Prithvi Academic Journal
[A Peer-Reviewed, Open Access Multidisciplinary Journal]
Indexed in NepJOL with JPPS Star Ranking
ISSN 2631-200X (Print); ISSN 2631-3252 (Online)
URL: https//ejournals.pncampus.edu.np/ejournals/paj/

Centre for Research & Innovation
Prithvi Narayan Campus
Tribhuvan University
Pokhara, Nepal
https://prnc.tu.edu.np/

RESEARCH ARTICLE

Problem Formulation and the Solution Strategies for Symmetric and Asymmetric Evacuation Networks

Iswar Mani Adhikari
Tribhuvan University, Prithvi Narayan Campus, Pokhara, Nepal

Submitted: 15 December 2025; Reviewed: 9 April 2026; Revised: 21 April 2026; Accepted: 3 May 2026
Corresponding Author : *Iswar Mani Adhikari,* **Email:** ishwor.adhikari@prnc.edu.tu.np
DOI: *https://doi.org/10.3126/paj.v9i1.94502*

Abstract

Dynamic flow formulations are the basic mathematical tools for evacuation planning problems. They can be formulated to maximize flow, minimize time, or minimize cost, depending on the nature and context of the problem. However, for the evacuation system, the focus is on minimizing time while maximizing flow, so that the large number of evacuees can be evacuated in the shortest possible time. Their effectiveness in evacuation systems mainly depends on network structures and available resources. In this research work, we consider the formulation of the evacuation planning problem and solution strategies to minimize clearance time in evacuation networks with symmetric and asymmetric topologies were considered. For this, we used the uniform path length network, grid network, and embedded network topologies to get the minimum clearance time on such specific networks. Depending upon the nature of the problem, different heuristic approaches are used to get their solutions.

Keywords: disaster management, evacuation management, network optimization, transportation system

Introduction

Disasters cannot be predicted. Natural calamities such as earthquakes, floods, landslides, avalanches, etc., are unavoidable, and their impacts are severe. The effectiveness of disaster operations is affected by the available network structures and resources. Due to geophysical conditions, it is difficult to modify the existing network. However, smart network concepts can be applied to develop a smart city for better logistics and supply chain management. After many disasters, saving and normalizing the situation is challenging.

Ford and Fulkerson introduced the first dynamic network flow problems in Ford & Fulkerson (1958) and -Ford & Fulkerson

(1962). The minimum-clearance-time evacuation-planning problem is one of the basic time-minimization problems for transshipping evacuees from disaster zones to safety shelters. Different variants of the evacuation planning problems mainly focus on time minimization, based on a bus-based evacuation network, and are formulated as network flow formulation techniques and are symmetric in nature. In most of them, the fundamental objective is to minimize the duration of evacuation as in (Bish, 2011; Goerigk & Grun, 2014; Goerigk, Grun, & Hessler, 2013). There has been a fair amount of work on time minimization problems, as referred to by (Adhikari & Dhamala, 2020; Adhikari & Dhamala, 2020a; Adhikari, Pyakurel, & Dhamala, 2020; Dhamala & Adhikari, 2018; Dhamala, Adhikari, Nath, & Pyakurel, 2018). Most of them are based on symmetric network structures.

Planning, preparedness, response, and recovery are four major sequential and interconnected components in disaster management. Based on these components, the network flow formulation of the evacuation planning problem connects operations research, transportation science, and emergency management with combinatorial mathematics. Such mathematical modeling, with its solution strategies and their applications to real-life problems, has great importance as it supports life-saving decisions and enhances disaster preparedness.

Methods

Documents related to network flows and their solution techniques, and their variants in various types of networks, including their formulations, designs, and solution strategies were reviewed and analyzed. Here, the focus is on variants of network topologies. Special types of grid network, uniform path length networks, and the embedded collection and assignment sub-networks were also explored with their problem formulations and solution strategies.

