Learning Mathematics through virtual mode: A review of literature

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Abstract
This paper identified the gap in existing literature in virtual learning environment in Mathematics. About twenty-five journal articles, two e-books, one conference papers and seven dissertations had been collected from different websites and electronic databases like as; Research gate, Elsevier, Wolfram Alpha, Directory of open access journals, Semantic scholar, Pro Quest Dissertations & Theses Global, Google Scholar, ERIC and Science Direct. The collected literature was reviewed thematically. The literature indicated that virtual learning is the useful learning process and an opportunity for higher education as well as school level to continue formal education from home, although it has many challenges. Although there are fewer numbers of studies on the virtual learning environment in Mathematics, some studies emphasise combined use of virtual manipulative and physical manipulative to increase the mathematical ability. Also, some literature indicates that online learning is more effective than traditional classroom learning but the rate of completion online mathematics course is lower than the classroom learning.

Keywords: Mathematics learning, virtual mode, perception, challenges, digitalization, synchronous

Introduction
Learning is the relatively permanent change in behaviour through experiences and training. It is a lifelong process. For learning, the experiences need not to be recent. Learning is a process that leads to change, which occurs as a result of experience and increases the potential for improved performance and future learning (Ambrose, Bridges, DiPietro, Lovett, & Norman, 2010). Learning occurs throughout life. Following the Bruner’s theory, learning is an active process. Learners interpret their situation through the similarities and the differences between their previous ideas and the present situation. A person’s socio-cultural background and the environment play a vital role in constructing knowledge (Pagan, 2006).

There are three main ways of acquiring knowledge: formal, non-formal and informal. Eaton (2010) defines the three ways of acquiring knowledge as formal learning is organised and guided by the formal curriculum and government at some level. Non-formal learning is usually organised to enhance certain knowledge, skill and capacity for some specific area in a short period. It may or may not be guided by the curriculum. But an informal learning is an
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accidental procedure that is possible anywhere at any time. Formal learning was considered as the face to face learning in the beginning. The procedure of acquiring knowledge in schools and colleges within classroom was called the formal learning because there were no alternative ways of formal learning. Nowadays, formal learning has been possible from home through electronic devices, software and the Internet.

Mathematics Learning

Mathematics learning depends on the active mental process of the learners and the learning environment. Interaction with mathematics teachers and the peers also affect to understand the new concepts. Lefa (2014) states that Mathematics is difficult directly from the everyday environment but it is possible through the other mathematicians and one’s own reflective intelligence. It means appropriate learning environment, the role of mathematics teacher and the interaction with other mathematicians are important factors in mathematics learning. Constructing relationship, extending and applying mathematical knowledge, reflecting experiences, articulating the knowledge and making new mathematical knowledge one’s own are the five forms of mental activities to develop mathematical understanding. Mathematical learning depends on the cognitive development of a child. A child can understand new mathematical concepts on the basis of previous knowledge and the age maturity. Piaget (1983 as cited in Lefa, 2014) argues that teacher should provide the opportunities to the learners to experience themselves. He further emphasises that the use of concrete materials and the hands on activities play a vital role in mathematics learning. Quilter and Haper (1988) stated that there are three main causes of difficulty in mathematics learning which are students’ intelligence, negative attitude towards the mathematics and the influence of learning environment. Fraser (1998) also focused on the influence of the learning environment in the mathematics achievement. He further emphasises that mathematics learning environment refers to a given space for the learning process in the social, psychological and pedagogical contexts which can influence students’ mathematics achievement. Teachers should create the environment and provide sufficient opportunity to the learners to classify and compare the new mathematical concepts to the previous concepts.

In the context of Nepal, SEE result indicates the failure rate of students in Mathematics every year. Most of the students feel mathematics as a difficult and abstract subject. Khanal(2015) mentions the cause of above two problems may be the mismatch between the teachers’ teaching strategy and the learners’ learning strategy in Mathematics. If the teachers know the mathematics learning strategies of their students, definitely they can change their teaching strategies to make mathematics learning effective and interesting.

Virtual Learning

With the rapid development of technology, e-learning has come into the practice for the past two decades (Pappas, 2015). He further argues that at the beginning of 21stcentury, technology is rapidly changing and different organisations and learning institutions have started a blended learning approach. Developed countries in the world had practised ICT integrated approach in learning from the beginning of 2000. After the worldwide terror of Coronavirus (Covid-19), most of the developing countries started online mode of learning from school to university level. Mostly, distance learning through radio and television, online learning via the Internet and virtual classes through different software like Zoom, Google
meet, Skype and Team has come into practice at the university level as well as at the school level in Nepal.

