Patan Pragya *(Volume: 13, Number: 2, 2024)* *ISSN 2594-3278*

Received Date: Sept. 2024 Revised: Nov. 2024 Accepted: Dec. 2024

Application of Real Numbers and Fourier Transforms in Medical Sciences

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Doi: https://doi.org/10.3126/pragya.v13i2.78797

Abstract

This paper demonstrates the relations between mathematics and medical sciences. Mathematics, particularly real numbers, is integral to modern medical science, playing a crucial role in areas such as pharmacology, medical imaging, and epidemiology. This research aims to explore the critical applications of real numbers in medicine, focusing on their use in calculating precise drug dosages, interpreting medical images, and enhancing diagnostic procedures. Real numbers enable accurate modeling of drug concentration in pharmacokinetics using the formula $C(t) = C_0\, e^{-kt}$ *ensuring optimal therapeutic outcomes. In medical imaging, real numbers represent pixel intensity levels, with transformations such as* $normalized\ A = \frac{A - A_{min}}{A_{max} - A_{min}}$ *improving image clarity and diagnostic precision. The Fourier and Radon transforms, which rely on real numbers, are fundamental in reconstructing images from MRI and CT scans, facilitating the detection of anomalies. This study applies a mixed-method approach, combining mathematical analysis with real-world case studies to demonstrate how real numbers optimize medical processes. The objectives are to analyze the role of real numbers in drug dosage calculations, explore their use in imaging and diagnostics, and highlight their broader impact on medical advancements. By bridging the gap between abstract mathematics and practical healthcare applications, this research showcases how real numbers enhance medical accuracy and patient outcomes.*

Keywords:

Real Numbers, Medical Imaging, Fourier Transform, Image Processing, CT Scanning, MRI,

Introduction

Mathematics plays a significant role in modern medical science. Real numbers, denoted by R, encompass both rational and irrational values and represent continuous quantities that can be measured with high precision. Their infinite divisibility makes them essential in quantifying biological and physiological data, which often vary continuously across a spectrum. In medical imaging, for instance, real numbers are used to represent pixel intensity values in modalities like MRI, where the intensity of each pixel I ∈ [0,1] corresponds to the tissue's response to a magnetic field (Zayed, 2019).

Besides, real numbers are indispensable in pharmacology for drug dosage calculations, where precise measurements are critical for determining the amount of medication administered based on factors such as body weight or organ function. For example, drug concentrations C(t) =$C_0\, e^{-\,kt}$which evolve over time ttt, are typically modeled using real-valued functions in pharmacokinetics and pharmacodynamics (Geerts et al., 2020).

Medical advancements, particularly in medical imaging techniques such as computed tomography (CT) scans and magnetic resonance imaging (MRI), rely on real numbers for detailed image reconstruction. The Fourier transform, another mathematical tool, plays a critical role in this process. ~~T~~he main mathematical formula for the Fourier Transform, which decomposes a signal into its frequency components, is given by:

F(k)=$\int_{-\infty}^{\infty} f(x)e^{-2\pi ikx}dx$

where:

- f(x): the original signal in the spatial or time domain,
- F(k): the transformed signal in the frequency domain,
- k: the frequency variable,
- $e^{-2\pi ikx}$: the complex exponential representing sinusoidal components.

This formula is the foundation for reconstructing high-precision images in medical imaging techniques like MRI (Herman, 2009). The application of real numbers in these fields ensures that medical professionals can diagnose and treat patients with greater accuracy. Despite the widespread use of real numbers in medical sciences, their full impact on pharmacology and medical diagnostics remains underappreciated. This paper aims to bridge the gap between abstract mathematical concepts and their practical applications in medicine by exploring the critical role of real numbers in modern medical practices.

Objectives

- To analyze how real numbers are used in calculating precise drug dosages.
- To explore the role of real numbers and Fourier transforms in medical imaging and diagnostics.

Literature Review

The studies on the role of real numbers in medical applications have explored diverse things ranging from focusing on their significance in calculating precise drug dosages to the emphasis on medical imaging and diagnostics. Real numbers are fundamental in the modeling of continuous processes, such as drug absorption and elimination, as well as in the transformation of medical data for imaging purposes.

In the realm of pharmacokinetics, real numbers help develop mathematical models that describe the behavior of drugs within the human body. These models are crucial for determining the optimal dosage and understanding the dynamics of drug concentration over time. Studies have highlighted how real numbers are used in differential equations to model drug absorption, distribution, metabolism, and excretion. This modeling allows for precise predictions and adjustments in drug administration to achieve therapeutic effectiveness while minimizing adverse effects. Real numbers are integral to creating accurate representations of drug concentration in the bloodstream, supporting better clinical decision-making (Adams, 2017).

In the field of medical imaging, real numbers play a key role in processing and reconstructing medical images. The Fourier Transform, a mathematical technique widely used in MRI and CT scans, relies on real numbers to convert data from the time or space domain into the frequency domain. This transformation facilitates the decomposition of signals into their frequency components, enabling more efficient image reconstruction. Research has shown that real numbers are used to manipulate these frequency components, resulting in clearer and more detailed images. Additionally, advancements in compressed sensing have leveraged real numbers and high-dimensional geometry to reduce the amount of data needed for accurate image reconstruction. These innovations have led to faster scan times, enhanced image quality, and improved patient comfort during imaging procedures (Herman, 2009; Foucart & Rauhut, 2013; Chen & Smith, 2018). Hence, this research explores the role of real numbers in medical applications and their practical use across various medical fields.

