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RESEARCH ARTICLE

Non-Conservative Maximum Flow in Belief Degree Approach

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Abstract

In many real-world problems, we have to face situations where there is not enough data or no data. The belief degree approach is one of the suitable techniques that can be applied to solve such real-life problems. Domain experts, utilizing uncertainty theory, excel in these situations. The goal of this study was to design the maximum flow model with deterministic excess flow storage at the intermediate vertices and uncertainty in arc capacities of a network at a predefined confidence level β. Through the application of an uncertain measure and the inverse distribution formed by the uncertain variable, the classical maximum flow model is converted into its deterministic counterpart. This problem is solved by developing a polynomial procedure. A numerical example is provided that illustrates the suggested model and the algorithm. Lastly, a graphic comparison is made between conservative and non-conservative maximum flows for different values of confidence level, which verifies the dominance of non-conservative flows over the conservative ones.

Keywords: flow maximization, intermediate storage, uncertainty programming.

Introduction

In network flow optimization, the flow maximization problems are widely studied. The first basic model of this issue was studied by Fulkerson and Dantzig (1955). Dantzig and Fulkerson (1956) solved it using the augmenting path procedure. Dinic (1970) introduced the concept of augmenting flows in the layered network. In a two-terminal network, Wilkinson's (1971) solution to the earliest arrival flow problem (EAFP) maximizes the flow at each time step. Edmonds and Karp (1972) noted the slow nature of augmenting flow due to the large number of augmentations. Minieka (1973) used a pseudo-polynomial running time algorithm to solve the EAFP and a polynomial running time approach to solve the lexicographic maximum(lex-max) static flow problem. The Preflow-push algorithm on layered networks reduced the number of augmentations, which was introduced by Karzanov (1974). Goldberg and Tarjan (1986) introduced push and relabel operations at active nodes to optimize the running time of the preflow-push algorithm. Using a genetic algorithm with flow matrices to represent each solution, Munakata and Hashier (1993) addressed the maximum flow problem (MFP). The lex-max dynamic flow problem was resolved in polynomial time by Hoppe and Tardos (1994).

Network optimization was examined by Dhamala et al. (2018) and Pangeni and Dhamala (2021). Pangeni and Dhamala (2023) investigated flow dynamics using average arc capacities in continuous time. Pyakurel et al. (2023) studied an abstract technique for the deterministic network evacuation problem with storage at intermediate vertices. Pyakurel et al. (2019, 2018, 2017a, 2017b) have also used partial, efficient, and continuous deterministic models to study the network flow problems utilizing the contraflow technique. To overcome the MFP, Pyakurel and Dempe (2020) in particular used a network with deterministic arc capacity and storage at intermediate vertices.

In classical network flow problems, every arc capacity and the vertex capacity of the network are known. Their values are assumed to be changed with respect to time and form a probability distribution. In this regard, the MFP in the probabilistic approach has been investigated widely. Frank and Hakimi (1965) came up with a communication network to obtain the probability of a flow between nodes. Frank and Frisch (1971) took arc capacity as a continuous random variable, whereas Doulliez (1971) considered discrete stochastic arc capacities in a multi-terminal network. Rather than the exact value of the maximum flow, some works are carried out on the value of flows within an interval, such as Onaga (1968), Carey and Hendrickson (1986), and Nagamochi and Ibaraki (1991).

But, in reality, probability theory does not work well in network problems, due to a lack of enough data on parameters to form a distribution. This happens when indeterminacy factors, such as weather conditions, earthquakes, or traffic congestion. In these situations, the belief degree approach is applied by inviting the related experts. To overcome the difficulty of solving the problems in such human uncertainty, Liu (2007, 2009a, 2009b) founded and refined uncertainty theory, and also proposed uncertain programming theory. Peng and Iwamura (2010) established sufficient and necessary conditions for uncertainty distribution. Liu and Ha (2010) got a formula to evaluate the expected values of monotone functions of uncertain variables. Research on uncertain multilevel and multi-objective programming are carried out by Liu and Yao (2012) and Liu and Chen (2013), respectively. Various methods, such as least squares, Delphi, and B-spline methods, are used to deal with uncertainty distribution [Liu (2010), Wang et al. (2012), and Chen and Ralescu (2012)]. Gao and Gao (2013), Zhang and Peng (2011), and Gao (2013) contributed to uncertain graphs and uncertain networks.