The problem itself is NP-complete; it is not imperative to find an exact solution. One of the universally acceptable formulations of such a problem is always challenging. However, better heuristics are expected in their formulations and solution strategies corresponding to different network topologies for urban areas. This research will be novel and applicable.

Network Flows and the Evacuation Planning Problems

A network N = (V, A) is formally defined as a directed graph, where V denotes the finite set of vertices (nodes), and A represents the set of directed arcs (edges). Each node corresponds to a physical intersection point within a spatial domain, and each directed edge (i, j) connects an ordered pair of nodes, modeling a directed roadway or pathway within the network. Commodities, such as evacuees or transported goods, traverse the network from origin to destination via feasible paths consisting of sequential arcs. Each arc is assigned a non-negative capacity u_a, representing the maximum allowable flow rate along that edge. Nodes where evacuees or commodities are initially present are designated as source nodes (denoted by S), while nodes representing safe destinations or final delivery points are classified as sink nodes (denoted by Z). Intermediate transfer or pickup points within the network are represented by set Y. The transit time associated with each arc, denoted as $\tau_a \forall a \in A$, quantifies the temporal cost or delay required for the flow to traverse a given edge.

Consider a dynamic evacuation network defined as $N = (V, A, u, \tau$, S, Z, $T)$, in which V corresponds to the set of vertices (nodes), A is the set of directed arcs (edges), u denotes the capacity function for each arc, τ represents the transit time function, S is the set of source nodes, Z is the set of sink nodes, and T specifies the finite time horizon during which evacuation is modeled. The capacity u_a for an arc $a \in A$ indicates the maximum flow that can traverse arc a per time unit, while τ_a defines the discrete time steps required for flow to move from the tail to the head of arc a. This formal structure enables the study of temporally dependent flows in evacuation scenarios. Given N, a transformation can be applied to obtain a modified network $N' = (V, A', u', \tau', S, Z, T)$, which incorporates adjustments to the set of arcs, their capacities, and the transit times to facilitate certain analytical approaches. Specifically, in N', the arc capacities are redefined as $u'_a = u_a + u_{a'}$, accounting for both the original arc and its reverse (if present), and the transit times become symmetric such that $\tau'_a = \tau_a$ if $a \in A$ and $\tau'_a = \tau_{a'}$ if $a \notin A$, where an arc $a' \in A'$ is in the transformed network, if $a \vee a' \in A$ in the network N. All other parameters, including the sets of nodes, sources, sinks, and the time horizon, remain unchanged to preserve the essential characteristics of the network while allowing for enhanced analytical tractability.

Evacuee flow from source to sink over time is captured by a non-negative function f on $A \times R_{>0}$ time steps T = {0, 1, 2, …, T-1}, subject to flow conservation and the capacity limits, (1-3). Flow conservation constraints permit waiting at intermediate nodes, but any flow entering an intermediate node must leave right away.

$$\sum_{a \in A_i^{in}} \sum_{\sigma=\tau_a}^{T} f(a, \sigma - \tau_a) - \sum_{a \in A_i^{out}} \sum_{\sigma=0}^{T} f(a, \sigma) = 0, \forall i \in V \backslash (S \cup Y) \quad (1)$$

$$\sum_{a \in A_i^{in}} \sum_{\sigma=\tau_a}^{\theta} f(a, \sigma - \tau_a) - \sum_{a \in A_i^{out}} \sum_{\sigma=0}^{\theta} f(a, \sigma) \geq 0, \forall i \in V(S \cup Y), \theta \in T \quad (2)$$

$$0 \leq f(a, \theta) \leq u_a \forall a \in A, \theta \in T (3)$$

For node $i \epsilon V$; outgoing arcs are denoted by, $A_i^{out} = a = (i, j) \epsilon A$; incoming arcs are $A_i \ = a = (j, i) \epsilon A$. For all $y \epsilon Y$ and $s \epsilon S$, where we assume that $A_i^{out} = A_i^{in} = \phi$ in the case without arc reversals. Here, the net flow becomes, $v_f(s) > 0$, and $v_f(y) < 0$, respectively, in the sources and pickups where $\sum_{i \in V} v_f(i) = 0$,

Flow reached Y for all time setup up to $\theta' \in R_{>0}$ is, $|v_f|_{\theta'} \sum_{\theta=1}^{\theta'} |(v(Y, \theta)|$ (4)

For the given time bound T, the flow value in the network is, $|v_f| = \sum_{\theta=1}^{T} |(v(Y, \theta)|$ (5) For details, we refer to Dhamala et al. (2018).

