Original Research Article

Quantum Teleportation (QT) with Qiskit: An Example

Ram Krishna Tiwari[1], **Harihar Paudyal**
Birendra Multiple Campus, Tribhuvan University,
Bharatpur, Chitwan, Nepal

Abstract

Quantum computing (QC) is a developing area that relies on fundamental aspects of quantum physics and the principles of traditional computing. This paves the way for creating systems capable of solving problems that a classical system cannot handle. This work defines logical operation related to quantum circuits using Qiskit in Python and explains the basic theoretical protocols for Quantum Teleportation (QT). As an example, this work demonstrates that QT enables the transmission of a quantum state (QS) across two locations, by using a mutually entangled state.

Introduction

Quantum computing (QC) is an emerging area of technology that uses the fundamental information-carrying unit known as the quantum bit, or qubit. These qubits can be understood as the quantum mechanical equivalent of a classical bit and are represented mathematically as a vector or point located on the surface of the Bloch sphere (Ropa Roy & Asoke Nath, 2021) (Figure 1).

Unlike regular bits, qubits can exist in a superposition state, enabling the system to be in both 0 state ($|0\rangle$) and 1 state ($|1\rangle$) at the same time. In addition to superposition, another important concept in quantum computing is entanglement that allows multiple qubits to be interconnected (Bouwmeester et al., 1997). Teleportation, the concept of moving objects between locations without physical transport, has a science fiction feel till 1993 before it was realized that something analogous can take place in the realm of quantum mechanics (Olofsson et al., 2020). Thereafter, it was possible to transfer a particle's QS from one location to another by utilizing the correlations between entangled particles. QT relies on an inseparable, entangled state, i.e. states of two

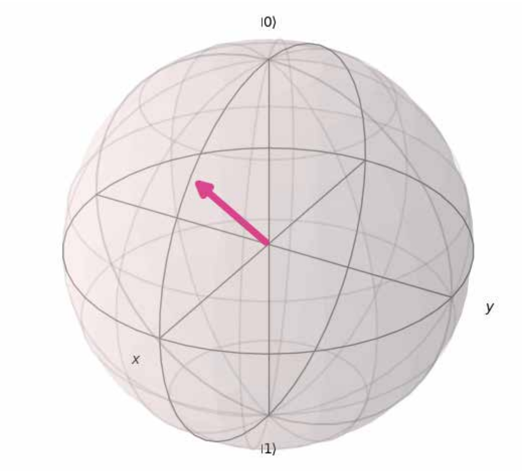
---

Corresponding author: ram.tiwari@bimc.tu.edu.np

quantum bits that share correlation. Classical 1 and a 0, are separable, but qubits can be assigned both a 1 and a 0 at the same time (Zeilinger, 2018).

Figure 1

A Bloch sphere



Qiskit is a freely available quantum computing toolkit created by International Business Machines (IBM) that sits between quantum algorithms on one side and physical quantum devices on the other side (Anaya & Delgado, 2022). It is developed using Python and converts the Python language into quantum machine language, allowing anyone outside of the IBM Quantum Lab (a cloud-based platform) to program a quantum computer (Shaik & Rangaswamy, 2020). It offers a universal gate set, including the Hadamard gate (H) and the CNOT gate, and supports fundamental quantum gates (Pauli X, Y, and Z gates). Once the quantum circuit is constructed, the next step is to execute it to gather results. The results of a quantum computation are probabilities of measuring each possible bitstring, which can be obtained by measuring the qubits on a computational basis (Dey & Mukhopadhyay, 2019). The measurement process collapses the QS into a classical bitstring, which is stored in classical bits.

