Understanding Mega-paradigms in the History of Western Thought, Euro-centrism, Indian Mathematical Proofs and Gödel's Incompleteness Theorems

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ABSTRACT

This paper explains the evolution of western thought through three major paradigms (pre-modern, modern, and post-modern) and their profound implications for knowledge, education, and mathematical philosophy. The pre-modern paradigm emphasized cosmic harmony and essentialist truths, while the modern era ushered in a mechanistic and empirical worldview. The post-modern shift, influenced by quantum physics and Gödel's incompleteness theorems, embraces uncertainty, contextuality, and socially constructed realities. Within this intellectual framework, the traditional Eurocentric narrative of mathematical history is critically examined. Contrary to the dominant view that centers Greek and European contributions, the paper highlights the significant advancements made by Indian mathematicians, particularly those from the Kerala school, who developed foundational ideas in calculus and infinite series long before similar European discoveries. Their approach, rooted in intuitive reasoning and demonstrative methods (upapattis), contrasts with the formal deductive traditions of the West. Gödel's incompleteness theorems serve as a pivotal point in the discourse, revealing intrinsic limits within formal mathematical systems and challenging the absolutist belief in complete and consistent foundations. Together, these threads present a more inclusive and philosophically nuanced understanding of mathematics as a dynamic, culturally shaped, and epistemologically contested field.

Keywords: Eurocentrism, paradigms, intellectual thoughts, demonstrative methods (upapattis), incompleteness theorem.

Introduction

The evolution of western thought has undergone profound transformations through three major paradigms: pre-modern, modern, and post-modern; each redefining humanity's understanding of reality, knowledge, and education. The pre-modern paradigm emphasized cosmic harmony and essential truths, rooted in classical philosophy and geocentric worldview. The modern paradigm during the 17th to 19th centuries started to focus on objectivity, determinism, and mechanistic reasoning. However, the post-modern paradigm of 20th century questioned the certainty of knowledge, embracing ambiguity, cultural contexts, and the limitations of formal systems, as influenced by quantum theory and Godel's groundbreaking incompleteness theorems. Learning became participatory and dialogic, reflecting a fluid and socially constructed understanding of truth. This shifts also influence how mathematics is perceived and taught.

A historically Eurocentric narrative has framed Greek mathematics as the cornerstone of mathematical knowledge, marginalizing significant contributions from non-European civilizations. However, scholars increasingly recognize the profound innovations of Indian mathematics, particularly during the so-called "Dark Ages" of Europe. Indian mathematical philosophy emphasized intuitive understanding and practical demonstrations (upapattis), offering an alternative to the Greek formalist tradition.

This broader historical and philosophical context sets the stage for understanding the significance of Godel's Incompleteness theorems. These theorems exposed fundamental limitations in the formalist foundations of mathematics. These theorems underscored the incompleteness of logic-based mathematical worldviews and continue to influence contemporary in mathematics, logic and epistemology.

Objectives

This paper has addressed following objectives.

- To trace the evolution of western thoughts through three mega-paradigms and their impact on education and knowledge systems.
- To critique Eurocentric historiography in mathematics by acknowledging the overlooked contributions of Indian mathematicians and contrasting philosophical approaches.
- To introduce Godel's incompleteness theorems as a fundamental challenge to absolutist and formalist views of mathematics, illustrating the limits of formal reasoning.

Methods

This study was based on the review of the literature associated with the issues. The paper was prepared from the in-depth study of scholarly articles that were filtered on the basis of predetermined criteria for quality, relevance, and reliability for the study. The article associated to mega-paradigms in the history of western thoughts and its epistemology, eurocentrism and Indian mathematics development, and Godel incompleteness theorems. These articles were rigorously studied and insights were drawn to explain the issues critically. Finally, these essences and insight drawn from the study were analyzed logically and then explained critically.

Results and Discussion

The results of this theoretical paper are described under the separate headings as below.

Mega-paradigms in the History of Western Thought

Paradigm is a set of common beliefs or assumptions among mathematicians, scientists, researchers and the people in the world. These set of beliefs have been changed during the time span from ancient to now. Using science, particularly physics and astronomy, as an organizing framework, we categorize the development of western thought into three mega-paradigms: pre-modern, modern and post-modern thought.

The pre-modern thought of western society covers the long duration of time which begun from documented western history through the scientific and industrial revolutions of the 17^{th} and 18^{th} centuries. Among others small paradigms such as primitive, Greek, Christian, medieval, Renaissance and Humanist, the Greek thought had been dominated as recorded in the history. The Greek philosophers Plato and Aristotle separated ideas from objects, although they still believed that each category or quality needed the other.

Justice in society was viewed as the sense of balance for Plato while it was the mean between the excesses of extremes for Aristotle. Greeks developed their own epistemology, metaphysics, and a cosmology. They viewed reality and personal existence as the balance between the binary oppositions of good/evil, up/down, light/dark, hot/cold etc. Similarly, the person who could do balance between these binary oppositions was considered an educated or wise person. They believed that four is the perfect number as it symbolized the geometric shape of square, with its equal sides and balanced angles.

The sense of balance of Greek can also be seen in their statuary and architecture, especially in the Parthenon, according to the 'Golden mean' with the length of building approximately 1.6 times its width. This sense of balance even carried in justice and morality. Moreover, Plato emphasized on rational deduction and Aristotle on empirical induction, although they were agreed on the point of balance order.

