\|x\|_\Phi = \inf \left\{ r > 0 : \sum_{k=1}^{\infty} \Phi\left(\frac{|x_k|}{r}\right) \leq 1 \right\} [12].$$

The more extensive studies including the applications of Orlicz spaces are given by Rao and Ren [16]. In fact, Maddox [6] , Pokharel et.al.[7], Srivastava and Pahari[8], Ghimire [9], and many others have introduced and studied the algebraic and topological properties of various spaces using Orlicz function. In the next section, we define and investigate the class $\ell_\infty(V, W, \|.\|, \Phi)$ of Banach space $W-$ valued functions as a generalization of the previous works, constructed using an Orlicz function Φ.

2 The Class $\ell_\infty(V, W, \|.\|, \Phi)$ of Banach Space $W-$ Valued Functions

Let V be any non-empty set, which need not be countable. Let $\mathcal{F}(V)$ represent the family of finite subsets of V, arranged according to the inclusion relation. Let $\mathbb{C}$ be a complex field. Let $(W, \|\cdot\|)$ be a Banach space over $\mathbb{C}$.

In this work, we introduce and investigate a new class of W-valued functions of bounded type, defined on V, by employing an Orlicz function Φ.

$$\ell_\infty(V, W, \|.\|, \Phi) = \left\{\phi : V \longrightarrow W : \sup_{x\in V} \Phi\left(\frac{\|\phi(x)\|}{r}\right) < \infty \text{ for some } r > 0\right\} \tag{2.1}$$

Moreover, we define the subclass $\bar{\ell}_\infty(V, W, \|.\|, \Phi)$ of $l_\infty(V, W, \|.\|, \Phi)$ by

$$\bar{\ell}_\infty(V, W, \|.\|, \Phi) = \left\{\phi : V \hookrightarrow W : \sup_{x\in V} \Phi\left(\frac{\|\phi(x)\|}{r}\right) < \infty \text{ for all } r > 0\right\} \tag{2.2}$$

3 Linear Topological Structure of the Space $\ell_\infty(V, W, \|.\|, \Phi)$

This section is devoted to exploring various results that shed light on the linear topological structures of the bounded-type space $\ell_\infty(V, W, \|\cdot\|, \Phi)$ by equipping it with an appropriate norm.

To study the linear topological properties of the class $\ell_\infty(V, W, \|\cdot\|, \Phi)$ over the field $\mathbb{C}$, we define the vector space operations point-wise. That is, for any $\phi, \psi \in \ell_\infty(V, W, \|\cdot\|, \Phi)$, and for all $x \in V$, we set

$$(\phi + \psi)(x) = \phi(x) + \psi(x),$$

and for any scalar $s \in \mathbb{C}$,

$$(s\phi)(x) = s\,\phi(x).$$

Let θ denote the zero element of the space. It is defined by the function $\theta : V \to W$ such that

$$\theta(x) = \mathbf{0}, \quad \forall x \in V,$$

where $\mathbf{0}$ is the zero vector in W.

Theorem 3.1. *The class $\ell_\infty(V, W, \|.\|, \Phi)$ forms a vector space over the field $\mathbb{C}$.*

Proof. Let $\phi, \psi \in \ell_\infty(V, W, \|.\|, \Phi)$. Let $r_1 > 0$ and $r_2 > 0$ be the constants associated with ϕ and ψ, respectively. Let $s, t \in \mathbb{C}$. Then, by definition, there exist finite subsets $J_1, J_2 \in \mathcal{F}(V)$ such that

$$\sup_{x\in V} \Phi\left(\tfrac{\|\phi(x)\|}{r_1}\right) < \infty, \forall x \in V/J_1,$$

and

$$\sup_{x\in V}\Phi\left(\frac{\|\psi(x)\|}{r_2}\right)<\infty,\ \forall x\in V/J_2,$$

Let us choose

$$r_3=\text{Max}\{2|s|r_1, 2|t|r_2\}.$$

Since Φ is non-decreasing and convex, for all $x \in V/(J_1 \cup J_2)$,

$$\sup_{x\in V}\Phi\left(\frac{\|s\phi(x)+t\psi(x)\|}{r_3}\right)\leq \sup_{x\in V}\Phi\left(\frac{s\|\phi(x)\|}{r_3}+\frac{t\|\psi(x)\|}{r_3}\right)<\infty$$