Virtual learning is a technology-based learning strategy that occurs through different software via the Internet. Siemens (2004) suggested that the virtual learning situation occurs through network connection as individuals share their interests, knowledge, perspectives, expertise, and opinions through synchronous and asynchronous learning strategies. Similarly, Joseph and Ekemini (2014) stated that the Virtual Learning Environment (VLE) is human-directed with the machine processing process, enabling learners to participate from a distance in both synchronous and asynchronous modes. Therefore, it is a web-based teaching-learning platform to ensure all sorts of teaching-learning components and activities with content management, learner engagement, collaboration and assessment.

VLE is a means by which ICT enables educational statements between the teacher and the student during the teaching-learning process, accompanied by promoting the subject matter’s self-construction. The design of a VLE should include these five aspects knowledge (design of interactive digital content with pedagogical perspective), collaboration (student-student, student-teacher and teacher-teacher interaction), consulting (both synchronous and asynchronous), experimentation (simulation) and management (homework, evaluation and follow-up), all of this for technology to stimulate the required learning (Alarcia & Bravo, 2012).

Video-based learning environment with virtual debates; called virtual campus are practised in Alicanta where the teacher and the students can access using a password from any computer connected to the Internet. The website allows the teachers and the students to access the materials, watch the videos and download the document in text format anytime and anywhere (Llinares, 2007). Virtual debate is the space where the teachers and the students can share their misconceptions and the ideas to be clear on subject matter. Such type of debate helps the students to get clear concepts.

The education system of developed countries has been transformed with the help of new forms of technologies. Brick and Wilson (2015) described that technological knowledge and ICT infrastructure have become essential to work between students and teachers in the physical classroom as well as in the online learning community. Similarly, Nayar (2012) focused on applying ICT tools in the classroom in higher education opportunities. Based on the literature about virtual learning, I observed that developed countries have been using ICT as an incorporated part of the curriculum from the beginning. The developing countries are developing their infrastructures and have been facing several challenges in the fields of technological innovation.

Looking back to the history of e-learning in Nepal, the Ministry of Education (MoE) launched the Radio education teacher training project in 1978 with USAID’s technical and financial support. According to Dahal (2014), adio broadcasting started in 1980, which focused on enhancing the professional capabilities of in-service primary teachers with qualifications under the School Leaving Certificate (SLC). Distance Education Centre (DEC) was established under MoE in 1994. The centre conducted teacher training and education awareness programmes through radio broadcasting. After the unification of DEC with the National Centre for Education Development (NCED) in 2005, professional development
training courses for primary to secondary teachers, SLC support, and radio programmes on education information are being conducted (Dahal, 2014).

Tribhuvan University (TU) and Kathmandu University (KU) of Nepal have been launching some programmes at the Bachelor’s and the Masters’ level through distance and online mode. The Open and Distance Learning (ODL) programmes of these two universities are for teacher education (Pangeni, 2016). The government of Nepal has provided distance learning options to the SEE students also. According to Pangeni (2016), some schools, colleges, and universities also offer flexibility in schedule: evening, day or morning class. Students who are busy during the daytime can join the morning or evening classes as their interest. Pangeni (2016) stated the limited flexibility of higher studies is being expanded because every year new courses and programmes are offered by TU and KU through ODL. Since 2012, Open University Infrastructure Development Board had been working to establish Open University in Nepal. As a result, Nepal Open University (NOU) was established in 2017 to provide higher education through virtual mode in a different discipline.

After the pandemic of Covid-19, more schools and universities started a virtual mode in Nepal. Starting virtual mode at the school level was not an easy job but there was an option to engage the learners for a long pandemic period. The Government of Nepal had started the classes at the school level from Radio and television also. The Government had also requested to all the stakeholders to conduct virtual mode as the alternative way of teaching in the future in school level. In the pandemic period, most of the teachers were busy in preparing online classes in different subjects.