Problem Formulation and Solution Strategies

Dynamic flow models enable the solution of maximum flow, quickest transshipment, and earliest arrival problems within a time horizon T. Prior models by Bish (2011), Goerigk et al. (2013), and Pereira and Bish (2014) assumed deterministic or constant arrivals at pickup points, but real arrivals are stochastic, requiring robust optimization. Some works address uncertainty and bus capacity constraints, extending to robust formulations (e. g. Goerigk & Grun (2014). However, most existing models assume symmetric transit times in the evacuation network. Nevertheless,the formulations and the solution techniques corresponding to such a network can be extended to asymmetric and embedded network topologies. Here, our focus is on uniform, grid, symmetric, and asymmetric networks as well as their integrated embeddings.

Uniform Network.

Consider a fully connected dynamic network with unit arc capacities and transit times specified per arc (see Figure 1).

1. For all $i \neq z$, every i-z path has equal total transit time.
2. For all $i \neq z$, the minimum i-z cut is defined by arcs entering z from nodes reachable from i.

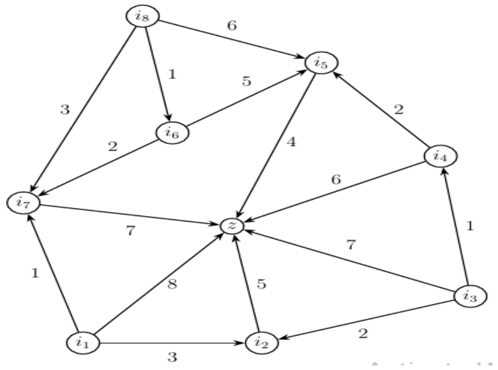
Example 1

In this setup, all i-z paths have uniform length.

Figure 1

A fully connected network with uniform

path lengths



In uniform path-length networks, directed cycles have zero transit time and do not affect evacuation progress. Since Zero-length cycles are unrealistic in real networks, they are excluded from practical models. Identical arc capacities and transit times allow explicit calculation of optimal evacuation schedules. Standardized paths and parameters are common in urban grids and enable network flow optimization. Homogeneous arc properties simplify modeling and solution procedures. Such uniformity reduces model complexity and is particularly suitable for a standardized environment like modern cities

Example 2

Consider a simple network of uniform path length with $\tau_{ij} = \tau \forall (i,j) \in E$. Here, every edge has the same travel time or travel cost; all shortest paths of the same length are equivalent in time; and spatial geometry does not matter. Some of the common examples have typical structures like a tree-like uniform network or a layered network having the layers in source S, intermediate layered nodes $v=\{V_1,V_2,...,V_n\}$; layered pickups as $P=\{P_1,P_2,...,P_n\}$; and finally, as the shelters $Z = \{Z_1, Z_1 ..., Z_n\}$ with each layer transition has a fixed cost. Consider a scenario with demand Q=120, bus capacity C=20, and the number of available buses B={2, 3, 4, 5, 6 } with the unit uniform path time per layer. For the demand Q, the wave capacity becomes B×C . Then the number of waves becomes, $W(B) = \lceil Q/BC \rceil$, and the evacuation time $T(B)=T_{fix}+2W(B)$. Here, we have $T_{fix}=3$, as the number of movement layers in the layered network. Then, the number of waves and the respective evacuation time can be computed as shown in Table 1.