The two-level qubit system is commonly used as the foundation for defining quantum algorithms (Arvind, 2001; Koch et al., 2019) so too has the general public's interest in testing some of the publicly available quantum computers. However, many might find learning all of the supplementary information that goes into quantum algorithms to be a daunting task, and become discouraged. This tutorial is a series of lessons, aimed to teach the basics of quantum algorithms to those who may have little to no background in quantum physics and/or minimal knowledge of coding in python. Each lesson covers select physics/coding topics needed for writing quantum algorithms, eventually building up a toolset for tackling more and more challenging quantum algorithms. This tutorial series is designed to provide readers from any background with two services: 1 represented with vectors as follows.

$$|0\rangle = \binom{1}{0}, |1\rangle = \binom{0}{1}$$

And the QS can be represented as,

$$|\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \qquad (1)$$

Here α and β are complex numbers such that,

$$|\alpha|^2 + |\beta|^2 = 1$$

Where, $|\alpha|^2$ is the likelihood that the qubit will be found in state $|0\rangle$ and $|\beta|^2$ is the likelihood that the qubit will be found in state $|1\rangle$. The gates needed for teleportation are Hadamard gate (H), Control (CNOT) gate.

Hadamard gate (H)

The Hadamard gate (Figure 2) converts a qubit into an equal probability distribution between the states $|0\rangle$ and $|1\rangle$, giving a 50-50 chance for each state (Ropa Roy & Asoke Nath, 2021). It maps $|0\rangle$ into $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|1\rangle$ into $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ by the relations,

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle),$$

$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Figure 2

A Hadamard Gate

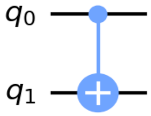
$$q\ - H -$$

Controlled Not Gate (CNOT)

The CNOT gate (Figure 3) can be used to create an entangled state into the two-qubit model i.e. an input qubit and a target cubit (Ropa Roy & Asoke Nath, 2021). It inverts the second qubit when the initial qubit is in the 1 state ($|1\rangle$). For instance, the CNOT gate changes a superposed QS of the form $a|00\rangle+b|01\rangle+c|10\rangle+d|11\rangle$ into a new state $a|00\rangle+b|01\rangle+c|11\rangle+d|10\rangle$.

Figure 3

A Controlled Not (CNOT) Gate



QT has been successfully achieved in various fields. Few examples are over a 100-kilometer optical fiber channel and a 1,400-kilometessr satellite-to-ground free space channel (Yin et al., 2012)the huge photon loss and decoherence in fibres necessitate the use of quantum repeaters for larger distances. However, the practical realization of quantum repeaters remains experimentally challenging. Free-space channels, first used for quantum key distribution, offer a more promising approach because photon loss and decoherence are almost negligible in the atmosphere. Furthermore, by using satellites, ultra-long-distance quantum communication and tests of quantum foundations could be achieved on a global scale. Previous experiments have achieved free-space distribution of entangled photon pairs over distances of 600 metres (ref. 14, the polarization states of photons (Bouwmeester et al., 1997), in photonic qubits (Lombardi et al., 2002; Marcikic et al., 2003), in trapped ions (Barrett et al., 2004), in nuclear spins (Nielsen et al., 1998) and in solid state qubits (Steffen et al., 2013).

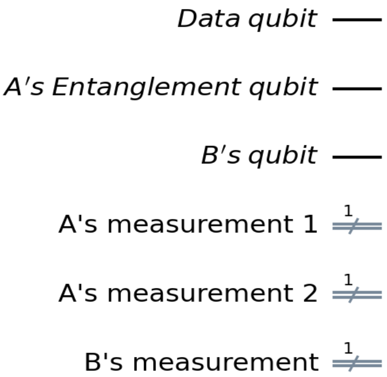
In this paper, we provide an example of QT using quantum gates, which will greatly deepen the understanding of quantum entanglement and have significant implications for quantum communications.

Protocol of QT

The first step involves conducting specific measurements on the original state and one half of an entangled pair. The sender (A) takes one of the entangled bits, while the receiver (B) takes the other. The sender then measures his part of the entangled pair along with the qubit he wants to transport, and this measurement result is sent to the receiver via a classical bit (Figure 4). The receiver obtains the measurement result of A's part of the entangled pair and the unknown qubit that A wants to send (Pirandola et al., 2015). Upon receiving this measurement, the receiver executes a quantum computing algorithm to manipulate his part of the entangled pair accordingly. Through this process, the receiver reconstructs the unknown qubit that the sender intended to send, without actually receiving the qubit itself (Zhao et al., 2023).

Figure 4

Three qubits and three classical bits.



The detail of the whole process is explained in the stepwise manner below.