Mathematics Learning through Virtual Mode

Teaching-learning Mathematics through virtual mode is challenging than other theoretical subjects in my own experience. Because it has its own different symbols, operators, complex numerical steps and the diagrams, it takes a long time to type and prepare slides and also difficult to draw directly in virtual class. Most of the Mathematics teachers use slides to teach Mathematics because they are not well trained in other mathematical software. Online learning is affected by teachers’ digital awareness. Mathematics teachers should know about different mathematical tools and software to make virtual mode of learning Mathematics effective. Teachers should know the local, national and international policies related to ICT to design their learning activities, to prepare lesson and to implement their strategies (Khanal, Belbase & Joshi, 2021).

I have been engaged in virtual mode of learning Mathematics since 2017 as a student of M. Phil. in Mathematics education from Nepal Open University. It was the programme of three semesters. During that period, I have faced different challenges as a learner. Similarly, in the Pandemic period of Covid-19, I have taught different papers of Mathematics at undergraduate level through virtual mode as a teacher of Mid-West University. In the experiences of three years, I have seen the virtual mode of learning Mathematics is a new research area in Nepalese context. Integrating technology in lesson improves the learners’ concepts and learning style. Moreover, the use of technology makes the teachers more confident in teaching. Technological classroom teaching support the learners’ learning (Bennison & Goos, 2010).

The main objective of this paper is to find the gap in existing literature in the field of virtual learning environment in Mathematics.
Methods

I have prepared this paper by reviewing different twenty-five journal articles, seven dissertations, two e-books and one conference papers. I have collected the literatures from different websites, search engines as well as the electronic databases. I visited the Research gate, Elsevier, Wolfram Alpha, Directory of open access journals, Pro Quest Dissertations & Theses Global, Google Scholar, ERIC, Semantic scholar and Science Direct to collect the literature. The collected literature is reviewed thematically. I have developed the following three different themes on virtual learning environment in Mathematics; perceptions towards virtual learning environment, challenges of virtual learning environment in Mathematics, and efficacy of virtual manipulative in Mathematics learning. To identify the gap in virtual learning environment in Mathematics, I read the theoretical and the empirical literature in downloaded pdf form or printed form as needed, categorised them according to the themes and mentioned them according to the proper theme. In the process of reviewing the literature, I emphasised the objectives, methodologies and the findings of the papers. Moreover, the literature was published between 2002 and 2021 in English medium.

Empirical Literature

There are different literature about the perception and challenges of the teachers and the learners towards virtual mode of learning in Global context although a few research studies are available in the field of Mathematics. Moreover, there are limited research studies on the virtual mode of learning Mathematics in the Nepalese context. Here, I have tried to review the literature on the themes; perception towards virtual learning environment, challenges of virtual learning environment in Mathematics and the efficacy of virtual manipulative in Mathematics learning.

Perceptions towards Virtual Learning

Amrtesand Jeayaram (2019) studied students’ perception towards virtual learning environment. It was a survey of 200 students from classes 8 to 12 who were learning Maths and Science through virtual mode. The researchers concluded that 85% of students strongly agreed that VLE makes it easy for teachers to teach practical modules; 70% of students strongly agreed that VLE helped to improve their Mathematics and Science scores. The researchers concluded that VLE helps the students to recollect the learning easily even after one month. It helps students to understand a concept in a much better way as compared to traditional labs.

A qualitative case study by Clarence (2017) entitled ‘Virtual learning Environment’s Impact on Adult Learners’ Motivation in the Workplace’ explored that visual learning, learner control, ease of use, technical competence, instructors support, and technical support are the factors that must be addressed when using a virtual learning environment to improve the talent development. The use of e-learning has a positive influence on motivation, autonomy, participation, mathematical concepts, results and grades. As the same way, e-learning leads to improvement in adult students who are studying the mathematical subject in the high school stage. E-learning is effective in implementation for adults (Guerrero ct. al, 2020). Content quality, problem-solving abilities and the Internet availability are the key factors for mathematical e-learning satisfaction. The e-learning tools help the students to self-regulate and discover their knowledge, which increased their chances of handling application type problems (Akugizibwe & Ahn, 2019). Virtual learning contributes to improve the study habit.
of learners. It makes the learners more active and interactive. It gives the opportunity to learn, relearn, and clarify the concepts. The study suggested that Mathematics courses need to be designed to integrate ICT and VLE to engage the learners emotionally, socially and cognitively (Dhakal & Sharma, 2016).