Maintaining order in the society is another aspect of pre-modern thought. The concept of order in pre-modern thought conveyed a strong feeling of closure and stasis. Its boundaries were fixed and unchanging. The individual was viewed in terms of these boundaries. Crossing boundaries or moving beyond one’s assigned role or social class meant tempting fate and, in mythology, risking the wrath of the gods. Euclid was too much rationalist who also favored finite and closed in his geometry of closed circles and line segments. His shapes in geometry were straight, balanced and closed. Similarly, Ptolemaic astronomy and cosmology, drawing on Euclid's work, imagined the universe as finite and circular.

Socially and educationally, the closeness of this view meant that individuals were not to overstep their bounds or rise above their class. In a more positive manner, it means that harmony and integration should pervade all one did. Life and learning should be balanced. This noble ideal flourished in the Renaissance with its views on courtly behavior and well-rounded gentlemen and ladies. Ultimately, the pre-modern society provided a foundation for true justice as a modern period of legislative court.

With the above description, the pre-modern society was an ordered society which lay a belief in the reality of ideals. In this realm everything had an essential or internal quality which became Christian soul for the neo-Platonists of the Renaissance.

Considering Plato and Aristotle were the great thinkers of pre-modern mega-paradigm. The Plato's educational and social theories were based on each individual fulfilling a pre-set role for the common good. The roles for rulers, guardians, artisans were prescribed and nonvariant. The forms that these classes copied were absolute, permanent and unchanging. Plato believed, in the soul of the person that which partook of the world-soul. This view allowed Plato to think of knowledge as remembrance of past experiences. In the similar vein, Socrates well-known method of questioning focuses on recollection rather than being open ended and progressive like genuine dialogue. Its purpose is to uncover eternal, pre-existing truths.

The Plato's concept of world soul was rejected by Aristotle. However, he accepted the concept of essence and claimed that transferring essence from the externality of idealized Forms to norm-referenced categories. Thus, in Aristotelian taxonomic classification systems there are a normative ideal for each category, even for virtue. In Aristotelian physics, fire rose because its essence was lightness, water coalesced because of its essence, and object thrown into the air strove actively to return to their natural home, the earth where earth was considered the centre of the universe. So, the pre-modern thought is geocentric.

The modern paradigm adopted the mechanistic cosmology begun with the work of mathematicians Copernicus, Kepler, Galileo and Brahe. They found ordered beauty in mathematics and discovered Nature's order- its natural laws. Galileo linked the universe to a grand book which was written in the language of mathematics including the concepts of triangles, circles and other geometric figures. Newton (empiricist) discovered the mathematical rule of gravitational force ($F = G\frac{Mm}{r^2}$) which showed that all Nature was consonant, "conformable to Herself and simple", world is mathematical, mechanistic and following the Nature's order. The formula also justified that apples falling from trees and planets revolving around the sun follow the same rules. For Newton, reality is both straightforward and observable. His metaphysical perspective on nature and order sees beauty in the uniformity of its simple symmetry within which lies a set of essential, linear, causal relationships that can be precisely described through mathematics.

With this discovery of Nature's order, those scientists, philosophers, and other intellectuals following Newton adopted the new vision. The discovery of Nature's law gave humanity control over Nature. The shifting from pre-modern to modern paradigm were the changes from female nature to men machines, from an organic and holistic view to a system composed of lifeless, inactive particles driven by outside forces instead of their own internal energy. Also, the sense of balance and harmony has lost where the shifting emphasis from internal direction to external force. The modern science and industrial revolution introduced not just material advantages but also ideas of progress, liberty, and personal achievement.

In the similar vein, by adopting Galileo sense of experimentation, Descartes (rationalist) method of right reason, and Newton's principles, it was now seen as possible to bend, first Nature and then other people, to the will of those experts who knew what should be. Sociology and psychology were born in the aftermath of this vision, and the 'scientific method' took on a mystical aura. Industrial productions become the source of wealth. Thus engineers, builders, planners and these technocrats used and improved Nature's law.

At deeper level, the vision in modern paradigm was closed because Descartes methodology of right reason was as certain and dogmatic and the Newton's mechanistic science was predicted on a stable, uniform, cosmological order. At the core of this modern worldview was cause-and-effect determinism, quantifiable through mathematics, which relied on a closed, unchanging, and linearly progressing universe. Stability was presumed, with nature consistently conforming to itself and being fundamentally simple. The disciplines were arranged in a reductionist hierarchy, from mathematics and physics down to sociology and psychology.

Moreover, Descartes' Cartesian method was based on achieving certainty, while Newtonian predictability relied on the idea of a universe that is stable, symmetrical, and simply structured. Descartes method focused for rightly conducting reason for seeking truth where faith in an external order. Descartes' statements also reflected to support Copernicus' positing of a sun-centered (heliocentric) universe. This was a great shift in the world view of pre-modern society. Thus, it was established that the universe is lawful and regulated terms. Also, planet movement was measured with clockwork precision in modern period.

Descartes method of right reasoning was connected with Euclid's geometrical reasoning where obvious and unmistakable truths are directly derived from self-evident, geometric principles. His long chains of reasoning are the deductive steps as Euclid's used in his geometry. In both Euclid's and Descartes' methods there is the assumption of an external reality, set by a rational, geometrical, undeceiving God, unaffected by our personal ruminations and activities.