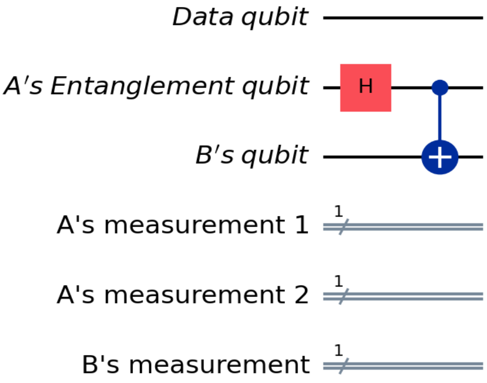
Step 1: An entangled Bell pair (Figure 5) is formed between two qubits by first applying a Hadamard gate to one qubit to transfer it to the X-basis, and then using a CNOT gate on the other qubit, with the qubit in the X-basis serving as the control (Mastriani, 2018). Let us consider the random state (a quantum bit to be transferred) be $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$ and let Bell pair or entangled qubit pair or quantum channel for teleportation is

$$|\phi\rangle = \frac{1}{\sqrt{2}}(\alpha|00\rangle + \beta|11\rangle) \quad (1)$$

Then $|\Psi\rangle \otimes |\phi\rangle = \frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle +)\beta|111\rangle$ (2)

Figure 5

Bell states consisting of Hadamard gate and CNOT gate



Step 2: Next, a CNOT gate is applied to the two qubits, succeeded by a Hadamard gate on the initial qubit to entangle the random qubit with the Bell pair (Figure 6).

$$(H \otimes \mathrm{I} \otimes \mathrm{I})(\mathrm{CNOT} \otimes \mathrm{I})|\Psi\rangle \otimes |\phi\rangle = (H \otimes \mathrm{I} \otimes \mathrm{I})(\mathrm{CNOT} \otimes \mathrm{I})\frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle +)\beta|111\rangle$$

Or $(H \otimes \mathrm{I} \otimes \mathrm{I})(\mathrm{CNOT} \otimes \mathrm{I})|\Psi\rangle \otimes |\phi\rangle = (H \otimes \mathrm{I} \otimes \mathrm{I})\frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle +)\beta|101\rangle$ (3)

Since,

$(H \otimes \mathrm{I} \otimes \mathrm{I})(\alpha|000\rangle) = (H \otimes \mathrm{I} \otimes \mathrm{I})\alpha(|0\rangle \otimes |0\rangle \otimes |0\rangle) = \alpha\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right) \otimes |0\rangle \otimes |0\rangle = \frac{\alpha}{\sqrt{2}}(|000\rangle + |100\rangle)$ (4)

$(H \otimes \mathrm{I} \otimes \mathrm{I})(\alpha|011\rangle) = (H \otimes \mathrm{I} \otimes \mathrm{I})\alpha(|0\rangle \otimes |1\rangle \otimes |1\rangle) = \alpha\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right) \otimes |1\rangle \otimes |1\rangle = \frac{\alpha}{\sqrt{2}}(|011\rangle + |111\rangle)$ (5)

$(H \otimes \mathrm{I} \otimes \mathrm{I})(\beta|110\rangle) = (H \otimes \mathrm{I} \otimes \mathrm{I})\beta(|1\rangle \otimes |1\rangle \otimes |0\rangle) = \beta\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) \otimes |1\rangle \otimes |0\rangle = \frac{\beta}{\sqrt{2}}(|010\rangle - |110\rangle)$ (6)

$(H \otimes \mathrm{I} \otimes \mathrm{I})(\beta|101\rangle) = (H \otimes \mathrm{I} \otimes \mathrm{I})\beta(|1\rangle \otimes |0\rangle \otimes |1\rangle) = \beta\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) \otimes |0\rangle \otimes |1\rangle = \frac{\beta}{\sqrt{2}}(|001\rangle - |101\rangle)$ (7)

Using equation (3), (4), (5), (6), (7)