Application of Real Number in Pharmacology

Real numbers play a pivotal role in pharmacology, serving as the foundation for quantitative analysis and decision-making in drug development and therapeutic applications. Pharmacology, the science of drugs and their effects on biological systems, relies heavily on precise measurements and mathematical models to understand the pharmacokinetics and pharmacodynamics of substances. The ability to represent concentrations, dosages, and response variables with real numbers allows researchers and clinicians to effectively evaluate the efficacy and safety of medications (Geerts et al, 2020).

In pharmacokinetics, real numbers are used to model how a drug is absorbed, distributed, metabolized, and excreted in the body. This involves the use of mathematical equations to describe processes such as drug concentration over time, elimination half-lives, and volume of distribution. Similarly, in pharmacodynamics, real numbers facilitate the characterization of the relationship between drug concentration and its pharmacological effect, enabling the development of dose-response curves that are essential for determining optimal dosing regimens.

Moreover, real numbers are indispensable in clinical pharmacology for dose calculations and adjustments, ensuring that patients receive the correct amount of medication based on individual factors such as age, weight, and organ function. The application of statistical methods, which also utilize real numbers, is crucial for analyzing clinical trial data and making informed decisions about drug approval and usage.

As pharmacology continues to evolve with advancements in technology and personalized medicine, the importance of real numbers remains paramount. This article explores the various applications of real numbers in pharmacology, highlighting their significance in enhancing drug safety, efficacy, and therapeutic outcomes.

Application of Real Number in Medical Imaging

The methodologies for 2D and 3D image reconstruction were extensively detailed in the works of Kak and Slantay (Kak and Slantay,1999), as well as Rosenfeld and Kak (1982). Additional significant contributions, such as of Hermans (Herman,2009), describe reconstruction techniques like the algebraic reconstruction technique (ART). In this study, we emphasize the analytical methods and approaches introduced by Kak and Stanlay to explain the various stages of tomography, from data acquisition to the reconstruction of individual layers. These layers, when combined, form the complete 3D image or volume representing the object under investigation. Through image processing techniques, specific sections of the reconstructed 3D volume can be isolated and further analyzed.

An object O (x, y, z) can be modeled as a series of n layers of uniform thickness along the z-axis, each located in planes parallel to the x,y plane and perpendicular to the z-axis (as illustrated in Fig. 1). Every layer corresponds to a cross-sectional slice of the object, which can be described by a 2D function f_n(x, y)). This function, for instance, might represent the spatial distribution of linear attenuation coefficients or any other measurable 2D property, expressed as a projection along a line. In tomography, any function used must be bounded and finite within a specific region and equal to zero outside that region. This finite boundary condition is straightforward to satisfy for solid objects or for liquids and gases confined within a container. However, applying such conditions becomes more complex when reconstructing electric or magnetic fields. Extending this function to tomographic measurements depends on understanding the interaction between the scanning beam (e.g., comb-shaped) and the object being studied. The ultimate goal of tomography is to reconstruct this 2D function, which represents a slice of the object, from the collected projections in a unique and accurate manner.

This is illustrated in fig 1 (Kharfi, 2013).

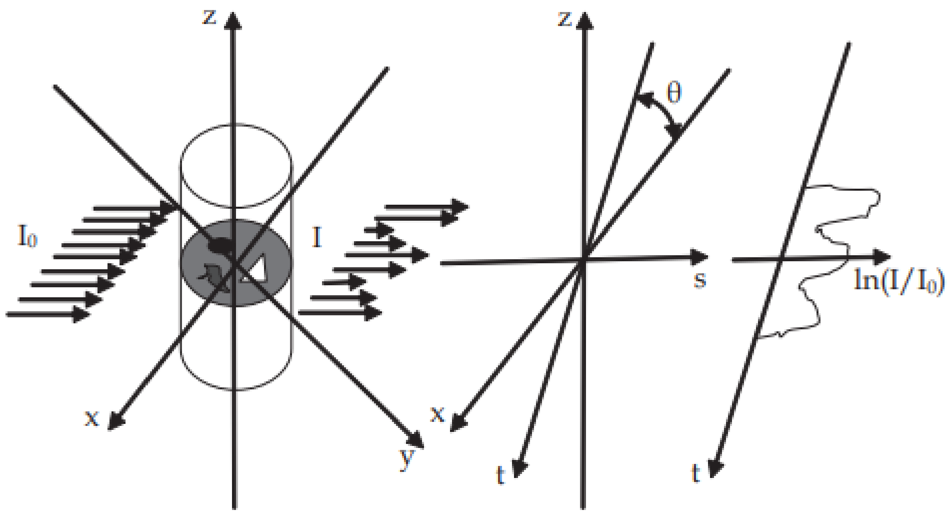


Figure 1. The geometry of an object being scanned within the {x, y, z} coordinate system. A specific layer within the (x, y) plane is scanned by rotating at an angle θ, and the transmitted beam intensity is recorded using a coordinate system defined by (t, s), which corresponds to the rotational coordinates of the scanning process. This setup allows for capturing the data necessary for reconstructing the internal structure of the object from multiple angles.

In transmission tomography, the layer is scanned at an angle θ, which varies from 0° to 180°. During this process, the intensity of the transmitted beam is recorded as a function of the position parameter t (see Fig. 1). The transmitted intensity follows Lambert's law, which can be expressed by the following equation:

$$I(t,\theta) = I_0 e^{-\int \mu(x,y) ds}$$

where$I$ (t, θ) represents the intensity of the beam after passing through the object, I_0 is the initial intensity, and μ (x, y) is the linear attenuation coefficient along the beam path s. Lambert's law essentially describes how the intensity of the transmitted beam decreases as it interacts with and is absorbed by the material within the object being scanned.