Holmstrom and Pitskanen (2012) performed a qualitative field study to examine the Bolivian teachers’ beliefs about e-learning in higher education. It describes and understands teachers’ beliefs about e-learning in higher education at Universidad Mayor de San Andres. Qualitative semi-structured interviews and observations were used to identify 10 teachers’ beliefs about e-learning. The Technological Pedagogical Content Knowledge framework was used for analysing the interviews and observations. The study showed different levels of knowledge of teachers about technology, pedagogy and content, as well as different combinations of these three knowledge domains. Most teachers believed that e-learning is beneficial for themselves and their students. Likewise, Krishnan (2016) says that face-to-face instruction enables the learners to learn and understand the mathematical concepts better. Learners preferred the face to face learning mode for communication, discussion and understanding of mathematical concepts in improving their mathematics learning. More than half of the students believed that Mathematics courses should be taught in hybrid mode (Krishnan, 2016). On the other hand, Masriyah et al. (2019) found that students' responses using virtual learning are positive (the percentage of students' positive responses was more than 70%) and 90.03% of the final grades of students joining in the Foundation of Mathematics courses through virtual mode were B, B+, A- or A, and the percentage of positive responses of students after joining in Foundation of Mathematics courses through virtual mode was 80.0%.

Tootoonchi (2014) investigated the perceptions of college students of the learning environment in the online Mathematics classes. After analysing the qualitative data, it was found that college students’ perceptions of all three components (social component, pedagogical component and psychological component) were negative. The only area that contributed to a more positive reaction among college students were convenience and flexibility, limited to online Mathematics classes, which have been a common outcome of numerous studies on online education and, therefore, not only in Mathematics.

The above mentioned literature has indicated that the learners have mixed perception on virtual/online Mathematics learning. Most of the literature has shown that blended learning environment is better than traditional learning approach in Mathematics. Moreover, some literature has found the negative perception of the learners on virtual learning in Mathematics.

**Challenges of Virtual Learning Environment in Mathematics**

Teaching-learning Mathematics through online mode is a challenging job. The main challenges of online Mathematics learning are the availability of network, lack of contact with teachers and the peers, lack of easily manipulative Mathematics objects, lack of skill on ICT, lack of proper designing of modules and the lack of administrative support (Gadanidis et al., 2002). Likewise, computer manipulative (virtual manipulative) plays an important role to make virtual mode of teaching-learning Mathematics more effective. Virtual manipulative makes the learners more active and engaged (Durmus, 2006). The main challenges that the learners perceived on virtual mode of learning Mathematics were accessibility, availability and the students’ ICT skill. The institutions and the instructors need to identify
the perceived challenges and opportunities of e-learning and provide practical support (Joseph & Ekemini, 2014).

The main challenges of virtual learning were internet connectivity, personal computing devices and the teachers’ efficiency (Dhakal & Sharma, 2016). Similarly, Gunga (2010) reviewed the challenges of implemented e-learning in Mathematics, Science and Technology Education in African schools. The study indicated that understanding mathematical and scientific concepts is a challenge in ordinary/traditional pedagogy. It would require a renewed revolutionary approach to implement effective e-learning strategies for Maths and Science in African nations.

In the context of Nepal, the geographical diversity, technological infrastructure, lack of awareness and attitude, economic condition, lack of readiness and the trained facilitators are the challenges of e-learning in Mathematics (Dahal & Dahal, 2015). Likewise, e-learning is the most desired, effective and cheaply available tool for the learners because they can acquire more information from the internet. The learners of rural areas were still facing the problem of infrastructures and the internet. However, there is a rapid growth trend of e-learning in higher education in Nepal (Shakya, Sharma, & Thapa, 2018).

After reviewing the challenges of virtual learning, I concluded that lack of technological knowledge, lack of infrastructure, irregular electricity and unstable internet connection are the common challenges of virtual learning in the global context. Poor ICT infrastructure and limited communication opportunities are the main challenges in developing countries. Moreover, lack of software and proper training for teachers and the lack of practical and ICT integrated curriculum are the challenges of virtual learning in Mathematics, especially in Nepalese context.