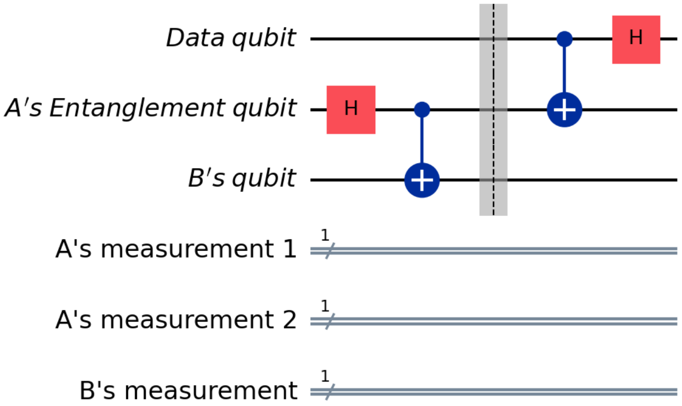
It can be written as,

$$(H \otimes \mathrm{I} \otimes \mathrm{I})\frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle +)\beta|101\rangle = \frac{1}{2}\big(\alpha(|000\rangle + |100\rangle + |011\rangle + |111\rangle) + \beta(|010\rangle - |110\rangle + |001\rangle - |101\rangle)\big)$$

Or, $(H \otimes \mathrm{I} \otimes \mathrm{I})\frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle +)\beta|101\rangle = \frac{1}{2}(|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(\alpha|1\rangle - \beta|0\rangle)$ (8)

Figure 6

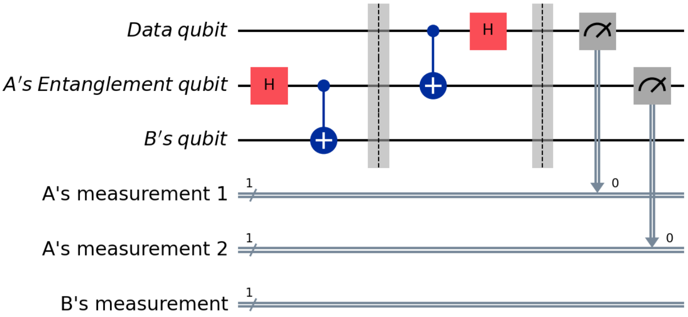
Bell state with barrier followed by CNOT gate on two qubits and Hadamard gate



Step 3: Now the first two qubits are measured, and the measurements are sent as two classical bits to the desired place (Figure 7). The results of the measurements are one of the four bits () an associated with standard two qubit basis states $\alpha|0\rangle + \beta|1\rangle$, $\alpha|1\rangle + \beta|0\rangle$, $\alpha|0\rangle - \beta|1\rangle$, and $\alpha|1\rangle - \beta|0\rangle$ with equal probability (Steffen et al., 2013).

Figure 7

Measurements on two qubit and sent as classical bits to the classical gates.



Step 4: The original state can be achieved by applying appropriate transformation on the received qubit (Figure 8). For the bit |00⟩ Identity gate (I) is applied, for the bit |01⟩ Pauli X gate is applied, for the bit |10⟩ Pauli-Z gate is applied, and for the bit |11⟩, ZX gate is applied. As an example, for the state $\alpha|0\rangle - \beta|1\rangle$, Pauli-Z gate is applied as $Z(\alpha|0\rangle - \beta|1\rangle) = \alpha|0\rangle + \beta|1\rangle$ which is the state to be teleported (Yin et al., 2012).

Figure 8

Pauli-X gate and Pauli-Z gate are applied to qubit to achieve the original state.

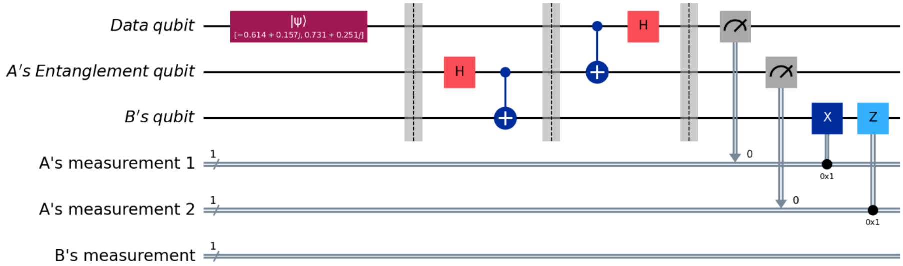


Figure 9

The result of the measurement for each basis (Bloch vector of teleportation).