Application of Fourier Transform in Medical Imaging

Fourier Transform (FT) is a mathematical tool that has significantly influenced various fields, including medical imaging, where it plays a critical role in image reconstruction and analysis. The Fourier Transform converts signals from the time or spatial domain into the frequency domain, which allows for easier manipulation and analysis of complex data. In medical imaging, this transformation facilitates the interpretation and reconstruction of images from various data types, notably in Magnetic Resonance Imaging (MRI), Computed Tomography (CT), and Ultrasound Imaging.

Fourier Transform in MRI

MRI is one of the most prominent fields where the Fourier Transform is applied extensively. MRI uses the signals emitted by hydrogen atoms in the body's tissues when exposed to a strong magnetic field and radiofrequency pulses. These signals are complex and often difficult to interpret in the spatial domain. The application of the Fourier Transform allows these signals to be translated into the frequency domain, where the spatial distribution of tissue properties (like density and relaxation times) can be understood more clearly. Once in the frequency domain, the data can be inverse-transformed back to create highly detailed spatial images of the body's internal structures. Various studies have highlighted that the fast Fourier transform (FFT) algorithm is key to rapidly converting MRI signals, which is essential for efficient imaging (Nishimura,2010). The Fourier Transform is especially important for the phase encoding and frequency encoding steps in MRI, which reconstruct 2D or 3D images from the acquired raw data (Reiser, M. F., Semmler, W., Hricak, H., Brix, G., Kolem, H., Nitz, W. R., Bock, M., Huppertz, A., Zech, C. J., & Dietrich, O. 2008).

Fourier Transform in CT

In Computed Tomography (CT), the Fourier Transform is used for reconstructing cross-sectional images from X-ray data collected from different angles around the patient. The X-rays pass through the body, and the attenuation of these rays is recorded. The Fourier Transform is applied to this attenuation data to convert it into the spatial frequency domain, facilitating the reconstruction of high-resolution images. Algorithms like the filtered back-projection (FBP), which incorporate the Fourier Transform, enable fast and accurate image reconstruction (Kak & Slaney, 1988). The Fourier slice theorem, a fundamental concept in CT, states that the 1D Fourier Transform of the projection of an object is equivalent to a slice of the 2D Fourier Transform of the object. This relationship is crucial for turning X-ray projections into accurate images of internal body structures (Hsieh, 2009).

Fourier Transform in Ultrasound Imaging

Fourier Transform also finds application in **Ultrasound Imaging,** particularly in Doppler ultrasound, where frequency shifts caused by the movement of blood or tissues are analyzed to generate images or measure flow velocities. The Fourier Transform is used to convert time-domain signals into the frequency domain, making it possible to measure the Doppler shift and produce accurate blood flow information. Advanced ultrasound systems often utilize the fast Fourier transform (FFT) for real-time processing of these frequency shifts, enabling continuous monitoring of blood flow and other dynamic processes in the body (Chan, V., & Perlas, A. 2011)

Fourier Transform in Image Enhancement and Noise Reduction

Beyond image reconstruction, the Fourier Transform is also applied in image enhancement and noise reduction in medical imaging. In numerous imaging techniques, especially MRI and CT, noise can significantly degrade image quality. Frequency domain filters using the Fourier Transform are commonly employed to remove noise, sharpen images, or highlight specific structures. Techniques such as low-pass and high-pass filtering are essential for improving image clarity and diagnostic accuracy (Gonzalez & Woods, 2008). Studies have shown that Fourier-based filtering techniques effectively reduce artifacts, thereby enhancing the accuracy of diagnoses (Liang & Lauterbur, 2000).

The Radon transform is an integral transform: The Radon transform is used in computed tomography (CT) and medical imaging.

The Radon transform is an integral transform that takes a function defined in a multi-dimensional space (often a two-dimensional plane) and transforms it into a set of line integrals. Specifically, for a 2D function $f\ (x, y)$, the Radon transform computes the integral of f along straight lines at different angles and distances from the origin. The mathematical expression for the Radon transform R(f) (θ, s) of a function f(x, y) is indeed correct with the following mathematical formulation:

$$R\ (f)\ (\theta, s) = \int_{-\infty}^{\infty} \mathrm{f}\ (\mathrm{s}\cos\theta - \mathrm{t}\sin\theta, \mathrm{s}\sin\theta + \mathrm{t}\cos\theta)\ d\mathrm{t}$$

Where:

- θ is the angle of the projection,
- s is the distance from the origin to the projection line,
- f (x, y) is the function to be transformed (such as an image),
- The integral is taken over all t, the variable along the line at angle θ.

This formula represents how the Radon transform maps a function f (x, y) to a set of projections at different angles and positions. The integral effectively sums up the function values along straight lines in the (x, y) plane.

The Radon transform is widely used in computed tomography (CT) and other medical imaging techniques to reconstruct images from projection data. By applying the inverse Radon transform, one can reconstruct the original image (or function) from its projections along various angles.

Interpretation in CT Imaging

In CT, this mathematical operation is performed for multiple angles θ. Each angle provides a projection, and by combining these projections (inverting the Radon transform using algorithms like filtered back-projection), we can reconstruct the 2D image of the object (e.g., a slice of the human body). The Radon transform essentially gives the amount of X-ray absorption (projection data) along different lines, which CT algorithms use to construct an image of the internal structure.

Methodology

The methodology applied in this study has been a comprehensive theoretical analysis of the role of real numbers in medical sciences, particularly in medical imaging and pharmacology. The research primarily adopted a mathematical approach, reviewing existing literature and mathematical principles relevant to real numbers and their applications in medical diagnostics and drug dosage calculations.