Liu (2009a, 2009b, 2010, 2013) connected uncertainty theory to network optimization through project scheduling problem and also to random networks. The relation between the uncertain and deterministic shortest path was studied by Gao (2011). A numerical solution method to the uncertain maximum flow problem was studied by Han et al. (2014). In conservative formulations, Ding (2015) proposed the α-maximum flow model of an uncertain network, and Shi et al. (2017) proposed the models of the maximum flow problem for uncertain random networks. The interdiction problem in an uncertain network was studied by Pangeni and Dhamala (2024a). They used the order-guided non-conservative flow feature to maximize the flow value with limits in the interdiction budget. They also proposed the maximum flow and minimum cost to such a flow feature in an uncertain network, Pangeni and Dhamala (2024b). The flow maximization in a hybrid network and in the uncertain network, introducing the centiles method, are also introduced (Pangeni and Dhamala 2024c, 2024d).

In this work, we develop a maximum flow model for a network with uncertain arc capacities and present the deterministic capabilities for flow storage at intermediate vertices. As we know, this is done for the first time. This type of flow is called *non-conservative flow*. To evaluate the model's effectiveness, an executing algorithm is created, and a random example is considered.

This article has structure as follows. In Section 2, notions of uncertainty are discussed. In Section 3, the β maximum flow model with storage at the intermediate vertices is proposed and then converted into a deterministic model. A solution algorithm for optimality is developed in Section 4. The efficiency of the proposed algorithm is checked in Section 5. The paper concludes in Section 6.

Preliminaries

A nonvoid collection of subsets of the real number R, which is closed under countable unions as well as complements, is a σ-algebra on R. The smallest σ-algebra on R having all open sets (or closed sets also) is known as the Borel σ-algebra. The members of the Borel algebra are said to be Borel sets or measurable sets. The following notions are according to Liu (2007, 2009a, 2010).

Consider a nonvoid set Γ and a σ-algebra L over Γ. Every member $\Lambda \in L$ is known as an event. A set function $M: L \to [0,1]$ is known as an uncertain measure if it satisfies:

Normality: $M\{\Gamma\}=1$, where Γ is the universal set

Duality: $M\{\Lambda\} + M\{\Lambda^c\}=1$, to all events Λ

Sub-additivity: For all countable sequences of events $\Lambda_1, \Lambda_2, \ldots$ we have

$$M\{\cup_{i=1}^{\infty}\Lambda_i\} \le \sum_{i=1}^{\infty} M\{\Lambda_i\}$$

Product: Let uncertainty spaces be $(\Gamma_k, L_k, M_k), k = 1,2,\ldots,n$. The product uncertain measure M is an uncertain measure on the product σ-algebra $L_1 \times L_2 \times \ldots \times L_n$ such that

$$M\{\prod_{k=1}^{n} \Lambda_k\} = \min_{1\le k\le n} M_k\{\Lambda_k\}$$

An *uncertain variable* is a measurable function $\xi : (\Gamma, L, M) \to R$, where R is the set of real numbers such that $\{\xi \in B\} = \{\gamma \in \Gamma \mid \xi(\gamma) \in B\}$ is an event for any Borel set B of real numbers. An uncertain variable is depicted in Figure 1[1].

Figure 1

An uncertain variable

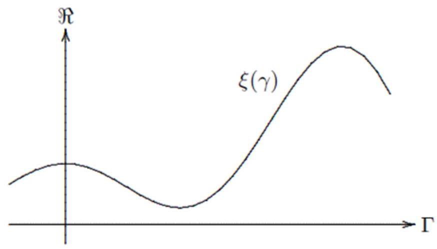


Figure 2

An uncertainty distribution

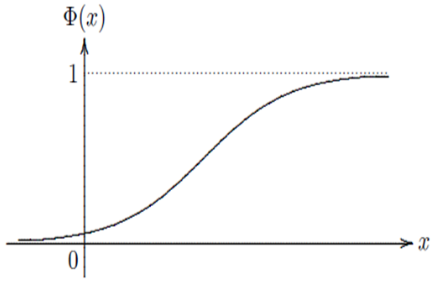
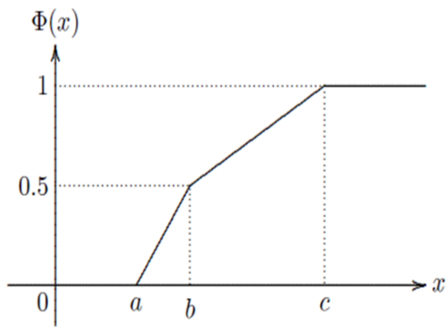