**Efficacy of Virtual Manipulative in Mathematics Learning**

In an experimental study, Mendiburo (2010) examined the technology’s impact on fraction learning. The participants (students) of this study spent two weeks learning about fractions using virtual and physical manipulative and other variables were constant. After experiment, the assessment of the learners indicated that virtual manipulative are at least as effective as physical manipulative. But, when the amount of time spent practicing is held constant, students complete more practice activities using virtual rather than physical manipulative. Similarly, Doias (2013) conducted a quasi-experimental study to examine the relationship between concrete and virtual manipulative in seventh grade Mathematics classroom. Using students’ Mathematics composite scores on standardised and teacher-created assessments, it compared the effectiveness of using concrete manipulative alone versus using a combination of concrete and virtual manipulative. The results of this year-long study indicated that the combination of these two types of manipulative enabled the students in this to accomplish a measurable change in tested mathematical ability. The study recommended that Mathematics educators should incorporate both types of manipulative to increase the mathematical ability of the learners.

Smith (2004) examined the efficacy of online Mathematics courses in California. The study followed the quantitative survey and the qualitative study of opinions to answer the research questions. The result indicated that the students who engaged in online Mathematics courses obtained higher grade than their peers in traditional classroom format. But the rate of completion students in online course was lower during the same time period.
Similarly, the study found that the number of female students and the age of enrolled students were higher in online course than the general one. Likewise, the ethnic distribution of online Mathematics students was consistent with the ethnic distribution of the general student population. Similarly, Bayrak and Bayram (2010) indicated that computer aided teaching methods have positive effect on students’ achievement in science and technology.

The above experimental studies represent the efficacy of online Mathematics learning. The findings of some studies emphasis on combined used of virtual manipulative and physical manipulative to increase the mathematical ability. On the other hand, some study indicate the online learning is more effective than traditional classroom learning but the rate of completion online Mathematics course is lower than the classroom learning.

Different studies have indicated that the virtual mode of learning is an alternative learning approach in the 21st century. Due to the rapid development of technology, learning has been possible from home. In the context of Nepal, the geographical diversity, technological infrastructure, lack of awareness and attitude, economic condition, lack of readiness and the trained facilitators as the challenges of e-learning in Mathematics. Most of the studies have focused that irregular electricity supply, unstable internet connection, lack of ICT knowledge and infrastructure are the main challenges of e-learning in Nepal. In the same line, Bhusal and Rimal (2020) emphasised the coordination of education governing authorities, electrical authority and telecommunication to make online classes more useful and accessible. They also focused that there is more chance of disturbing internet wires as well as electrical wires in the windy season that affect online classes. They further argued that a free data pack should be available to the learners to make online classes accessible for all.

The above mentioned literature indicates that virtual learning is a relevant area of research in education. Different studies have been conducted about the challenges of e-learning in different countries. Some studies have found that ICT integrated approach is better in Mathematics learning. Virtual environment in Mathematics is still a new area because no more studies have been done in virtual mode of learning Mathematics. From the findings of the above mentioned literature, I have concluded that a participatory action research on virtual mode of learning Mathematics is necessary to make it better. From the literature, I found the gap on conducting a participatory action research on virtual mode of learning Mathematics to identify the actual ways of making virtual mode of learning Mathematics more effective. Moreover, participatory action research on virtual learning environment in Mathematics through the perspectives of transformative learning theory will fill the gap in the field of virtual learning in Mathematics in University level.

**Conclusion**

Virtual learning is an alternative way of learning at the university level as well as at the school level. Not only in the pandemic period, it is useful in other situations for the learners who could not attain their classes physically due to different problems. In most of the developing countries like Nepal, continuing higher education without job/work is not easy for all. Most of the job holders should stop their higher education after joining a job. Same case is seen for the girls after marriage. So, in different situations, virtual mode of learning is a need and an opportunity for higher education. In my perspective, face to face and virtual both procedures of learning should be available at the university level.
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The above mentioned literature indicates that the teachers and the students have taken virtual mode of learning as an alternative way of learning but it has many challenges. Irregular electricity and unstable internet, expensive data pack and other alternative resources, lack of the knowledge on ICT, lack of sufficient software and training for the teachers are found as the common challenges of e-learning specially in the Nepalese context. Some studies emphasise combined used of virtual manipulative and physical manipulative to increase the mathematical ability. On the other hand, some study indicate that online learning is more effective than traditional classroom learning but the rate of completion online Mathematics course is lower than the classroom learning. So, a participatory action research is needed to identify the actual problems on virtual mode in Mathematics learning to make it more effective.

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