Table 1

Number of waves and evacuation time.

No. of buses available: |B|	Wave capacity: BC	No. of waves required: W(B)	Evacuation time $T(B)=T_{fix}+2W(B)$.
2	40	3	9
3	60	2	7
4	80	2	7
5	100	2	7
6	120	1	5

Here, the critical value for the threshold behavior becomes, $B^* = \frac{Q}{C}$. Then, the evacuation behavior can be represented as in Table 2.

Table 2

Evacuation behavior

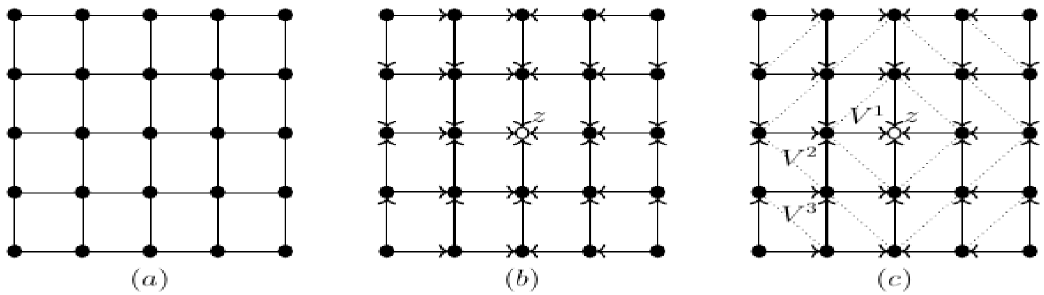
Scenarios	Evacuation behavior	Remarks
B≥ B*	Single wave regime	Fast evacuation system
B < B*	Multi-wave regime	Delays in the evacuation system

Grid Network

A grid network is formed by N×N vertices at integer lattice coordinates, where each vertex is connected to its immediate neighbors (Manhattan distance one), Figure 2(a). Every arc has a uniform capacity c and transit time τ. Arcs are directed from node i to i' if moving to i' brings the path closer to the sink Z, that is $d(i', z) < d(i', z)$, Figure 2(b). The network can be represented as N = (D, c , Z), where c and τ are constants. The structure accurately models environments like city streets or building corridors, where connectivity and capacity are standardized from the pickup locations to shelters.

Figure 2

Grid Network



For any vertices i and j in the grid, consider l(i, j) to be the sum of transit times of arcs on the path. Let the vertex set V be partitioned into different layers according to their distances from z; then, such a directed graph becomes a layered graph, and the network on such a layered graph is called a layered network. It consists of different layers with partitions of V into the subsets as $V^0, V^1, V^2 \ldots$ for $V^0=\{i, j\}$ such that the vertices $i \in V^p$ and $j \in V^q$ are connected by a directed arc (i, j) only if p-q=1, and are in the V^r layer, which denotes the set of all vertices satisfying l(i, z)=rτ, as in Figure 2(c). For details, we refer to the text by Ahuja et al. (1988).

Example 3

Consider a 4 by 4 grid network having the source at S = (1, 1), intermediate nodes as the internal grid intersections, pickup nodes as the collection points, like P_i= {(2,2), (2, 3), (3, 2), (3, 3)} respectively as P_1,P_2,P_3,P_4 and the remaining grid nodes as the transit nodes and the shelters as Z_i = {Z_1=(4,2), Z_2=(4,3)}. For symmetrical edges, each grid node is bidirectional, that is (i, j) ↔ (i+1, j) and (i, j) ↔ (i, j+1). Consider the Manhattan distance as the shortest path distance as a unit, then the horizontal and vertical moves each have a travel time of 1. In such a grid network, the distance traveled from source S to pick up, that is $D(S, P_i) = |x_1-x_2|+|y_1-y_2|$ is given in Table 3.

Table 3

Distance traveled from the source to the pickup

Pick up	P_1	P_2	P_3	P_4
Distance	2	3	3	4

The pickup P_i to shelters Z_i distance can also be computed as shown in Table 4.