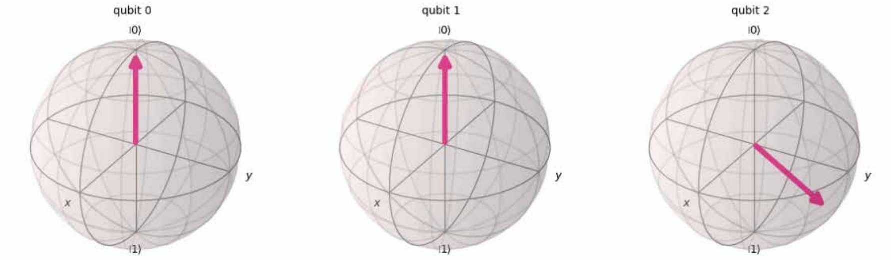
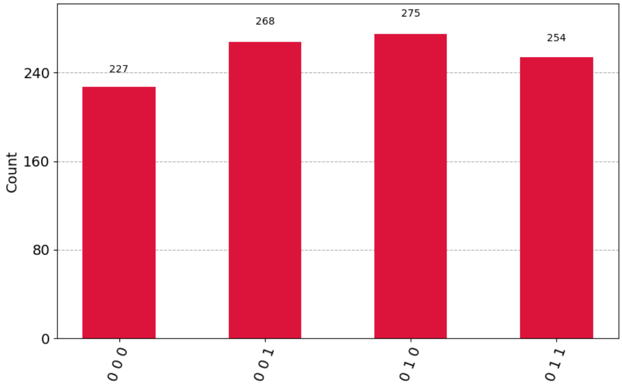


Figure 9 depicts the state that has been teleported. The north pole of the Bloch sphere is the 0 state (|0⟩) state, while the south pole is the 1 state (|1⟩) state. Any other point on the sphere represents a superposition of |0⟩ and |1⟩ (Ropa Roy & Asoke Nath, 2021).

Figure 10

Histogram of teleportation's results (Count versus computational basis vectors)



The measurement of the Bell state above 1024 times is depicted in figure 10. On the x-axis, all the results are orderly presented and due to quantum entanglement effect, we can see that two qubits end up with |00>, |01>, |10> |11> with 22.1%, 26.1%, 26.9%, and 24.8% of total of shots (1024 times) we asked for in the measurements. The Bell-measurement outcome counts (227, 268, 275, 254) are consistent with the expected 25% probabilities within normal experimental variability. This uniformity suggests that the teleportation circuit is functioning correctly with a maximally entangled channel, supporting teleportation of the qubit. In practice, teleportation success is confirmed by measuring the teleported state's fidelity (which should exceed the classical 2/3 limit. Here, the nearly equal outcome distribution and its agreement with theory is a positive indicator that the underlying entanglement and teleportation operations are meaningful and that no gross error has occurred.

Conclusion

The purpose of QT is to reliably transport an unknown QS from one place to another, without the qubit itself moving through the intervening area. In this work, we demonstrated the implementation of quantum logic circuits (quantum gates) to understand QT. By employing a pair of entangled qubits along with classical communication, we successfully achieved QT and built the concept of entanglement. This teleportation phenomenon relies on the curious logic of quantum mechanics and has powerful implications for quantum communication. This paper explains the example of QT, which will promote the practical development of quantum technology.

Acknowledgements

The authors are grateful to Birendra Multiple Campus, Bharatpur (Tribhuvan University), Nepal for necessary supports.

Disclosure of interests

The authors state that they have no conflicts of interest.