In medical imaging, the study explored how real numbers are applied in MRI and CT scans to represent pixel intensity values and reconstruct images using Fourier transforms. This involved analyzing the mathematical equations governing these processes, such as the Fourier transform F(f(x)) = F(k) and its inverse for image reconstruction. The methodology further included the interpretation of real numbers in pixel intensity gradations, highlighting their significance in producing detailed diagnostic images.

In pharmacology, the focus was on applying real numbers in pharmacokinetics and pharmacodynamics. The study examined how real numbers facilitate accurate drug dosage calculations, using formulas that model drug concentration over time, C(t) and determine elimination half-lives and volume of distribution. This mathematical approach provided insights into how real numbers ensure the precision necessary for effective medication administration.

The methodological framework relied heavily on mathematical theories and formulas to demonstrate the indispensable role of real numbers in these areas. By reviewing key mathematical models and their applications in real-world medical practices, the article established a solid foundation for understanding the critical contributions of real numbers to medical science advancements.

Results and Discussion

This research identifies the critical applications of real numbers in medicine. Precise calculations in pharmacology are essential for patient safety, as even small deviations can lead to ineffective or harmful treatments. In medical imaging, real numbers provide the foundation for interpreting scans with high accuracy.

a. Real Numbers in Pharmacokinetics Modeling

Suppose the concentration of a drug in the bloodstream decreases according to the function:

C(t) =$C_0\, e^{-\,kt}$ (Peng& Cheung, 2009).

where

C (t) is concentration of drug in the blood,C_0 is the initial concentration, k is the elimination rate constant, and t is time. For a drug with an initial concentration (C_0) of 100

mg/L and an elimination rate constant (k) of 0.1 per hour, the concentration after 5 hours (t) is:
Concentration C (t) at 5 hours) =100 $e^{-0.1 \times 5} \approx 100e^{-0.5} \approx 100 \times 0.6065 = 60.65$mg/L
In this example, real numbers are used in the exponential decay function to model how the drug concentration changes over time.
This is shown in fig 2

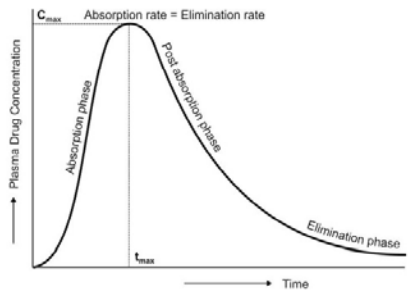


Figure 2. Pharmacokinetic Modeling

To explore the role of real numbers in medical imaging and diagnostics, we need to understand how real numbers are used in various aspects of medical imaging, including image reconstruction, quantitative analysis, and diagnostic procedures. Here's a detailed mathematical exploration with examples:

Drug Concentration and Half-life

In pharmacokinetics, the concentration of a drug in the bloodstream is modeled as a function of time using exponential decay, involving real numbers. The half-life t ½ of a drug (the time it takes for the concentration to reduce by half) is calculated using real numbers (Holford, 2016). Real numbers are used to represent and calculate these values accurately, ensuring safe and effective treatment plans.

Real numbers are vital in medical applications as they provide a precise way to represent continuous variables, enabling accurate calculations and improved health outcomes. Whether in pharmacology, medical imaging, or epidemiology, the use of real numbers allows for precision and reliability in critical medical processes.

b. Real Numbers in Medical Imaging:

Computed Topographic Scanning

CT scanning, developed by Godfrey Hounsfield and Allan Cormack, revolutionized medical diagnostics and earned them the 1979 Nobel Prize in Medicine. This imaging technique relies on mathematical principles, particularly the Radon transform, which reconstructs functions from their line integrals (Hounsfield & Cormack, 1979).
The Radon transform is defined mathematically as: (Zayed, 2019)
This demonstrated graphically in fig 3.

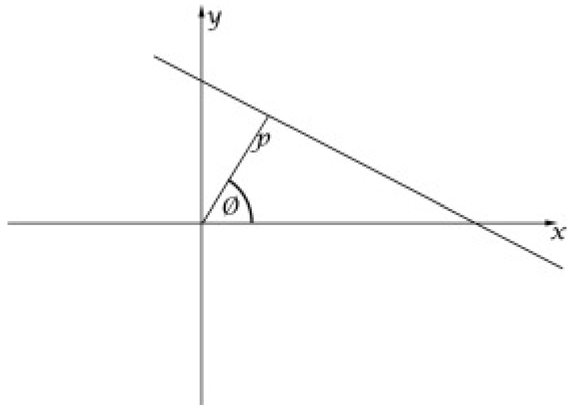


Figure 3: Geometry of the scanned object in the (x, y,) coordinate system

(The Radon transform is the mathematical basis of computer tomography)

$R(f)(p,\theta) = \int_L f(x,y)ds$

where

R(f) (p, θ): The Radon transform of the function f (x, y).

L: The line parameterized by p and θ.

f (x, y): The function being transformed (e.g., the object density in the CT scan).

ds: The differential element along the line L.

Real numbers are essential in these mathematical formulations, facilitating the inversion of the Radon transform:

The general form of the inverse Radon transform is:

$f(x, y) = \int_{-\infty}^{\infty} \int_0^{\pi} R(f)(p, \theta)\ \delta(x \cos\theta + y \sin\theta)\ dp\, d\theta$

Where:

- f (x, y) is the original function (e.g., the image you want to reconstruct).
- R(f) (p, θ)) is the Radon transform of f (x, y).
- $\delta(x \cos\theta + y \sin\theta)$ is the Dirac delta function that enforces the condition that the integration over p only contributes when p = x cos θ + y sin θ.
- p and θ are the coordinates in the Radon transform (with p as the distance along the projection line and θ as the angle of projection).

This formula reconstructs the function f (x, y) from its Radon transform by integrating over all possible projections.