Thus for $\phi, \psi \in l_\infty(V, W, \|.\|, \Phi)$, we have $s\phi+t\psi \in l_\infty(V, W, \|.\|, \Phi)$. Therefore, $l_\infty(V, W, \|.\|, \Phi)$ forms a vector space over field $\mathbb{C}$. □

Theorem 3.2. *The class $\ell_\infty(V, W, \|.\|, \Phi)$ forms a normed space with respect to the norm defined by*

$$\|\phi\|_\infty=\inf\left\{r>0:\sup_{x\in V}\Phi\left(\frac{\|\phi(x)\|}{r}\right)\leq 1\right\}$$

.

Proof. By definition,

$$\|\phi\|_\infty=\inf\left\{r>0:\sup_{x\in V}\Phi\left(\frac{\|\phi(x)\|}{r}\right)\leq 1\right\} \tag{3.1}$$

We see that $\Phi(0) = 0$.

Therefore for $\phi = \theta$ we easily get

$$\|\phi\|_\infty = 0.$$

Thus, we have $\|\phi\|_\infty \geq 0$, for all $\phi \in l_\infty(V, W, \|.\|, \Phi)$.

Conversely suppose that $\|\phi\|_\infty = 0$, i.e

$$\inf\left\{r>0:\sup_{x\in V}\Phi\left(\frac{\|\phi(x)\|}{r}\right)\leq 1\right\}=0.$$

Then, for $\epsilon > 0$, there exists some $r_\epsilon (0 < r_\epsilon < \epsilon)$, such that

$$\sup_{x\in V}\Phi\left(\frac{\|\phi(x)\|}{r_\epsilon}\right)\leq 1.$$

Thus

$$\sup_{x\in V}\Phi\left(\frac{\|\phi(x)\|}{r}\right)\leq \sup_{x\in V}\Phi\left(\frac{\|\phi(x)\|}{r_\epsilon}\right)\leq 1.$$

Suppose $\phi(x) \neq 0$ for some $x \in V$. Clearly $\epsilon \longrightarrow 0$ implies that $\frac{\|\phi(x)\|}{\epsilon} \longrightarrow \infty$. This contradicts that $\sup_{x\in V}\Phi(\frac{\|\phi(x)\|}{r_\epsilon}) \leq 1$. Thus $\phi(x) = 0$ for each $x \in V$ and hence $\phi = \theta$.

For absolute homogeneity property, if $s = 0$, then obviously

$$\|s\phi\|_\infty = |s|\|\phi\|_\infty$$

So, suppose $s \neq 0$, we have

$$\|s\phi\|_\infty = \inf\left\{r > 0 : \sup_{x\in V}\Phi\left(\frac{\|s\phi(x)\|}{r}\right) \leq 1\right\} = \inf\left\{\frac{|s|r}{|s|} > 0 : \sup_{x\in V}\Phi\left(\frac{\|\phi(x)\|}{r/|s|}\right) \leq 1\right\} = |s|\|\phi\|_\infty.$$

For the triangle inequality, let $r_1 > 0$ and $r_2 > 0$ be such that

$$\sup_{x\in V}\Phi\left(\frac{\|\phi(x)\|}{r_1}\right) \leq 1,\ \forall x \in V$$

and

$$\sup_{x\in V}\Phi\left(\frac{\|\psi(x)\|}{r_2}\right) \leq 1,\ \forall x \in V$$

Then for $r_3 = r_1 + r_2$, we have

$$\Phi\left(\frac{\|\phi(x)+\psi(x)\|}{r_3}\right) \leq \Phi\left(\frac{\|\phi(x)\|+\|\psi(x)\|}{r_r+r_2}\right) \leq \frac{r_1}{r_1+r_2}\Phi\left(\frac{\|\phi(x)\|}{r_1}\right)+\frac{r_2}{r_1+r_2}\Phi\left(\frac{\|\psi(x)\|}{r_2}\right)$$

This implies that

$$\sup_{x\in V}\Phi\left(\frac{\|\phi(x)+\psi(x)\|}{r_3}\right) \leq \frac{r_1}{r_1+r_2}\sup_{x\in V}\Phi\left(\frac{\|\phi(x)\|}{r_1}\right) + \frac{r_2}{r_1+r_2}\sup_{x\in V}\Phi\left(\frac{\|\psi(x)\|}{r_2}\right)$$