Table 4

Distance travelled from pickup locations to shelters

$P_i \rightarrow Z_i$	$P_1 \rightarrow Z_1$	$P_2 \rightarrow Z_1$	$P_3 \rightarrow Z_1$	$P_4 \rightarrow Z_2$
Distances	2	3	1	1

Consider the demands at the pickup nodes at P_1,P_2,P_3,P_4 to be 25, 30, 20, and 25, respectively, with the shelter capacities at Z_1 and Z_2 be 50 and 60, respectively. Then the travel time per evacuation path can be computed by T_i= d (S, P_i) + d (P_i,Z_i) as shown in Table 5.

Table 5

Travel time per evacuation path

Node	Route	Travel time
P_1	$S \to P_1 \to Z_1$	4
P_2	$S \to P_3 \to Z_1$	4
P_3	$S \to P_2 \to Z_2$	6
P_4	$S \to P_4 \to Z_2$	5

Clearly, the longest path is 6. Hence, even with the infinite buses, T_{min} =6. Moreover, for bus capacity C=20 and the total demand Q=100, the possible number of buses with the number of waves of evacuation can be computed as in Table 6.

Table 6

Number of waves and evacuation time

No. of active buses: |B|	No. of waves	Evacuation time
5	1	6
4	2	9
3	2	9
2	3	12

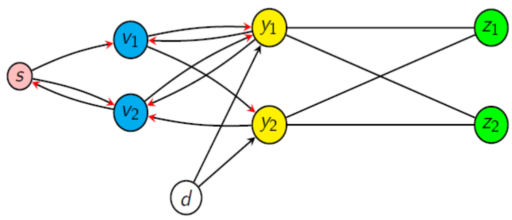
Note that, in such a grid network, even with the infinite buses, the evacuation time cannot be reduced to below 6. Here, the budget only affects the additional delay on the system.

Embedded Network Topology

The network consists of two embedded sub-networks: the primary sub-network for evacuee collection and the secondary sub-network for bus assignment. The primary sub-network connects source nodes, auxiliary nodes, and pickup points, which are linked to bus depots and sink nodes. Pickup locations become source nodes for the secondary sub-network, which aims to minimize overall evacuation time. Evacuees are first moved to pick up sites, then assigned to buses for transport to safe zones. The collection phase uses earliest arrival flows and partial arc reversal techniques while bus assignments are based on evacuation duration dominance. The collection sub-network includes bidirectional arcs with possible arc reversals, whereas the assignment sub-network features directed one-way arcs from depots to pick up points and symmetric connections among pickup points for routing. This integrated approach enables efficient, flexible evacuation by combining two-way and one-way routing within a unified framework. For details, we refer to Adhikari (2021) and the references therein.

Figure 3

Embedding, an integrated network



Example 4

Consider an embedded network for minimum time clearance of evacuees. In each embedding, consider the symmetric collection sub-network and the asymmetric assignment sub-network. In such a fixed network structure, let S denote the source; V$\{v_1,v_2\}$ as the intermediate nodes; P$\{p_1,p_2\}$ as the pickup nodes; and finally, Z$\{z_1,z_2\}$ as the sinks. Let the demands of the pickup nodes be 50 and 70, respectively, with the total demand Q = 120. For computational simplicity, assume the buses of uniform capacities with C=20, where the evacuation buses are |B|=6 and the cost of each bus activation is 1 unit. Consider the travel time τ on the collection sub-network with a symmetric network structure, be fixed as 2. That is the travel time for the route $S \to p_i$ as well as $p_j \to S$ be 2.

A wave is one complete cycle of evacuation using available bus capacity at a given time. Thus, it is a batch dispatch of buses that leaves the pickup nodes within a short time window to carry evacuees to the shelter. So, if the bus capacity is C and the available active buses are B, then the wave capacity becomes B×C that is each wave can evacuate at most B×C evacuees.