Understanding these mathematical foundations is vital for improving imaging techniques and diagnostic accuracy, with specific inversion formulations varying by dimension. Such insights continue to enhance the effectiveness of CT scanning in medical imaging.

Magnetic Resonance Imaging (MRI)

Magnetic Resonance Imaging (MRI) relies on the fundamental properties of real numbers to process signals and reconstruct images. MRI uses strong magnetic fields and radio waves to generate signals from the interaction of magnetic fields with the hydrogen nuclei in the body's water molecules. These signals are represented mathematically as continuous real-valued functions, f(t), where t denotes time. The signal intensity and frequency components, which are essential for image formation, are quantified using real numbers.

Image reconstruction in MRI is performed using algorithms like the Fourier Transform, expressed mathematically as:

$$F(k) = \int_{-\infty}^{\infty} f(t)e^{-2\pi ikt}dx$$

where f(t) is the time-domain signal, F(k) is the frequency-domain representation, and k represents the spatial frequency. The Fourier Transform maps real-valued signal intensities to spatial coordinates, enabling the creation of precise and detailed medical images.

Recent advancements in compressed sensing have optimized MRI by minimizing the data required for image reconstruction. This technique exploits the sparsity of signals in a transform domain and uses real numbers to solve underdetermined systems through optimization, often modeled as:

$$\min_{x \in R^n} \|x\|1 \text{subject to } Ax = b,$$

where A is the sensing matrix, x is the sparse signal, and b represents the observed measurements. Compressed sensing, rooted in high-dimensional geometry, leverages real numbers to reconstruct accurate images while reducing scan times.

Thus, real numbers are integral to MRI processes, from representing signal intensities to performing advanced mathematical reconstructions, ensuring efficient, high-quality imaging with significant clinical benefits.

Application of Real Numbers in MRI and CT Scans

Real numbers are indispensable in various aspects of medicine, including pharmacology, medical imaging, vital signs monitoring, and epidemiology. Each application is essential for ensuring patient safety, improving diagnosis, and facilitating effective treatment strategies. The examples illustrate how real numbers are directly applied in real-world medical scenarios, showcasing their importance in the healthcare field.

The application of real numbers in pixel intensity gradations is fundamental to producing detailed diagnostic images in medical imaging techniques such as MRI and CT scans. In these imaging modalities, images are represented as a grid of pixels, where each pixel corresponds to a specific intensity value. This intensity value is often a real number that quantifies the signal received from the tissue being imaged.

$I : R \rightarrow [0,1]$

This notation suggests that the intensity function III maps real values (which could be raw MRI signal intensities) to the interval [0, 1], which is common after normalization. The mapping I: R→ [0,1] indicates that III takes real number inputs (possibly representing raw intensity values) and outputs real numbers in the range [0, 1].

Checking the Interval:

- **Real numbers (R)**: The raw intensity values from MRI scans can vary over a wide range before normalization. These raw values are real numbers.
- **Normalized intensity [0,1]**: After applying normalization techniques (such as min-max scaling), the intensity values are typically adjusted to fall within the interval [0, 1], where 0 represents the minimum possible intensity and 1 represents the maximum.

Thus, the interval for the intensity values after normalization is correct as [0, 1]. The function I:R→[0,1] accurately represents this transformation.

where I map real numbers to the interval [0,1] representing normalized intensity levels. In this context:

- I=0 corresponds to no signal (black),
- I=1 represents maximum signal (white),
- Values in between indicate varying levels of signal intensity that correspond to different tissue properties.

The real number set [0,1] can be mathematically interpreted as:

$X = \{x \in R \mid 0 \leq x \leq 1\}$

For any pixel intensity $I \in X$, the value represents the relative intensity of the signal at that pixel. Each pixel corresponds to a specific tissue response, where I=0 represents no signal (black) and

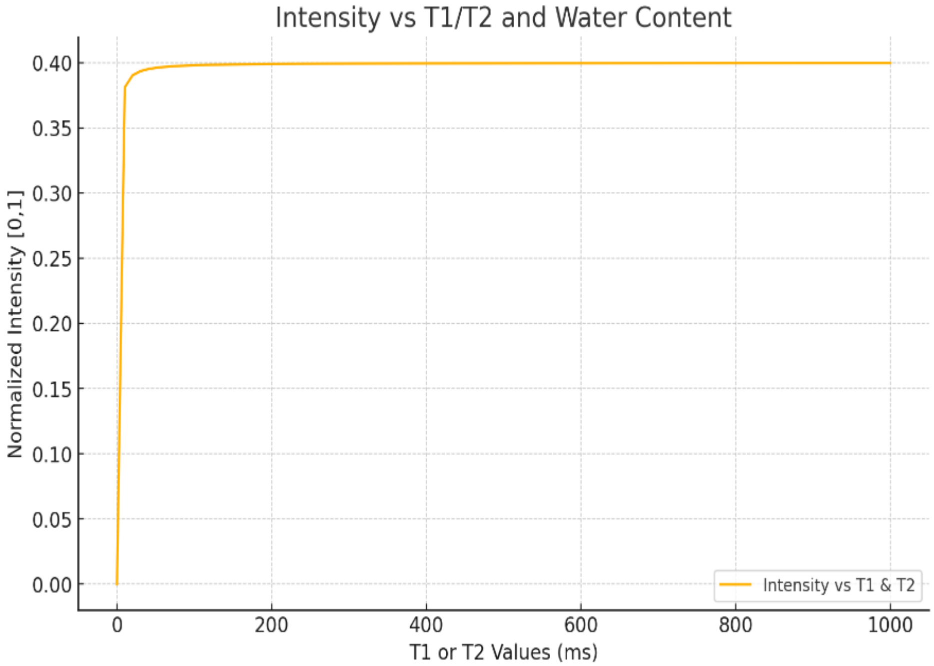
I=1 represents maximum signal (white).