Thus

$$\inf\left\{r > 0 : \sup_{x\in V}\Phi\left(\frac{\|\phi(x)+\phi(x)\|}{r}\right) \leq 1\right\} \leq r_3 = r_1 + r_2$$

Hence

$$\inf\left\{r > 0 : \sup_{x\in V}\Phi\left(\frac{\|\phi(x)+\psi(x)\|}{r}\right) \leq 1\right\}$$
$$\leq \inf\left\{r_1 > 0 : \sup_{x\in V}\Phi\left(\frac{\|\phi(x)\|}{r_1}\right) \leq 1\right\} + \inf\left\{r_2 > 0 : \sup_{x\in V}\Phi(\frac{\|\psi(x)\|}{r_2}) \leq 1\right\}$$

and by (3.1), we get

$$\|\phi+\psi\|_\infty \leq \|\phi\|_\infty + \|\psi\|_\infty$$

This shows that $\ell_\infty(V, W, \|.\|, \Phi)$ forms a normed space. □

Theorem 3.3. *The normed space $\ell_\infty(V, W, \|.\|, \Phi)$ is complete with respect to the norm $\|.\|_\infty$ defined by*

$$\|\phi\|_\infty = \inf\left\{r > 0 : \sup_{x\in V} \Phi\left(\frac{\|\phi(x)\|}{r}\right) \leq 1\right\}.$$

Proof. Let $\{\phi_n\}$ be a Cauchy sequence in $\ell_\infty(V, W, \|.\|, \Phi)$. Let $t_0 > 0$ and $p > 1$ be fixed. Then for $\frac{\epsilon}{pt_0} > 0$, $\exists N \in \mathbb{N}$ such that

$$\|\phi_n - \phi_m\|_\infty < \frac{\epsilon}{pt_0}, \quad \forall n, m \geq N. \tag{3.2}$$

By the definition of the norm in $\ell_\infty(V, W, \|.\|, \Phi)$, we obtain

$$\sup_{x\in V} \Phi\left(\frac{\|\phi_n(x) - \phi_m(x)\|}{\|\phi_n - \phi_m\|_\infty}\right) \leq 1, \quad \forall n, m \geq N.$$

In particular, for each $x \in V$, it follows that

$$\Phi\left(\frac{\|\phi_n(x) - \phi_m(x)\|}{\|\phi_n - \phi_m\|_\infty}\right) \leq 1, \quad \forall n, m \geq N.$$

Hence, there exists a constant $p > 1$ such that

$$p\left(\frac{t_0}{2}\right) \cdot q\left(\frac{t_0}{2}\right) \geq 1,$$

where q is the complementary function associated with Φ. Then for each $x \in V$ and for all $n, m \geq N$, we have

$$\Phi\left(\frac{\|\phi_n(x) - \phi_m(x)\|}{\|\phi_n - \phi_m\|_\infty}\right) \leq p\left(\frac{t_0}{2}\right) q\left(\frac{t_0}{2}\right).$$

Using the integral representation of the Orlicz function Φ, it follows that

$$\frac{\|\phi_n(x) - \phi_m(x)\|}{\|\phi_n - \phi_m\|_\infty} \leq pt_0, \quad \forall x \in V,\ \forall n, m \geq N.$$

Therefore, for each $x \in V$, and by (3.2), we have

$$\|\phi_n(x) - \phi_m(x)\| < \epsilon, \quad \forall n, m \geq N,$$

which shows that the sequence $\{\phi_n(x)\}$ is Cauchy in $(W, \|\cdot\|)$. Since $(W, \|\cdot\|)$ is complete, there exists $\phi(x) \in W$ such that

$$\|\phi_n(x) - \phi(x)\| \to 0 \quad \text{as } n \to \infty.$$

Further, for any $\epsilon > 0$, there exists a natural number $N_0 \geq 1$ such that

$$\|\phi_n - \phi_m\|_\infty < \epsilon, \quad \forall n, m \geq N_0.$$

Choose $r > 0$ such that

$$\|\phi_n - \phi_m\|_\infty < r < \epsilon.$$

Then r satisfies

$$\sup_{x\in V} \Phi\left(\frac{\|\phi_n(x) - \phi_m(x)\|}{\|\phi_n - \phi_m\|_\infty}\right) \le 1,$$

and hence, for all $n, m > N_0$ and $x \in V$,

$$\Phi\left(\frac{\|\phi_n(x) - \phi_m(x)\|}{r}\right) \le 1. \tag{3.3}$$

Taking the limit as $m \to \infty$ in (3.3) and then taking the supremum over $x \in V$, we get

$$\sup_{x\in V} \Phi\left(\frac{\|\phi_n(x) - \phi(x)\|}{r}\right) \le 1, \quad \forall n > N_0. \tag{3.4}$$