Here, the total evacuee capacity per wave becomes Capacity=20×B. Then the number of waves becomes, $W(B) = \lceil 120/20B \rceil = \lceil 6/B \rceil$, as the upper ceiling function. Then, the evacuation times become, $T(B) = 4 \times W(B)$. The evacuation time stands for the total time required until the last evacuee reaches shelter. Then, corresponding to different scenarios, the required number of waves and their respective evacuation time T(B) can be computed as shown in Table 7.

Table 7

Number of waves and evacuation time

Scenarios	No. of active buses: |B|	No. of waves: W(B)	Evacuation Time: T(B)
Scenario-1	6	1	4
Scenario-2	5	2	8
Scenario-3	4	2	8
Scenario-4	3	2	8
Scenario-5	2	3	12

Here, the evacuation time is 4, 8, and 12 depending on |B|≥6, 3≤|B|≤6 and |B|<3, respectively. It represents a step function of the budget. It is not linear but a piecewise function. Here, we are using the budget as the number of available active buses. Note that, even if the network capacity is sufficient, the evacuation is dominated by the fleet size constraint. Moreover, the evacuation time is not smooth, and it changes as a step function. This tells that the system suddenly shifted from an efficient evacuation to a bottleneck-dominated evacuation system. For this, we can discuss it in terms of the threshold.

For a small change in the budget,that is, the fleet size, a sudden change occurs in the system as the threshold effect, and the key threshold is given by $B^* = \frac{Q}{C}$. Then, the evacuation behavior can be represented in Table 8.

Table 8

Evacuation behavior

Scenarios	Evacuation behavior	Remarks
B ≥ B*	1 wave equation	Fast evacuation system
B < B*	Min-wave equation	Delays on the evacuation system

The extended sensitivity analysis measures how the output varies on their input in the system. In this example, we can define the elasticity as, Elasticity, $E_B = \frac{\frac{\Delta T}{T}}{\frac{\Delta B}{B}}$

Depending upon the input in the system, we get the situations as in Table 9.

Table 9

Elasticity and system analysis

Variation in |B|	Variation in T	Elasticity: EB	Remarks
|B|:6→ 5	T:4→ 8	$EB = \frac{\frac{4}{4}}{\frac{-1}{6}} = -6$	Highly sensitive region
|B|:3→ 2	T:8→ 12	$EB = \frac{\frac{4}{8}}{\frac{-1}{3}} = -1.5$	Less sensitive region

Table 9 shows how fragile the evacuation performance is under the budget constraints or the resource changes in the system. Notably increasing the budget beyond a certain threshold does not improve evacuation performance. Therefore, overinvestment should be avoided as it may lead to inefficiencies caused by the delay in the system with the bottleneck in buses, road networks, or shelters.

Conclusion

The topology of the evacuation network is different. Most urban areas exhibit a complex network topology, like a connected uniform path-length, a symmetric type of network, or an asymmetric network. In addition, evacuation-vehicles often have heterogeneous capacities and models. Developing mathematical formulations and corresponding solution strategies for such an integrated complex network topology is therefore essential. Since the problem is NP-complete, efficient heuristics should be developed for their solution. Moreover, due to high computational complexity, exact solutions may not be practical.

Despite its strengths, network flow formulations for evacuation planning problems have several limitations. Key limitations include deterministic assumptions (e.g., fixed capacities, travel time), predefined network structures, simplified representation of human behavior, and high computational complexity. Bottlenecks, time dependency, demand uncertainty, network degradation, and multi-modal conflicts are additional challenges and key issues in the network flow formulations of evacuation planning problems.

There is a strong potential to extend network flow formulation in evacuation planning by incorporating uncertainties in demand, travel time, and infrastructural failure. Such models can integrate behavioral dynamics within multimodal evacuation systems. Hybrid and integrated evacuation approaches, such as contraflow strategies with real-time adaptability in a large-scale urban context and making evacuation systems more realistic, resilient, applicable and equitable, are always challenging.