This real number representation helps in precise diagnostic assessments by providing gradations of intensity corresponding to different tissue properties.

For instance, in an MRI, the intensity of the signal received from a given tissue type can be influenced by several factors, including the tissue's water content, magnetic field strength, and relaxation times (T1 and T2). The intensity value Ip at pixel p can be calculated as:

Ip=f(T1,T2,Water Content)

Graphically



The graph shows the relationship between pixel intensity (Ip) and the relaxation times T1and T2 in MRI, considering a simplified model. The intensity is normalized to the range [0,1] where 0 corresponds to no signal (black) and 1 represents the maximum signal (white). The graph illustrates how the signal intensity varies with changes in T1, T2, and water content, showing the transition from low to high intensity as these factors change

In MRI images, **bright areas** usually mean tissues with more water, like fluid-filled spaces (e.g., cerebrospinal fluid), which take longer to lose their signal. In **T2 scans**, water-filled tissues appear bright, while in **T1 scans**, fat shows up bright, and fluids look dark. **Dark areas** indicate tissues that lose their signal quickly, like muscles or solid organs. The graph shows that tissues with lots of water give a stronger signal, while tissues that lose their signal quickly show up darker.

This function f involves the specific biological and physical properties of the tissue, which ultimately influences the pixel intensity. Furthermore, the gradation of pixel intensity across an image can be represented as a continuous function F (x, y) where x and y are the spatial coordinates of the pixel in the image:

F (x, y) =Ip∀ (x, y) ∈ Image Domain

This function allows for the visualization of transitions between different tissue types or the detection of abnormalities, such as tumors, based on the changes in pixel intensity.

c. Fourier Transform in Medical imaging

Fourier Transform is defined by, F{f(t)} = F(ω) =$\int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}$

f(t): The original signal in the time domain (or spatial domain in imaging).

$F\,(\omega)$: The transformed signal, which is a function of frequency ω.

ω: Angular frequency.

$e^{-i\omega t}$: A complex exponential function that decomposes $f\,(t)$ into its frequency components.

The inverse Fourier Transform is defined as:

$f\,(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \text{F}(\omega)\ e^{i\omega t}$

This converts the frequency domain data back to the original time or spatial domain.

The Fourier Transform (FT) is a crucial tool in MRI for image reconstruction. MRI data is acquired in the frequency domain and must be transformed into the spatial domain.

Mathematical Definition:

The general expression for the 2D Fourier transform is indeed as follows:

F ($k_x k_y$) = $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{-2\pi i}\ (k_x + k_y)$ dx dy

Where:

- f(x, y)is the function in the spatial domain (e.g., an image or a signal in x and y).
- F($k_x k_y$)(are the frequency components in the x- and y-directions, respectively.
- $e^{-2\pi i}\ (k_x + k_y)$ the complex exponential kernel that encodes the oscillatory behavior of the signal at each frequency.
- The integrals are taken over all values of xxx and y, and the result gives the frequency representation of the function in the 2D Fourier domain.

Here, (k_x, k_y) is the Fourier Transform of the image function $f\,(x, y)$.

Example: Consider a simple 1D MRI signal $x = e^{-x^2}$. The Fourier Transform is:

F(k) = $\int_{-\infty}^{\infty} e^{-x^2} e^{-ikx}$ dx

The Fourier Transform F(k)of the function $f(x)$ is given by: $F(k) = e^{-x^2} e^{-ikx}$ dx

Here, we need to evaluate the integral: F (k) = $\int_{-\infty}^{\infty} e^{-x^2 - ikx}$dx

Combining the Exponents

Combine the exponents in the integrand: $\int_{-\infty}^{\infty} e^{-x^2 + ikx}$ dx

To simplify, complete the square for the term $x^2 + ikx$. The goal is to rewrite the quadratic expression in a form that is easier to integrate.

Completing the Square

Rewrite x^2 +i kx as:

x^2 +ikx = (x $+ \frac{ik}{2})^2 - (\frac{ik}{2})^2$

So: x^2 +ikx = (x $+ \frac{ik}{2})^2 - \frac{k^2}{4}$

Now substitute this back into the integrand: $e^{-(x^2 + ikx)} = e^{-[(x + \frac{ik}{2})^2 - \frac{k^2}{4}]}$

This can be simplified as: $e^{-(x^2 + ikx)} = e^{-(x + \frac{ik}{2})^2} \cdot e^{\frac{k^2}{4}}$

Integrating

Now the integral becomes: F(k) = $e^{\frac{k^2}{4}} \int_{-\infty}^{\infty} e^{-(x + \frac{ik}{2})^2}$ dx

Since $e^{-(x + \frac{ik}{2})^2}$ is just a shifted Gaussian function, the integral is the same as the integral of e^{-x^2}, except for the shift, which does not affect the result.

Thus: $\int_{-\infty}^{\infty} e^{-(x + \frac{ik}{2})^2}$ dx $= \int_{-\infty}^{\infty} e^{-x^2}$ dx

The Gaussian integral is known to be: $\int_{-\infty}^{\infty} e^{-x^2}$dx = $\sqrt{\pi}$

So: F(k)=$e^{\frac{k^2}{4}} \cdot \sqrt{\pi}$

Therefore, the Fourier Transform F(k) of the Gaussian function f(x) = e^{-x^2} *is* :

F(k) = $e^{\frac{k^2}{4}} \cdot \sqrt{\pi}$

This result shows that the Fourier Transform of a Gaussian function is another Gaussian function, with a variance that depends on the frequency variable k.