Descartes separated realities as primary and secondary. The primary qualities are mathematical and objective such as size, shape, motion and position. The secondary qualities are subjective in natures which are less real that are recognized through sense like color, odor, taste, texture, sound. This separation of the qualities was the fundamental change in western thought. But the belief of reality was objective where the nature became objects of manipulation by reason and ultimately established that personal emotions, intuitions, and experiences were not considered source of knowledge. Knowledge was seen as external, unchanging, and embedded in the fundamental laws of nature. It could be uncovered but not invented.

The third mega-paradigm in the western thought is the post-modern paradigm which was generally originated in the twentieth century with in the development of thermodynamics in physics. The thermodynamics has explained three types of system which are isolated, closed and open. From these systems we can classify the western thought as in mega-paradigms.

The isolated system (pre-modern) exchanges neither energy nor matter which may move as universe and thus the movement is purely cyclical within a set frame. It is this system that Socrates envisioned in his concept of knowledge is recycled; Plato's vision of reality and Aristotelian empirical process of thought.

Similarly, the modern system is closed where exchange energy but not matter as in gears in mechanical system. There is only transference and concentration of energy but no transformation of matter into energy.

The open system which is the post-modern thought where exchange both energy and matter. If matter is to be transformed into energy, at least the energy present is focused and harnessed, thus yielding increased results. Open systems, very much predicted on Einstein's $E = mc^2$, "exchange both energy and matter". It means that the reality is transformational, not static and objective. This is the thought of post-modern era.

This is the post-modern thought that the certainty does not and cannot exist in the micro world of the subatomic which is claimed by Werner Heisenberg in early of twentieth century in the interpretation of quantum physics. Later, Kurt Gödel (1931/1963) showed that the foundation of mathematics could not be proven in terms of consistency and completeness. He further claimed that any mathematical system rests on basic assumptions that seem intuitively correct but are logically unprovable. We are now in twenty first century, the third millennium, which is the age of doubt and fear. If we have faith then it is faith based on doubt, not on certainty. It means that the right reason and empirical observation cannot work correctly in this time because this is the age of uncertainty.

The reality for post- modernist is not absolute and single, but it is contextual and local. That is reality is multiple. The intellectual outlook of postmodernism is grounded in practical skepticism- a doubt arising from decisions based not on overarching meta- narratives but on individual experiences and specific local histories. Embracing this perspective encourage us to become more skillful negotiators with ourselves, our ideas, our surroundings, and others. To reach near to reality, dialogic communication is necessary with each other.

Simultaneously, post-modernism aims for a diverse but context-specific blending of subject and object, mind and body, curriculum and individual, teacher and student, as well as self and others. We have a responsibility for our futures as well as for the future of others. Thus, individual and personal reality is meaningless; reality is only in terms of imaginations. Both self and reality are found in relation.

Epistemological Views of Western Intellectual Thoughts

Learning is the life long process for any one. Generally, learning is guided by some theories, called learning theories which are developed and understood from the philosophical, psychological and social development in the world. The philosophical and social understandings are developed from the contemporary social, religious, cultural beliefs, etc. and scientific development. During this mega-paradigmatic shift, the scientists, mathematicians, philosophers and other social thinkers developed their own social, personal and intellectual vision.

From the history, the western thought can be divided by pre-modern, modern and post-modern paradigmatic thoughts. Each of these paradigms has developed and posited their own epistemological, metaphysical and methodological views in western thought. Consequently, different theories of learning were developed, which are underpinning in mega-paradigms thought. Here, I am going to sketch the theories of learning as developed and adopted in three mega-paradigms of western thought with respect to their epistemologies and from the posited social, personal and intellectual vision of each paradigm.

Learning and life were viewed as balanced, ordered, closed and circular in pre-modern society. Thus, the learning theories could be viewed under these assumptions. They viewed justice in society as a sense of balance (for Plato) and a mean between the excess of extremes (for Aristotle). Reality and personal existence were made up the struggles or balance of binary oppositions like good versus evil, high and low, light and dark, and hot and cold. In this sense the pre-modern learning was focused to maintain the balance between these oppositions and who can maintain these conditions become educated and wise person in the society.

Maintaining order in the society is another aspect of learning. The concept of order in pre-modern thought conveyed a strong feeling of closure and stasis. Its boundaries were fixed and unchanging. The individual was viewed in terms of these boundaries. Crossing boundaries or moving beyond one's assigned role or social class meant tempting fate and, in mythology, risking the wrath of the gods.

Socially and educationally, the closeness of this view meant that individuals were not to overstep their bounds or rise above their class. Developing social harmony and integration is the learning. It means that the pre-modern society is guided from some closed rule and regulations. With the above description, the pre-modern society was an ordered society which lay a belief in the reality of ideals.

The Plato's educational and social theories were based on each individual fulfilling a pre-set role for the common good. The roles for rulers, guardians, artisans were prescribed and nonvariant. The forms that these classes copied were absolute, permanent and unchanging. Plato believed, in the soul of the person that which partook of the world-soul. This view allowed Plato to think of knowledge as remembrance of past experiences. Then experiences from past were considered the sources of knowledge constructions and formations which is the fundamental aspects of the development of experiential learning theory.