Figure 3

A zigzag uncertainty distribution



1 Adopted from Liu (2010), Figure 1.2

The uncertain variables $\xi_1, \xi_2, \ldots, \xi_n$ are called *independent* if,

$$M\{\bigcap_{i=1}^{n} (\xi_i \in B_i)\} = \bigwedge_{i=1}^{n} M\{\xi_i \in B_i\}$$

for Borel sets B_i of real numbers,
$i = 1, \ldots, n$.

The *uncertainty distribution* Φ of an uncertain variable ξ is given by
$\Phi(x) = M\{\xi \leq x\}; \forall x \in R$.
An uncertainty distribution is depicted in Figure 2[2].

An uncertain variable ξ is said to be *zigzag*, if it has a zigzag uncertainty distribution

$$\phi(x) = \begin{cases} 0, & x \leq a \\ \dfrac{x-a}{2(b-a)}, & a \leq x \leq b \\ \dfrac{x+c-2b}{2(c-b)}, & b \leq x \leq c \\ 1, & x \geq c \end{cases}$$

and is denoted by Z(a, b, c), where a, b, c $\in R$ with a < b < c.

A zigzag uncertainty distribution is depicted in Figure 3[3].

The *inverse uncertainty distribution* of the zigzag uncertain variable Z(a, b, c) is

$$\phi^{-1}(\beta) =$$

$$\begin{cases} (1-2\beta)a + 2\beta b, & \beta < 0.5 \\ (2-2\beta)b + (2\beta-1)c, & \beta \geq 0.5 \end{cases}$$

An uncertainty distribution $\phi(x)$ is called regular if it is a continuous and strictly increasing function with respect to *x*,
$0 < \phi(x) < 1$, and

$$\lim_{x\to-\infty} \phi(x) = 0, \lim_{x\to\infty} \phi(x) = 1$$

To transform an indeterminacy flow model into a deterministic one, we can use the following theorem.

Theorem 1. Liu (2010)

Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. If a function $g(x_1, x_2, \ldots, x_n)$ is strictly increasing, then the uncertain variable $\xi = g(\xi_1, \xi_2, \ldots, \xi_n)$ has an inverse uncertainty distribution $\Psi^{-1}(\beta) = g(\Phi_1^{-1}(\beta), \Phi_2^{-1}(\beta), \ldots, \Phi_n^{-1}(\beta))$

Model Formulation

Consider a doublet $N = (V, A)$, in which V is a set of finite vertices and A is the set of arcs of the directed network N, such that the vertices are the crossing-over of arcs, i.e., $A \subseteq V \times V$. Let $f_{ij} \geq 0$ be a flow in an arc (i, j). Also, u_{ij} be the capacity of the arc (i, j). Let $s_{ij} \geq 0$ and , respectively, be the storage and the storage capacity at the vertex i. Also, suppose that F is the maximum flow from the starting vertex s to the sink vertex d, which is a function of u_{ij}, for all $(i, j) \in A$. Then,
Deterministic flow maximization model:

$$\max\,[F + \textstyle\sum s_i],\ i \in V \setminus \{s, d\} \quad (1)$$

$$\sum_{j:(s,j)\in A} f_{sj} - \sum_{j:(j,s)\in A} f_{js} = F \quad (2)$$

$$\sum_{j:(i,j)\in A} f_{ij} - \sum_{j:(j,i)\in A} f_{ji} + s_i = 0,\ i \in V\setminus\{s, d\} \quad (3)$$

$$\sum_{j:(d,j)\in A} f_{dj} - \sum_{j:(j,d)\in A} f_{jd} = -(F - \sum_{j\in V\setminus\{s,d\}} s_j) \quad (4)$$

$$0 \leq f_{ij} \leq u_{ij}, (i, j) \in A \quad (5)$$

$$0 \leq s_i \leq v_i \quad (6)$$

The objective function (1) aims to maximize total flow. Equation (2) gives the total mass supplied by the source *s*. (3) gives the balance of mass to the intermediate vertices. (4) is the total mass demanded by the target vertex *d*. (5) and (6) are the capacity constraints of the arcs and vertices,

2 Adopted from Liu (2010), Figure 1.3
3 Adopted from Liu (2010), Figure 1.5

respectively.