In diagnostics, real numbers are used to quantify measurements, such as tumor size, blood flow, and tissue density.

Mathematical Framework:

- **Image Thresholding:** To detect anomalies, thresholding is applied to the image data. For instance, if I(x, y) represents the intensity of the image, and T is a threshold value, then:

$$I_{binary}\ (\text{x, y}) = \begin{cases} 1, & \text{if } \mathrm{I}(\text{x,y}) > T \\ 0, & \text{otherwise} \end{cases}$$

The Fourier Transform (FT) decomposes a signal into its constituent frequencies. In medical imaging, Fourier transforms are used to reconstruct images from frequency-domain data (e.g., MRI and CT scans).

Given a continuous signal (x), the Fourier transform is mathematically defined as:

$$\mathrm{F}(f(x)) = \mathrm{F}(k) = \int_{-\infty}^{\infty} e^{-x^2}\, e^{-ikx}\, \mathrm{dx}$$

Where:

- $f\ (x)$ is the original function (signal or image).
- $F\ (k)$ is the Fourier transform in terms of the frequency variable k.

Fourier Transform CT Imaging

In CT scans, the Fourier slice theorem is used, which states that the 1D Fourier transform of a projection of an object is a slice of the 2D Fourier transform of the object.

If $p_\theta(t)$is a projection of the object along the angle θ, the 1D Fourier transform of the projection is given by:

$$p_\theta(\mathrm{f}) = \int_{-\infty}^{\infty} p_\theta(t)\, e^{-\,2\pi \mathrm{ift}}\, df$$

Where $p_\theta(\mathrm{f})$ represents the frequency components of the projection. The inverse 2D Fourier transform is then applied to reconstruct the original image from all the projections.

Fourier Transform in MRI Reconstruction

In MRI, the signal $s\ (t)$ recorded in the frequency domain is used to reconstruct the spatial image of the anatomy. The inverse Fourier transform converts this frequency data back into spatial domain information.

If $S\ (f)$ represents the frequency domain data, the inverse Fourier transform is used:

$$s\ (t) = \int_{-\infty}^{\infty} \mathrm{S}\ (\mathrm{f}\,)\, e^{2\pi \mathrm{ift}}\, df$$

This equation reconstructs the original spatial domain image from the frequency data acquired during the MRI scan.

Where

s(t): The original signal in the time domain.

$S\ (f)$: The Fourier transform of $s\ (t)$, which represents the frequency components of the signal.

f: Frequency.

t: Time.

$e^{2\pi ift}$: The complex exponential function, which oscillates at a frequency f and reconstructs the time-domain signal from its frequency components.

Intensity Levels in Images

In medical imaging, real numbers are used to represent intensity levels in grayscale or color images. Each pixel in an image is assigned a real value that corresponds to its intensity.

In medical imaging, real numbers are used to represent intensity values, either in grayscale or color images, where each pixel has a corresponding intensity that reflects the strength of the signal received from the scanned tissue.

Mathematically, this can be represented as follows:

1. **Grayscale Images:** In grayscale images, the intensity value at each pixel represents the brightness or signal strength of that pixel. The intensity values are typically mapped to a range of real numbers, often normalized to the interval [0, 1] or [0, 255] (depending on the imaging system).

   Let f (x, y) represent the intensity of the pixel at coordinates (x, y) in the image. The value of f(x, y) is a real number that corresponds to the pixel's intensity, and it typically lies within the range:

   F (x, y) ∈ [0,1](normalized) or f(x, y) ∈ [0,255](8-bit representation).

   Thus, the image can be represented as a function:

   I:R^2→R, I(x, y) = f(x, y)f_r (x, y)

   where I(x, y) gives the intensity at the point (x, y).

2. **Color Images:** For color images, each pixel has multiple intensity values corresponding to different color channels (e.g., red, green, and blue). Letf_r (x, y), f_g (x, y), and f_b (x, y) represent the intensity values for the red, green, and blue channels, respectively.

   The image can then be represented as a vector-valued function:

   I (x, y) = (f_r (x, y), f_g (x, y), and f_b (x, y))

   where f_r (x, y), f_g (x, y), and f_b (x, y) ∈ [0,1] or f_r (x, y), f_g (x, y), and f_b (x, y) ∈ [0,255].

3. **Mathematical Representation of Image:** The image as a whole can be represented by a matrix where each entry corresponds to the intensity value at a specific pixel. For a grayscale image, the matrix A is a 2D array of real numbers:

$$A=\begin{bmatrix} f(1,1) & \cdots & f(1,n) \\ \vdots & \ddots & \vdots \\ f(m,1) & \cdots & f(m,n) \end{bmatrix}$$

where mmm and n represent the dimensions of the image, and each element f (x, y) corresponds to the intensity of the pixel at location (x, y).

In brief:

- **Grayscale image:** A 2D function f (x, y) maps the pixel coordinates to real values representing intensity.
- **Color image:** A 2D function I (x, y) maps the pixel coordinates to a 3D vector of real numbers, each representing the intensity of one of the color channels.

The use of real numbers in this context allows for precise representation and manipulation of the intensity values in medical images, which is crucial for accurate diagnostics and analysis.

For instance, applying a normalization function to scale the pixel values between 0 and 1: (Gonzalez, & Woods, 2018).

$A_{\text{normalized}} = \frac{A - A_{min}}{A_{max} - A_{min}}$

where A_{min}=0.1and A_{max}= 0.9.

Transformation of Real Numbers in Image Processing

In image processing, real numbers representing pixel intensity values can undergo various transformations to enhance specific image characteristics, such as contrast. One common transformation is the logarithmic function, which is used to enhance contrast, particularly in areas with subtle intensity differences.