In the similar vein, the method of questioning of Socrates was focused on recollection rather than being open ended and progressive. Its aim was to uncover eternal, pre-existing truths. This shows that the Socratic method of questioning became a method of learning in pre-modern society. Thus, it is concluded here is that the Socrates methods of questioning has become base for the development of dialogic method.

The pre-modern society believed on geo-centric universe which means the earth is the centre of the universe. Earth was round around which others had moved to make centre at origin. Thus, everything was considered circular and similarly learning was considered circular, repetitive and memorable past experiences. This method of learning is called traditional learning method where teacher is active, knowledgeable and discipline person who transmit their knowledge to students.

Modernism is the philosophical perspective which posits the positivistic reality, mechanistic cosmology and rational scientific methodology. The intellectual vision of modernism is predicted on positivistic certainty based on scientific method of discovery. The knowledge and reality are external which are independent from human experiences, perceptions and feeling. The modernists believe on the objective reality and scientifically derived knowledge. Thus the, there is strong separation between knowledge and knower. The tools for measuring knowledge should be valid, reliable, precise and measurable. The science and its theory became the guiding tools for the development of social, personal and intellectual vision. Reality is assumed universal and thus individual and personal knowledge need to correlate with this universal one. In the western intellectual thought of modern paradigm, the Newton and Descartes had played important roles.

Modernism developed definite social and epistemological visions as westerners, which are based on thought of Cartesian certainty (of Descartes) and Newtonian stability with the union of industrialization in seventeenth and eighteenth centuries. The intersection of these visions lay in the concept that improvement, progress, betterment for all would come through technology and right reason. These visions of social-epistemological metaphysical visions held sway during the nineteenth century and even well in to the twentieth century. The modernist has believed that certainty was attainable through right reason and that once attained it would be lasting. Once the real structures (such as of mathematics and the sciences, of social and psychological situations, or reality itself) were understood, the stability of the cosmos was such that one could be certain forever.

Moreover, it is Newton's metaphysical and cosmological views which had long prevailed in modern thinking, forming the basis in the social sciences for causal predictability, linear ordering and a closed approaches. These concepts were becoming the foundation for the scientific curriculum and the development of discovery methods of learning theory. It means that the modernism has believed on the discovery of knowledge rather than invention.

Causality is an important concept for modernists where they believed that for every effect there must be a priori cause; effect do not happen spontaneously and the same cause will always produce the same effect. These beliefs of modernism had become for the foundation of behaviorist learning theory. Within this scientific mechanistic paradigm, the behaviorists developed the learning theory within the principles of stimulus-response such as operant conditioning learning theory, Pavlov learning theory etc. of behaviorism.

The theory of measuring IQ (intelligence quotient) of students is also the product of modernist concepts which is used to ensure the quality of education in school level to colleges. The other learning theory such as problem-solving method of

George Polya, discovery method, experimental methods etc. are the concepts of learning/teaching emerged from the modernist perspective.

Descartes method of right reasoning became an ample tool for the attainment of certainty. His method focused for rightly conducting reason for seeking truth where faith in an external order. Descartes' statements also reflected to support Copernicus' positing of a sun-centered (heliocentric) universe. This had helped to change the philosophical thought of learning about the universe. In addition to these metaphysical concepts, Descartes' method of right reasoning in connection with Euclid's geometrical reasoning had provide the foundation for our modern system of deductive method of proving/learning mathematical theories.

The reality for post- modernist is not absolute and single, but it is contextual and local. That is realities are not certain and universal. This paradigm posits quite different social, personal and intellectual vision than pre-modernism and modernism. The intellectual vision of post-modernism is predicted on pragmatic doubts based on human experiences, reflections and local history. The knowledge comes from negotiations with ourselves, our concepts, our environments and others, which the process of reflexivity. Another process of generating knowledge is a dialogue between each other. True dialogue only exists if there is democratic discussion between dialoguers where communication is always two sided or multi-sided.

Such type of dialogue and communication can lead to different social vision which is applicable in teaching and research. Thus, it recognizes the right of others which rejects the concepts one best or right way. It acknowledges the uncertainty that comes with complexity and diverse viewpoints. It means that viewing the issue from different outlooks using different tools of measurement and interprets the result form contextual perspective. At the same time post-modernism seeks a selective but context-specific integration of subject and object, mind and body, curriculum and individual, teacher and student, as well as self and others. This integration depends in part on us and our actions. Active participation and actions to mediate the understanding in learning at the classroom is necessary. We all have responsibility for our futures as well as for the future of others. So, the vision of post-modernism may bring us to an ecological perspective and cosmology.

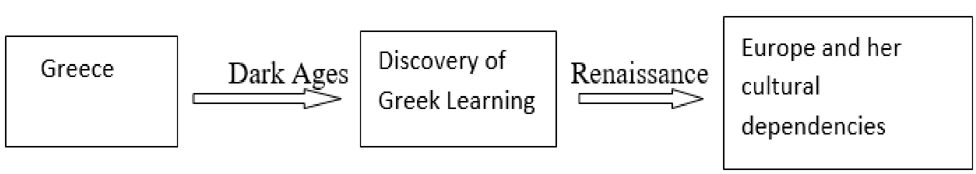
And within this perspective we may find a personal vision, on which helps us recognize that our sense of self and reality as independent object is meaningless. We are able to discern ourselves only in terms of others, reality only in terms of imagination. Both self and reality are found in relation.