Acknowledgment

The Author is grateful to the University Grants Commission, Nepal, for the SRDIG award, SRDIG-81/82-S & T-03, for the research support.

Funding Statement

The author received no financial support for the research, authorship, and/or publication of this article.

Availability of Data and Materials

"Not applicable"

Conflict of Interest

The author declares that there is no conflict of interest in relation to this manuscript.

Ethical Compliance

"Not applicable"

Consent for Publication

"Not applicable"

Plagiarism and AI Use

The manuscript is free from plagiarism and improper use of AI-generated content. Any permitted use of AI tools (if applicable) was limited to language support and has not replaced the original scholarly contribution.

References

Adhikari, I. M. (2021). *Evacuation optimization with minimum clearance time* (Unpublished doctoral dissertation). Institute of Science and Technology, Department of Mathematics, Tribhuvan University.

Adhikari, I. M., & Dhamala, T. N. (2020). Minimum clearance time on the prioritized integrated evacuation network. *American Journal of Applied Mathematics, 8*(4), 207–215. https://doi.org/10.11648/j.ajam2020804.15

Adhikari, I. M., & Dhamala, T. N. (2020a). On the transit-based evacuation strategies in an integrated network topology. *The Nepali Mathematical Sciences Report, 37*(1–2), 1–13. https://doi.org/10.3126/nmsr.v37i1-2.34063

Adhikari, I. M., Pyakurel, U., & Dhamala, T. N. (2020). An integrated solution approach for the time minimization evacuation planning problem. *International Journal of Operations Research, 17*(1), 27–39. 10.6886/IJOR.202003_17(1).0002

Ahuja, R. K., Magnanti, T. L., & Orlin, J. B. (1993). *Network flows: Theory, algorithms, and applications*. Prentice Hall.

Bish, D. R. (2011). Planning for a bus-based evacuation. *OR Spectrum, 33*, 629–654. https://doi.org/10.1007/s00291-011-0256-1

Dhamala, T. N., & Adhikari, I. M. (2018). On evacuation planning optimization problems from a transit-based perspective. *International Journal of Operations Research, 15*(1), 29–47.

Dhamala, T. N., Adhikari, I. M., Nath, H. N., & Pyakurel, U. (2018). Meaningfulness of OR models and solution strategies for emergency planning. In *Living under the threat of earthquakes* (pp. 175–194). Springer. https://doi.org/10.1007/978-3-319-68044-6_12

Dhamala, T. N., Pyakurel, U., & Dempe, S. (2018). A critical survey on network optimization algorithms for evacuation planning problems. *International Journal of Operations Research, 15*(3), 101–133. 10.6886/IJOR.201809_15(3).0002

Ford, L. R., & Fulkerson, D. R. (1958). Constructing maximal dynamic flows from static flows. *Operations Research, 6*(3), 419–433.https://www.jstor.org/stable/167028

Ford, L. R., & Fulkerson, D. R. (1962). *Flows in networks*. Princeton University Press.

Goerigk, M., & Grün, B. (2014). A robust bus evacuation model with delayed scenario information. *OR Spectrum, 36*, 923–948. https://doi.org/10.1007/s00291-014-0365-8

Goerigk, M., Grün, B., & Hessler, P. (2013). Branch-and-bound algorithms for the bus evacuation problem. *Computers & Operations*

Research, 40, 3010–3020.https://doi.org/10.1016/j.cor.2013.07.006

Pereira, V. C., & Bish, D. R. (2014). Scheduling and routing for a bus-based evacuation with a constant evacuee arrival rate. *Transportation Science, 49*(4), 853–867. 10.1287/trsc.2014.0555

Pyakurel, U., Goerigk, M., Dhamala, T. N., & Hamacher, H. W. (2015). Transit-dependent evacuation planning for Kathmandu Valley: A case study. *International Journal of Operations Research Nepal, 5*, 49–73.