Taking the infimum over such r's yields

$$\|\phi_n - \phi\|_\infty = \inf\left\{r > 0 : \sup_{x\in V} \Phi\left(\frac{\|\phi_n(x) - \phi(x)\|}{r}\right) \le 1\right\} \le r < \epsilon, \quad \forall n \ge N_0,$$

i.e.,

$$\|\phi_n - \phi\|_\infty < \epsilon.$$

From (3.4), it follows that $\phi_n - \phi \in \ell_\infty(V, W, \|.\|, \Phi)$ for all $n > N_0$.

Since each $\phi_n \in \ell_\infty(V, W, \|.\|, \Phi)$, and the space is linear, we have

$$\phi = \phi_n + (\phi - \phi_n) \in \ell_\infty(V, W, \|.\|, \Phi).$$

Thus, $\phi \in \ell_\infty(V, W, \|.\|, \Phi)$, proving that $\ell_\infty(V, W, \|.\|, \Phi)$ is complete. □

Theorem 3.4. *The normed space $\ell_\infty(V, W, \|.\|, \Phi)$ is normal.*

Proof. Suppose that $\phi \in \ell_\infty(V, W, \|.\|, \Phi)$. Let $r > 0$ be a constant associated with ϕ. By definition,

$$\sup_{x\in V} \Phi\left(\frac{\|\phi(x)\|}{r}\right) < \infty.$$

Now, choose a scalar-valued function $\alpha : V \to \mathbb{C}$ such that $|\alpha(x)| \le 1, \forall x \in V$. Consider the function $\alpha\phi$ defined by $(\alpha\phi)(x) = \alpha(x)\phi(x)$. Then,

$$\sup_{x\in V} \Phi\left(\frac{\|\alpha(x)\phi(x)\|}{r}\right) \le \sup_{x\in V} \Phi\left(\frac{|\alpha(x)| \cdot \|\phi(x)\|}{r}\right) \le \sup_{x\in V} \Phi\left(\frac{\|\phi(x)\|}{r}\right) < \infty.$$

This shows that $\alpha\phi \in \ell_\infty(V, W, \|.\|, \Phi)$. Hence, the space $\ell_\infty(V, W, \|.\|, \Phi)$ is normal. □

Theorem 3.5. *Let Φ satisfy the Δ_2-condition. Then*

$$\ell_\infty(V, W, \|.\|, \Phi) = \bar{\ell}_\infty(V, W, \|.\|, \Phi).$$

Proof. In view of definitions (2.1) and (2.2), it is obvious that

$$\bar{\ell}_\infty(V, W, \|.\|, \Phi) \subseteq \ell_\infty(V, W, \|.\|, \Phi) \tag{3.5}$$

Hence, to prove the claim, we show

$$\ell_\infty(V, W, \|.\|, \Phi) \subseteq \bar{\ell}_\infty(V, W, \|.\|, \Phi).$$

Let $\phi \in \ell_\infty(V, W, \|.\|, \Phi)$. Then for some $r > 0$, we have

$$\sup_{x \in V} \Phi\left(\frac{\|\phi(x)\|}{r}\right) < \infty.$$

Let $s > 0$. If $r \leq s$ then clearly

$$\sup_{x \in V} \Phi\left(\frac{\|\phi(x)\|}{s}\right) < \sup_{x \in V} \Phi\left(\frac{\|\phi(x)\|}{r}\right) < \infty$$