Thus, the learning theory related to critical pedagogy may support the social, personal and intellectual vision of above post-modernism. This pedagogy enforces the dialogic communication among teachers and students in the classrooms. The teacher's role is to facilitate, motivate, empower and emancipate to the students from the perspective of social justice and use cultural friendly pedagogy to address the cultural aspects of learning. Similarly, the social constructivist learning theory as developed by Vygotsky is also underpins in this post-modernist intellectual and social thought.

Euro-centrism in Mathematics

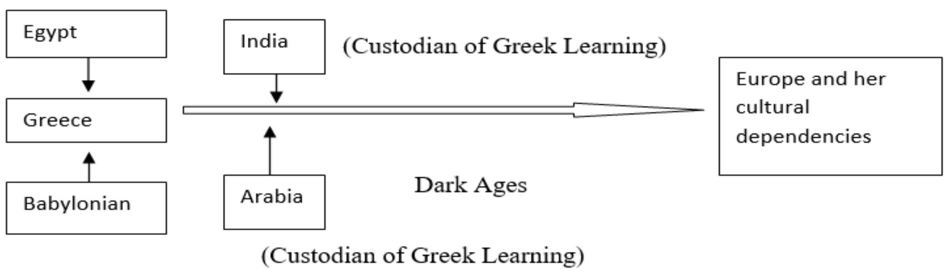
The mathematics as we know today is due to the contribution of European mathematicians from ancient period, specifically from Greek, in creation and development of mathematical knowledge in the history of mathematics indicated the Eurocentric view. Ernest (2009) has explained "this is euro-centrism, the raciest bias that claims that the European 'mind' and its cultural products are superior to those of other people and races" (p.198). According to Bernal (1987), over the past two centuries, Ancient Greece has been increasingly emphasized as the origin of modern European thought, while the Afro-Asiatic influences on classical civilization have been overlooked, dismissed and denied (as cited in Ernest, 2009, p.198). From these statements it is clear that European mathematicians are claiming the origin and roots of all mathematical knowledge are the product of the thought and works in European civilization. They rarely believe on the development of mathematical knowledge in other non-European civilizations. Eurocentric histories of mathematics often asserts that it was largely created by ancient Greeks. Their era concluded nearly 2000 years ago, after which came roughly 1000 years of Dark Ages. It was only with the European renaissance, sparked by the revival of Greek knowledge, that modern scientific and mathematical development emerged in Europe. This trajectory is shown in the following figure.

Fig.1. Eurocentric chronology of mathematics history



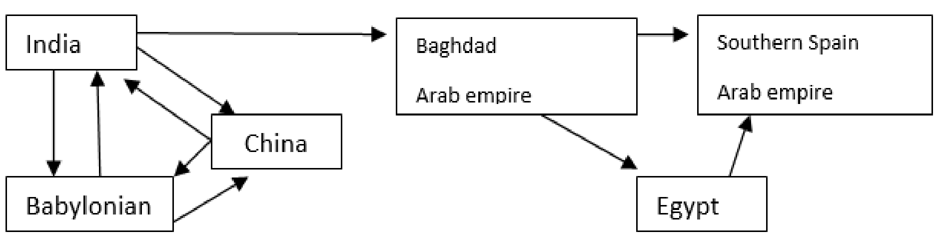
The above trajectory is called the traditional model of Eurocentric view of the development of knowledge of mathematics from ancient to till now. However, the following modified model of Eurocentric view is illustrated in fig.2 which acknowledge the influence of earlier Egyptian and Babylonian mathematics on the ancient Greek development, along with the later, often minimized contributions of Indian and Arabic mathematicians during the Dark Ages – who are frequently portrayed mainly as preservers of Greek knowledge – has often been overlooked in the history of mathematics in Europe.

Fig.2 Modified Eurocentric Model



Moreover, due to the so-called dark-age during the fifth to fifteenth century, the European did not do more contribution in the development of mathematics. But there were more contributions in the development of mathematics which was generally by non-European mathematicians. The following figure shows the trajectory of the development of non-European mathematicians during dark-age.

Figure 3. Non-European mathematics during the dark ages



The above diagram explains the development of mathematics during the dark ages in India, Babylonian, China, Arab, Egypt and then spread to southern Spain. But the Eurocentric histories of mathematics have been neglecting the non-European roots of mathematics development, explained by Joseph (2000) (as cited in Ernest, 2008, p.199).