References

Anaya, A., & Delgado, F. (2022). Simulating molecules using the VQE algorithm on Qiskit. http://arxiv.org/abs/2201.04216

Arvind. (2001). Quantum entanglement and quantum computational algorithms. Pramana - Journal of Physics, 56(2–3), 357–365. https://doi.org/10.1007/s12043-001-0130-9

Barrett, M. D., Chiaverini, J., Schaetz, T., Britton, J., Itano, W. M., Jost, J. D., Knill, E., Langer, C., Leibfried, D., Ozeri, R., & Wineland, D. J. (2004). Deterministic quantum teleportation of atomic qubits. Nature, 429(6993), 737–739. https://doi.org/10.1038/nature02608

Bouwmeester, D., Pan, J. W., Mattle, K., Eibl, M., Weinfurter, H., & Zeilinger, A. (1997). Experimental quantum teleportation. Nature, 390(6660), 575–579. https://doi.org/10.1038/37539

Dey, S., & Mukhopadhyay, S. (2019). All-optical integrated square root of Pauli-Z (SRZ) gates using polarization and phase encoding. Journal of Optics (India), 48(4), 520–526. https://doi.org/10.1007/s12596-019-00568-4

Koch, D., Wessing, L., & Alsing, P. M. (2019). Introduction to Coding Quantum Algorithms: A Tutorial Series Using Pyquil. 1–129. http://arxiv.org/abs/1903.05195

Lombardi, E., Sciarrino, F., De Martini, F., Popescu, S., & Popescu, S. (2002). Teleportation of a Vacuum–One-Photon Qubit. Physical Review Letters, 88(7), 4. https://doi.org/10.1103/PhysRevLett.88.070402

Marcikic, I., De Riedmatten, H., Tittel, W., Zbinden, H., & Gisin, N. (2003). Long-distance teleportation of qubits at telecommunication wavelengths. Nature, 421(6922), 509–513. https://doi.org/10.1038/nature01376

Mastriani, M. (2018). Simplified Protocol of Quantum Teleportation. Journal of Quantum Information Science, 08(03), 107–120. https://doi.org/10.4236/jqis.2018.83007

Nielsen, M. A., Knill, E., & Laflamme, R. (1998). Complete quantum teleportation using nuclear magnetic resonance. Nature, 396(6706), 52–55. https://doi.org/10.1038/23891

Olofsson, E., Samuelsson, P., Brunner, N., & Potts, P. P. (2020). Quantum teleportation of single-electron states. https://doi.org/10.1103/PhysRevB.101.195403

Pirandola, S., Eisert, J., Weedbrook, C., Furusawa, A., & Braunstein, S. L. (2015). Advances in quantum teleportation. Nature Photonics, 9(10), 641–652. https://doi.org/10.1038/nphoton.2015.154

Ropa Roy, & Asoke Nath. (2021). Introduction to Quantum Gates : Implementation of Single and Multiple Qubit Gates. International Journal of Scientific Research in Computer Science, Engineering and Information Technology, 3307, 385–392. https://doi.org/10.32628/cseit217697

Shaik, E. H., & Rangaswamy, N. (2020). Implementation of quantum gates based logic circuits using IBM qiskit. Proceedings of the 2020 International Conference on Computing, Communication and Security, ICCCS 2020, 1–6. https://doi.org/10.1109/ICCCS49678.2020.9277010

Steffen, L., Salathe, Y., Oppliger, M., Kurpiers, P., Baur, M., Lang, C., Eichler, C., Puebla-Hellmann, G., Fedorov, A., & Wallraff, A. (2013). Deterministic quantum teleportation with feed-forward in a solid state system. Nature, 500(7462), 319–322. https://doi.org/10.1038/nature12422

Yin, J., Ren, J. G., Lu, H., Cao, Y., Yong, H. L., Wu, Y. P., Liu, C., Liao, S. K., Zhou, F., Jiang, Y., Cai, X. D., Xu, P., Pan, G. S., Jia, J. J., Huang, Y. M., Yin, H., Wang, J. Y., Chen, Y. A., Peng, C. Z., & Pan, J. W. (2012). Quantum teleportation and entanglement distribution over 100-kilometre free-space channels. Nature, 488(7410), 185–188. https://doi.org/10.1038/nature11332

Zeilinger, A. (2018). Quantum teleportation, onwards and upwards. Nature Physics, 14(1), 3–4. https://doi.org/10.1038/NPHYS4339

Zhao, J., Jeng, H., Conlon, L. O., Tserkis, S., Shajilal, B., Liu, K., Ralph, T. C., Assad, S. M., & Lam, P. K. (2023). Enhancing quantum teleportation efficacy with noiseless linear amplification. Nature Communications, 14(1). https://doi.org/10.1038/s41467-023-40438-z