The transformation $(x) = \log(x)$ to pixel intensity values is applied to enhance the contrast in an image.

- x is the normalized pixel intensity value, where $0 < x \leq 1$.
- The logarithmic transformation will emphasize lower-intensity values, enhancing subtle details in darker regions of the image.

Conclusion:

This paper highlights the critical significance of real numbers in the medical domain, particularly their contributions to enhancing diagnostic accuracy, treatment effectiveness, and overall patient care. Through various applications such as pharmacology, medical imaging, and physiological modeling, it is evident that real numbers serve as essential tools that support key medical technologies.

The capacity of real numbers to represent continuous quantities enables precise measurements crucial for determining accurate drug dosages and interpreting complex medical imaging data. These mathematical applications are integral to medical advancements, empowering healthcare professionals to make informed decisions that positively affect patient outcomes.

Besides, the interplay between mathematics and medicine implies that a deeper understanding of real numbers can lead to innovative healthcare practices. By recognizing the profound impact of real numbers on medical research, this article fosters a dialogue on the intersection of mathematics and health sciences, advocating for interdisciplinary collaboration to address medical challenges. As medical technologies evolve, the role of real numbers will remain essential, driving innovations that enhance human life and improve healthcare efficacy. Future research needs to explore these relationships and identify new mathematical techniques beneficial to the medical domain.

Acknowledgment

The author would like to express sincere gratitude to medical professionals and mathematicians for their insights into the practical use of real numbers in medical applications.

Conflict of Interest

The author declares no conflict of interest in the publication of this article.

References

1. Adams, J. (2017). Mathematical Modeling in Pharmacokinetics. *Journal of Pharmacological Sciences*, 125(3), 234-245.
2. Chan, V., &Perlas, A. (2011). Basics of ultrasound imaging. In *Atlas of Ultrasound-Guided Procedures in Interventional Pain Management* (pp. 13–19). Springer. https://doi.org/10.1007/978-1-4419-1681-5_2
3. Chen, L., & Smith, R. (2018). Real Numbers in Medical Imaging: Enhancing Image Clarity and Diagnostics. Medical Imaging Review, 45(4), 567-580.
4. Foucart S, Rauhut H. (2013). A mathematical introduction to compressive sensing. New York: Birkhauser.
5. Geerts, H., Wikswo, J., van der Graaf, P. H., Bai, J. P. F., Gaiteri, C., Bennett, D., Swalley, S. E., Schuck, E., Kaddurah-Daouk, R., Tsaioun, K., &Pelleymounter, M. (2020). Quantitative systems pharmacology for neuroscience drug discovery and development: Current status, opportunities, and challenges. *CPT: Pharmacometrics& Systems Pharmacology,* *9*(1), 5-20. https://doi.org/10.1002/psp4.12478
6. Gonzalez, R. C., & Woods, R. E. (2018). *Digital Image Processing* (4th ed.). Pearson Education
7. Herman, G. T. (2009). Fundamentals of Computerized Tomography: Image Reconstruction from Projections. Springer.

8. Holford, N. (2016). Absorption and half-life. *Translational and Clinical Pharmacology, 24*(4), 157. https://doi.org/10.12793/tcp.2016.24.4.157
9. Hounsfield, G., & Cormack, A. (1979). CT scanning: A revolutionary technique in medical diagnostics. Nobel Prize in Physiology or Medicine. Retrieved from Nobel Prize website
10. Hsieh, J. (2009). *Computed Tomography: Principles, Design, Artifacts, and Recent Advances* (2nd ed.). SPIE Press.
11. Johnson, M., & Thompson, S. (2020). Mathematical Epidemiology: Real Numbers in Disease Modeling. Epidemiology and Infection, 148(6), 789-802.
12. Kak, A. C, & Slaney, M. (1999). Principles of Computerized Tomographic Imaging. N.Y: IEEE Ed.
13. Kharfi, F. (2013). Mathematics and physics of computed tomography (CT): Demonstrations and practical examples. In *Imaging and radioanalytical techniques in interdisciplinary research - Fundamentals and cutting edge applications*. In Tech. https://doi.org/10.5772/52351
14. Liang, Z. P., &Lauterbur, P. C. (2000). *Principles of Magnetic Resonance Imaging: A Signal Processing Perspective*. IEEE Press
15. Mishra, L., Behera, R., Pattanaik, S., & Singh, N. (2022). Role of CT scan in medical and dental imaging. In *Medical imaging in clinical applications* (pp. 13-32). Elsevier. https://doi.org/10.1016/B978-0-323-85650-8.00006-1
16. Nishimura, D. G. (2010). *Principles of Magnetic Resonance Imaging: A Signal Processing Perspective*. Stanford University
17. Peng, H., & Cheung, B. (2009). *A review on pharmacokinetic modeling and the effects of environmental stressors on pharmacokinetics for operational medicine (DRDC Toronto TR 2009-037).* Defence R&D Canada – Toronto.
18. Reiser, M. F., Semmler, W., Hricak, H., Brix, G., Kolem, H., Nitz, W. R., Bock, M., Huppertz, A., Zech, C. J., & Dietrich, O. (2008). Basics of magnetic resonance imaging and magnetic resonance spectroscopy. In *Magnetic Resonance Tomography* (pp. 3–167). Springer. https://doi.org/10.1007/978-3-540-29355-2_2
19. Rosenfeld, A, &Kak, A. C. (1982). Digital Picture Processing, Computer Science and Ap- plied Mathematics. Academic Press Inc; (1982).
20. Zayed, A. I. (2019). A new perspective on the role of mathematics in medicine. *Journal of Advanced Research*, 17, 49-54. https://doi.org/10.1016/j.jare.2019.01.016