Besides the above description, I am going to explain some contributions of mathematicians in Indian subcontinent, specifically the mathematics developed in Kerala. According to Ernest (2009) explained that the invention of zero by mathematicians of Indian subcontinent has long been acknowledged, even though the significance of this as the lynchpin of the decimal place value system is often underestimated (p.199). Pearce contends that the Indian invention of decimal numeration combined with the place value system represents not only the most extraordinary achievement in the history of mathematics but also one of the greatest intellectual accomplishments in the entire history of humanity. Also in Indian subcontinent, much attention has been given to very large numbers including power of ten up to near 50. Similarly, one may wonder whether the extension of the decimal place value system to include decimal fractions contributed to the conceptualization and development of the remarkable series expansions created in Kerala. It is also asserted that Kerala mathematicians used floating-point numbers to study the convergence of series. In fact, they discovered and developed numerous infinite series expansions, laying much of the groundwork for calculus, which is traditionally created to European mathematicians of the 17th and 18th centuries. But these remarkable contributions for calculus of Keralese mathematicians did work approximately 200 years ago than Newton and Leibnitz in Europe. The significant advancement from finite methods of ancient mathematics to the concept of limits, infinity, and the foundation of modern classical analysis is credited to Madhava of Sangamagramma (c.1340 – 1425), the Keralese mathematician-astronomer. He is also believed to have discovered many infinite series expansions for trigonometric functions, root expansions, and π, calculating its value accurately to 13 decimal places. Joseph (2000) claims that "we may consider Madhava to have the founder of mathematical analysis" (as cited in Ernest, 2009, p. 200). Some historians also contended that Kerala's contributions, particularly in the development of infinite series for numerical integration, preceded similar advances in Western European by several centuries. It is quite possible, they claim, that these discoveries were passed on to Europe by Jesuit missionaries and later claimed by European mathematicians as their own. While this evidence supporting this transmission and appropriation is compelling, it remains largely circumstantial.

Indian Mathematical Proofs

This is the sketch of the concepts of Indian mathematics development and notion of proofs or upapattis as described by Joseph (1993) in his paper. The word proof was used to mean a "thought experiment which suggest a decomposition of the original conjecture into sub conjectures or lemmas, thus embedding it in a possibly quite distinct body of knowledge" by Lakatos (1996). Thus, the proof has psychological, social and logical features. Psychologically, proofs need to lead the reader to agree with its findings. The system of symbols and the ways of arguments is formulated, organized, and presented determines whether the proof successes its tasks. Proofs establish particular properties of mathematical objects. Comprehending such statements requires education and practice, and for advanced mathematical concepts requires more and more training for the learners. Nowhere is this training more essential than in grasping the logical framework that underlies the proof. So, the proof needs to be logically also correct for understanding. The third feature of the proof is social and cultural. It means that the proof needs to be based on the intended audience and its culture, level etc. proofs are also context bound in terms of language, notation, reasoning and data according to the author of this article. Here, I consider the historical analysis of mathematical development in Kerala of South India as described by the Joseph (1993) in his article. According to him Kerelean mathematicians did work in infinite series and their finite approximation relating to circular and trigonometric functions in the fourteenth and fifteenth centuries. This occurred 200 years prior to the work of Newton, Liebniz and Gregory in Europe. The main driving force for Indian scholars was astronomical computations, which required highly accurate values for π as well as *sine* and *cosine* functions. Although Madhava's (c.1340 – 1425) proofs of these results may not meet the strict formal standards of modern proof, they are nonetheless valid. These types of proofs are called upapatti which are geometric and algebraic in nature. Today's we are not practicing such types of intuitive geometric proof in the world, but such type of proof may also be valuable in our classroom for the development of intuition.

In this paragraph, the Indian proofs (or Upapattis) of Pythagorean Theorem is described. The upapattis can be roughly translated as convincing demonstrations for each mathematical result by Indian mathematician around 2000 years ago. In fact, some of these upapattis were recorded by European scholars studying Indian mathematicians as late as the first half of 19^{th} century. For example, Colebrook (1817) gave the number of upapattis in footnotes from commentators which were the translation in English from the parts of works of Brahma Sputa Siddhanta of Brahmagupta (b. AD598) and of Lilavati and Bijaganita of Bhaskaracharya (b.AD1114). Similarly, Whish (1835) brought the work of Keralean mathematicians in infinite series for circular and trigonometric functions and revealed the sample of Upapattis related to Pythagorean theorem from Yuktibhasa (c.1550) of Indian mathematician's work. It is also the fact that the Indian astronomer of 15^{th} century, namely Bhaskaracharya and Niakantha wrote not only original treatises but also scholarly commentaries on their own writing. From their commentaries we can find the the detailed uppatis and processes of orginal texts as well as methodological and philosophical issues concerned by Indian mathematicians.

According to Ganesha, ganita is mainly of two types which are vykata and avyakta ganita. Vykata employs clear procedure well-known for general use and avykata (or bijganita) uses indeterminate in the process of solution. The unknown quantities were symbolized as yavat taval (for as much as), varna (for colours) and abbreviation such as ka (for kalaka or black), ni (for nilaka or blue) etc. One illustration of bijganita of Bhaskaracharya as follows.

Problem: Say what is the hypotenuse of a plane figure, in which the side and upright are equal to fifteen and twenty? And show the upapatti of the received mode of computation.

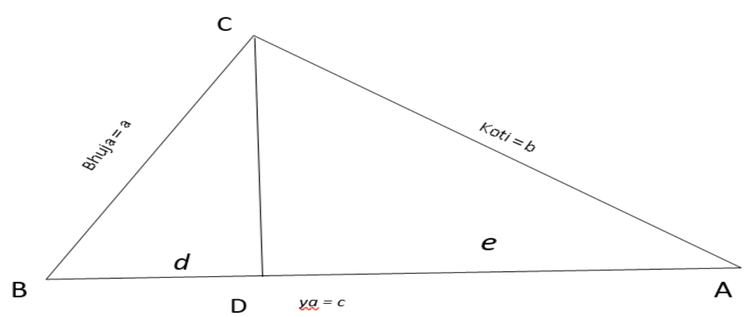
There are two-fold demonstrations for every such case: one is geometric (kshetra) the another is algebraic (avyakta). The geometric demonstration must be shown to those who do not understand the algebraic one, and vice versa.