The deterministic maximum flow problem assumes a fixed capacity of an arc. But in practice, it is not. Due to indeterminacy factors or lack of sufficient data, the probability distribution of arc capacity can not be created, and then the belief degree approach; uncertainty approach is used.

Let us denote an uncertain network $N^U = (V, A, \xi)$, where $\xi = \{\xi_{ij} / (i,j) \in A\}$ is the set of uncertain variables ξ_{ij}. The total flow is $F(\xi)$. Being a function of ξ, F is also an uncertain variable. For any feasible flow f' from source node s to sink node d, a flow f is the β-maximum flow, if

$$max\{F/M\{\xi \le f\} \le \beta\} \ge max\{F'/M\{\xi \le f'\} \le \beta\}$$

where β is a confidence level provided by the domain expert.

The chance constrained model with a given confidence level β optimizes the objective function (1) subject to the constraints (2) - (4), (6), and the constraints (7).

$$M\{\xi_{ij} \le f_{ij}\} \le \beta, (i,j) \in A \quad (7)$$

Classical deterministic flow maximization problem is solved by polynomial algorithms. So, if we could transform β-maximum flow problem with chance constraint into deterministic flow maximization problem, then the solution approach would be simplified. Therefore, we need to convert the chance constraints into its deterministic equivalent.

Let Φ_{ij} be the uncertainty distribution of ξ_{j}. Then for $0 < \beta < 1, (i,j) \in A$, Theorem 1 and the definition of uncertainty distribution give:

$$M\{\xi_{ij} \le \Phi_{ij}^{-1}(\beta)\} = \beta.$$

Using this in the chance constraint (7), we have

$$M\{\xi_{ij} \le f_{ij}\} \le \beta$$
$$= M\{\xi_{ij} \le \Phi_{ij}^{-1}(\beta)\} \Rightarrow f_{ij} \le \Phi_{ij}^{-1}(\beta)$$

So the deterministic model optimizes the objective function (1) subject to the constraints (2) - (4), (6), and the constraints (8).

$$f_{ij} \le \Phi_{ij}^{-1}(\beta), (i,j) \in A \quad (8)$$

Algorithm Development

Based on Theorem 1 and the model, the following polynomial algorithm is developed, and it performs well to get the maximum flow at a predetermined confidence level.

Algorithm 1: *β-Maximum Flow*

Input: Given *N = (V, A, f_{ij}, u_{ij}, Φ, s_i, v_i)*, $\beta \in$ *(0, 1).*

1. Evaluate $\Phi_{ij}^{-1}(\beta)$, $(i,j) \in A$
2. Set $u_{ij} = \Phi_{ij}^{-1}(\beta)$,
3. Take the length of each arc as a unit and find the shortest distance between the intermediate vertices from the source, where the flow to vertex i violates the capabilities of the succeeding arcs i.e., $\sum_{(j,i)\in A} f_{ji} > \sum_{(i,j)\in A} \Phi_{ij}^{-1}(\beta)$.
4. Prioritize the vertices of Step 3 more highly, which are farthest from the source.
5. Along with the provided sink, turn the vertices from Step 3 into virtual ones and use those as sinks.
6. Using the priority of Step 4, determine the maximum flow in the modified network with the single source, and super sink made from virtual vertices and given sink.
7. To acquire the maximum flow at the specified sink and storage at the specified intermediate vertices, turn on the network N while turning off the virtual vertices and arcs.

- **Output:** Uncertain maximum flow for different confidence levels β.

Experimental Verification

Consider $N^U = (V, A, \xi)$ as an uncertain network having five vertices and five arcs, as shown in Figure 4. The uncertain variables corresponding to the arcs of the network N^U are listed in Table 1. To calculate $\Phi_{ij}^{-1}(\beta)$ for the arcs of Figure 4, the value of β is arbitrarily taken as 0.2. and listed in Table 1. The arcs and the corresponding $\Phi_{ij}^{-1}(\beta)$ for every ξ_{ij} are calculated and listed in Table 2. For a constant value of ξ_{ij}, $\Phi_{ij}^{-1}(\beta)$ is taken as constant for any value of $\beta \in [0,1]$.