Let $s < r$. Then $\frac{r}{s} > 1$.
Using the Δ_2- condition for Φ, there exists K, a constant, such that

$$\Phi\left(\frac{\|\phi(x)\|}{s}\right) = \Phi\left(\frac{\|\phi(x)\|}{r}.\frac{r}{s}\right) \leq K\frac{r}{s}\Phi\left(\frac{\|\phi(x)\|}{r}\right) < \infty$$

for each $x \in V$, which implies that, for all $s > 0$,

$$\sup_{x \in V} \Phi\left(\frac{\|\phi(x)\|}{s}\right) < \infty$$

This shows that

$$\ell_\infty(V, W, \|.\|, \Phi) \subseteq \bar{\ell}_\infty(V, W, \|.\|, \Phi) \tag{3.6}$$

From (3.5) and (3.6), we obtain

$$\ell_\infty(V, W, \|.\|, \Phi) = \bar{\ell}_\infty(V, W, \|.\|, \Phi).$$

□

Remark 3.6. The Δ_2-condition on Φ in theorem 3.5 is essential for the equality

$$\ell_\infty(V, W, \|\cdot\|, \Phi) = \bar{\ell}_\infty(V, W, \|\cdot\|, \Phi)$$

to hold. If Φ does not satisfy the Δ_2-condition, the inclusion

$$\ell_\infty(V, W, \|\cdot\|, \Phi) \subseteq \bar{\ell}_\infty(V, W, \|\cdot\|, \Phi)$$

may not hold. We provide the following two examples to illustrate the situation.

Example 3.7. Let $V = \mathbb{N}$ and $W = \mathbb{C}$. Let us define a function $\phi : \mathbb{N} \to \mathbb{C}$ by $\phi(n) = \frac{1}{n}$. Consider an Orlicz function $\Phi(t) = t^2$. Clearly, $\Phi(t)$ satisfies Δ_2- condition.

For some fixed $r > 0$,

$$\sup_{n\in\mathbb{N}} \Phi\left(\frac{|\phi(n)|}{r}\right) = \sup_{n\in\mathbb{N}} \left(\frac{1/n}{r}\right)^2 = \frac{1}{r^2} < \infty.$$

Hence, $\phi \in \ell_\infty(\mathbb{N}, \mathbb{C}, |\cdot|, \Phi)$.

Also, for all $r > 0$,

$$\sup_{n\in\mathbb{N}} \Phi\left(\frac{|\phi(n)|}{r}\right) = \frac{1}{r^2} < \infty.$$

Therefore, $\phi \in \bar{\ell}_\infty(\mathbb{N}, \mathbb{C}, |\cdot|, \Phi)$.
Hence, the theorem 3.5 is verified as $\Phi(t)$ satisfies Δ_2- condition.

Example 3.8. If Φ does not satisfy the Δ_2- condition, then we may have

$$\bar{\ell}_\infty(V, W, \|\cdot\|, \Phi) \subsetneq \ell_\infty(V, W, \|\cdot\|, \Phi).$$

This is so because without Δ_2- condition, Φ can increase rapidly. Thus, for some ϕ and fixed $r > 0$,

$$\sup_{x\in V} \Phi\left(\frac{\|\phi(x)\|}{r}\right) < \infty,$$

but for smaller $s < r$, we have

$$\sup_{x\in V} \Phi\left(\frac{\|\phi(x)\|}{s}\right) = \infty.$$

This implies that $\phi \notin \bar{\ell}_\infty(V, W, \|\cdot\|, \Phi)$. In particular, consider an Orlicz function $\Phi(t) = e^t - 1$, which does not satisfy the Δ_2-condition. Let us define a constant function $\phi : V \to W$ by

$$\|\phi(x)\| = c > 0 \quad \text{for all } x \in V.$$

Then, for any fixed $r > 0$,

$$\sup_{x\in V} \Phi\left(\frac{\|\phi(x)\|}{r}\right) = e^{c/r} - 1 < \infty,$$

but as $r \to 0$, we have

$$e^{c/r} - 1 \to \infty.$$

Thus, $\phi \notin \bar{\ell}_\infty(V, W, \|\cdot\|, \Phi)$ even though $\phi \in \ell_\infty(V, W, \|\cdot\|, \Phi)$.

4 Conclusion:

The current study presents various findings that characterize the topological and algebraic structures of a newly introduced class of Banach space-valued function space of bounded type, denoted by $\ell_\infty(V, W, \|.\|, \Phi)$. These results allow for a broader generalization and unification of classical complex function and sequence spaces commonly examined in functional analysis. Moreover, they offer potential for investigating the linear and topological features of other emerging function and sequence spaces.

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