The Upapatti for the Avyakta Method

Let the hypotenuse as the base and denote it by *ya* as shown in the diagram. Consoder *bhuja* and *koti* are respectively 15 and 20. Draw the perpendicular CD on AB which divides the triangle into two similar triangles. Now use the rule of proportion. When *ya* is hypotenuse, the *bhuja* which is now the segment of the hypotenuse to the side of the original *bhuja* will be $15^2/ya$. i.e. BD = $15^2/ya$. Similarly, AD = $20^2/ya$. Thus ya = $15^2/ya + 20^2/ya$. This gives *ya*, the hypotenuse is 25.

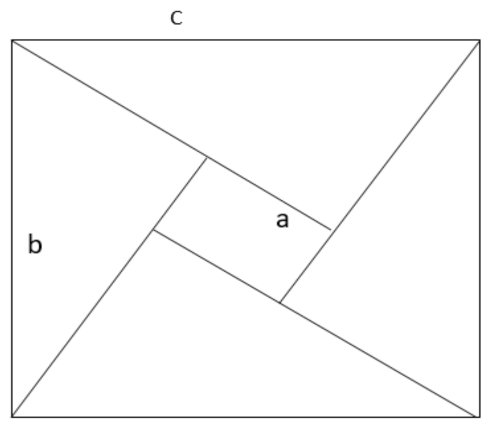
Modern Notation

Since CBD, BCA and CDA are similar triangle then $a/c = d/a \Rightarrow d = a^2/c$ and $b/c = e/b \Rightarrow e = b^2/c$. Thus $c = (a^2 + b^2)/c \Rightarrow c^2 = a^2 + b^2$. But $a = 15$ and $b = 20$. Therefore $ya = 25$



The Upapatti for the Kshetragata Method

ya = c



The above figure has four congruent triangles, each of which has area = (1/2) ab and the area of the interior square is $(b - a)^2$ and that of large square is c^2. Therefore, $c^2 = (b - a)^2 + 4(1/2)\, ab = a^2 + b^2$.

This shows that the notion of upapattis as given by Indian mathematicians have concept of proof that differs considerably from understanding found in Greek or even modern mathematical traditions. Saraswati Amma (1979) stated that the Indian aim was not to build up an edifice of geometry on a few self-evident axioms, but to convince the intelligent student of the validity of the theorem so that the visual demonstration was quite an accepted form of proof. Another key characteristic of that sets Indian mathematics apart from Greek mathematics is that the pursuit of knowledge for its own sake was not as appealing to the Indian mindset.

The upapatis are expressed in clear, precise language, outlining the key steps of the argument and highlighting the general principles used. With in this understanding, they are similar with proofs. But the proofs of Greek Mathematicians were more axiomatic, deductive, inductive as well as formal and informal which is similar to the modern tradition of proof. In fact, the Indian mathematicians did not much demonstrate for informal type of proof or upapatti. The Greek mathematicians view is that real mathematics is as formal derivations from formally stated axioms which linked with the western thought that mathematical knowledge is infallible and so they belief in absolute certainty of mathematical knowledge. This became a strong philosophical foundation of Greek mathematics.

The upapattis reveals that the epistemological position of Indian mathematics knowledge differs from that of Greek. Upapattis are required to get nirbhranta (free from misunderstanding) and to elevate the intellect (buddhi vriddhi). The indirect method was used only to show the non-existence of certain entities, for example, to show the negative number has no square root. Although, Indian mathematicians' proofs are not so logical and rarely accepting indirect method of proof, they adopted what is now called the constructivist perspective on the question of mathematical existence.

Gödel's Incompleteness Theorems

Absolutists have believed that mathematical knowledge are certain and unchangeable truths. They believed on absolute and objective reality of mathematical knowledge. Ernest (1991) stated that "mathematical knowledge is made up of absolute truths, and represent the unique realm of certain knowledge, apart from logic and statements true by virtue of the meaning of terms," is the absolutist view in the philosophy of mathematics (p.7). This view explains the epistemology of mathematics is made of with universal truths, in which some statements are true by virtue of the meaning of terms and others are established by logic and inferences. There are there schools of thought, namely, logicism, formalism and intuitionism (or constructivism), which are underpinning in absolutist view of mathematical knowledge. These schools have provided the foundations in the development of mathematics in the world.

According to Ernest (1991), the major proponents of logicism are Leibniz, Frege, Russell, etc. who claims all the concepts of mathematics can ultimately be reduced to logical concepts and all mathematical truths can be proved from the axioms and rules of inferences of logic alone (pp.8-9). So logicists have believed that pure mathematics is a part of logic. The another school of thought is formalist which has claimed that "pure mathematics can be expressed as uninterpreted formal systems, in which truth of mathematics are represented by formal theorems and the safety of these formal system can be demonstrated in terms of their freedom from inconsistency, by meta-mathematics"(Ernest, p.10).One of the major proponent of this thought is David Hilbert who established Hilbert formalist Program to developed these formal systems of mathematical knowledge. Intuitionist (or constructivist) school thought is another foundation for the absolutist view of mathematics, which claims that "both mathematical truths and existence of mathematical objects be established by constructive methods" (Ibid). Thus, constructivists have believed that all mathematical knowledge, are constructed by using constructive methods.