Figure 4

Uncertain network N^U, arc with capacities, when β = 0.2

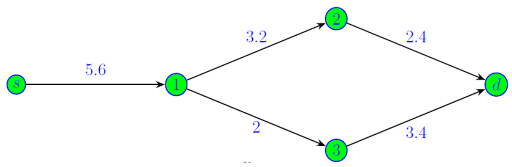


Table 1

List of arc capacities and $\Phi_{ij}^{-1}(0.2)$

arc	$\xi_{ij} \sim \mathcal{Z}(a,b,c)$	$\Phi_{ij}^{-1}(0.2)$
(s,1)	(4, 8, 9)	5.6
(1,2)	(2, 5, 8)	3.2
(1,3)	2	2
(2,d)	(2, 3, 4)	2.4
(3,d)	(3, 4, 6)	3.4

Figure 5

$\beta = 0.2$-max flow network, arc with flow value f_{ij}, dashed virtual arc with flow capacity, and vertex i with storage s_i.

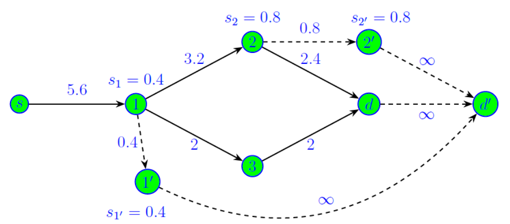


Table 2

List of arc capacities $\Phi_{ij}^{-1}(\beta)$ for different values of β

arc	$\xi_{ij} \sim \mathcal{Z}(a,b,c)$	$\beta=0$	$\beta=0.4$	$\beta=0.6$	$\beta=0.8$	$\beta=1$
(s, 1)	(4, 8, 9)	4	7.2	8.2	8.6	9
(1, 2)	(2, 5, 8)	2	4.4	5.6	6.8	8
(1, 3)	2	2	2	2	2	2
(2, d)	(2, 3, 4)	2	2.8	3.2	3.6	4
(3, d)	(3, 4, 6)	3	3.8	4.4	5.2	6

To see the varying trend of the maximum flow for different values of β, Table 3 is created. The flow pattern of $\beta = 0.2$ -maximum flow according to the β - max flow algorithm 1 is shown in Figure 5 and then plotted in Figure 6. The graph in Figure 3 depicts the variation of the optimum flow with and without intermediate storage for various values of β.

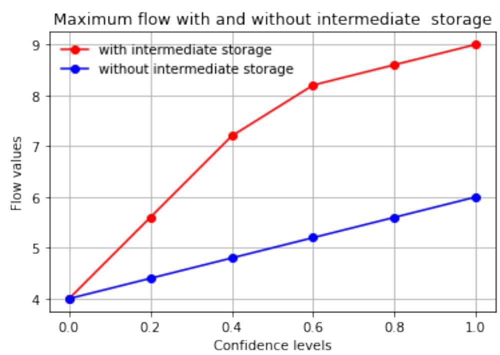
Table 3

List of β-maximum arc flow values for different values of β

β	f_{s1}	f_{12}	f_{13}	f_{2d}	f_{3d}	$F = \Psi^{-1}(\beta)$	$\sum s_i$
0	4	2	2	2	2	4	0
0.2	5.6	3.2	2	2.4	2	4.4	1.2
0.4	7.2	4.4	2	2.8	2	4.8	2.4
0.6	8.2	5.6	2	3.2	2	5.2	3
0.8	8.6	6.6	2	3.6	2	5.6	3
1	9	7	2	4	2	6	3

Figure 6

Uncertainty distribution of the maximum flow value



Conclusion

The capacity of arcs in a network becomes unknown due to indeterminacy factors or a lack of historical data. The lexicographic maximum flow problem in a network with enough storage facilities at the intermediate vertices and uncertain arc capacity is resolved in this research. First, uncertainty theory is applied to convert the chance-constrained flow model into its deterministic equivalent. As an explicit solution to priority-ordered optimal flow value, a β-max flow technique has been devised. To verify the algorithm's efficiency, an example is provided to demonstrate its functionality. Furthermore, it is noted that the non-conservative maximum flow value outweighs the conservative flow, which is depicted graphically. There are many uses for these kinds of issues in network optimization.

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