There are several criticisms about this old view of mathematical truths and knowledge and so it was challenged by mathematicians and philosophers including Kurt Gödel. The Australian mathematicians Gödel proved two theorems of mathematical logic in 1931, which are known as Gödel's incompleteness theorems in the history of mathematics. These theorems show the intrinsic constraints of all formal axiomatic system containing basic arithmetic. As stated in Raatikainen (2005), Gödel's first incompleteness theorems is "for any consistent formalized system *F*, which contains elementary arithmetic, there exist a sentence G_F of the language of the system which is true but unprovable in that system" and that of second is "no consistent formal system can prove its own consistency" (p.513). The second theorem means that if *S* is any consistent formal system, then it cannot be proving *S* by the axioms of *S*. Some properties of formal systems are completeness, consistency, and the existence of effective axiomatization. A formal system is deductive structure made up of a defined set of axioms and rules for symbolic manipulation or inference, which permit the derivation of new theorems from the axioms. For example, the Peano Arithmetic, Zermelo- Fraenkel set theory (ZFC) etc. A formal system is considered effectively axiomatized if its theorems from a recursively enumerable set. A set of axioms is consistent if there is no statement for which both the statement and its negation can be proven from the axioms; otherwise, it is inconsistent. A set of axioms is called complete if, for every statement expressed in the language of the axioms, either the statement or its negation can be proven from those axioms. The concept is central to Gödel's first incompleteness theorem and should not be confused with semantic completeness, which means that the axioms prove all the semantic tautologies in the language. Gödel's completeness theorem established that first order logic is semantically complete. However, it is not syntactically complete because there exist statements within first-order logic's language, such as "the flower is pretty" that cannot be proven or disproven solely based on the axioms of logic. For instance, the Euclidean geometry without the parallel postulate is incomplete, since the parallel postulate cannot be proven or disproven using the other axioms. The theory of first order Peano arithmetic is consistent, has an infinite yet recursively enumerable set of axioms, and contains sufficient arithmetic to satisfy the conditions required for the incompleteness theorem. The theorem provides a specific example of arithmetic statement that can neither proven nor disproven within Peano arithmetic. Ernest (1991) explains the notion of Gödel's first theorem which showed that not even all the truth of arithmetic can be derived from Peano's axioms (or any larger recursive axiom set) and second theorem showed that in the desired cases consistency proof require a meta-mathematics more powerful than the system to be safeguarded, which is thus no safeguard at all (pp.10-11).

Gödel's theorems hold great significance in both mathematical logic and the philosophy of mathematics. They are commonly seen as demonstrating that Hilbert's program, which aimed to establish a complete and consistent set of axioms for all mathematics, is unachievable. Feferman (n. d), viewed that the relevancy of Godel incompleteness theorems to mathematical logic is paramount; further their philosophical relevancy is significant and their mathematical relevancy out of logic is very much unsubstantiated (p.1). The incompleteness theorems concern formal provability within such systems, rather than informal notions of provability. They demonstrate that any system containing enough arithmetic cannot simultaneously have all three of these properties.

Gödel complained that the intuitionist notions of provability and constructivity are vague and indefinite and lack complete perspicuity and clarity (Raatikainen, 2005, P.4). It can be argued that Gödel's theorem presents a far greater challenge to many forms of intuitionism than has been acknowledged or recognized by intuitionist philosophers of mathematics. For the application of Gödel's theorem by Putnam (1967) (as cited in Raatikainen, 2005, P.4) consider the following principles which, in Putnam's words, "many people seem to accept":

(i) Even if some arithmetical (or set-theoretical) statements have no truth value, still to say that any arithmetical (or set-theoretical) statements that it has (or lacks) a truth value is itself always either true or false.

(ii) Only and all decidable statements possess a truth value.

Putnam demonstrates, using Godel's first theorem, that these two principles are mutually inconsistent.

Conclusions

The evolution of Western thought - from the essentialist harmony of the pre-modern era, through the mechanistic rationality of the modern age, to the relativistic pluralism of the post-modern paradigm - reflects profound shifts in how humanity conceives of knowledge, reality, and education. These paradigmatic transformations have not only redefined philosophical and scientific worldviews but have also deeply influenced the ways in which mathematics is taught and understood.

Challenging the traditionally Eurocentric history of mathematics, this overview emphasizes the pivotal contributions of Indian mathematicians, especially during Europe's so-called Dark Ages. The Kerala school's innovations in infinite series, calculus, and trigonometry demonstrate a sophisticated and intuitive mathematical tradition that predates similar developments in Europe. This constructivist and culturally embedded approach, centered on demonstrative reasoning (upapattis), contrasts sharply with the formalism of Greek logic, underscoring the global and diverse roots of mathematical thought.

Kurt Gödel's incompleteness theorems further deepen this philosophical inquiry by dismantling the notion of mathematics as a complete and self-contained system of absolute truths. His work reveals the inherent limitations of formal systems and challenges foundationalist ideals in logic and mathematics, reinforcing post-modern skepticism toward universal certainty.

Together, these narratives illustrate that mathematics is not merely a static body of truths but a dynamic, culturally influenced, and philosophically contested domain. Understanding its evolution requires moving beyond narrow, absolutist frameworks and embracing a broader, more inclusive and reflective vision of mathematical knowledge.

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