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The Surface Revolutionary Approach to Surface Area and Volume of a Sphere

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Abstract

The surface area and volume of sphere can easily be explained as a revolution of a sector of circle in space and can be derived from the area of the circle. This study depicts the amount of changes in the derivation of surface area and volume of solid from the area of plane geometric figures that are obtained after revolution. It begins with the derivation of the surface area and volume of cylinder, cone and then extending the idea up to the solid sphere as a revolution of a right sector of a circle in space. This is an alternative and revolution method of geometric figures (i.e. the method of surface revolution) for the derivation of surface area and volume of a solid. This method of computation facilitates to find the area of rectangular region on the surface of sphere and the volume of the pyramid having the rectangular base on the surface of the sphere and vertex at the center of the sphere. This article has presented completely new idea and different pattern of computation.

Keywords: *surface revolution, revolution of a sector, hemi-sphere, right sector, surface area of sphere, volume of sphere*

Introduction

The surface area and the volume of a solid can be studied as a surface revolution of geometric solid figures about a point, a line etc. There may be different revolutions according as the position of the point of revolution or line of revolution. Different revolution generates the different geometric solid. The revolution of a geometric figure falls under the topic of area revolution. Under this topic, we shall discuss about the revolution of triangle, rectangle etc. about a point or about a line.

The content of Mathematics course in school education level also covers the study and computation of surface area and volume of solid figure. The students' and teachers' requirement of this level were the presentation of those concepts geometrically in spite of presenting it analytically as given in the course of higher education system. The teachers, who working in the field of teaching in secondary level education, always faces the problems of representation of such an abstract concept geometrically, in their professional life. The integration method of interpretation of this content is out of the cognitive level of the students. Therefore, integration method of Calculus is not applicable in this level. So, there was a need to present a method of interpretation of such concepts geometrically.

So many studies can be found regarding the computation of surface area and volume of a solid figure in analytical geometry through the method of integration in Calculus of Higher Education Level. But there are very few studies can be found regarding the geometrical interpretation in computing the surface area and volume of a solid. Some of them have been presented those concepts magically. There is a stricture to the students to accept the concept as true. But as any student raises a question of 'how' and 'why' about the representation of computation formula for the surface area and volume of a sphere to his Mathematics teacher then it always becomes a big issue to the teacher to give the answer simply and meaningfully understandable way.

The main problem is the presentation of the surface area of a sphere because the volume (V) of sphere can easily be obtained on the basis of the surface area, say S, using the formula $V = \frac{1}{3} \times S \times R$, where R is the radius of the sphere. So the review of literatures mentioned below has mainly focused on finding the surface area of a sphere.

Literature Review

Bartol (1893) has mentioned the great circle on the surface of a sphere as a circle determined by two endpoints of a diameter of the sphere and the surface area of a sphere is equal to the area of four great circles (Bartol, W.C. 1893). A sphere has been generated by the revolution of a semicircle about its diameter. Then the diameter which has been used as the axis of revolution also becomes the diameter of the sphere hence generated. The part of the regular polygon, inscribed in the semicircle, determine the different frustums of cones after revolution and the surface area of the sphere is defined as the sum of the lateral surface areas of these frustums. Each of which has surface area equal to the product of side length and 2π times the perpendicular distance of its midpoint from the diameter. Hence, the surface area has been mentioned as the limiting position of the inscribed polygon as its number of sides' increases indefinitely. Then at this position, the surface area of the sphere equals$S = 2\pi r \times 2r = 4\pi r^2$. The volume of a sphere has been defined as the product of its surface by one third of its radius. Durell (1904) has mentioned the surface area generated by a straight line revolving about an axis in its plane is equal to the projection of the line upon the axis, multiplied by the circumference of a circle whose radius is the perpendicular erected at the midpoint of the line and terminated by the axis (Durell, F. 1904). The area of the surface of a sphere is equal to the product of the diameter of the sphere by the circumference of a great circle. Eves (1990) has mentioned Cavalier's principles to establish the surface area and volume of a sphere (Eves, 1990). Cavalier's principles has been mentioned as follows: (1) if two planar pieces are included between a pair of parallel lines, and if the lengths of the two segments cut by them on any line parallel to the including lines are always in a given ratio, then the areas of the two planar pieces are also in this ratio. (2) if two solids are included between a pair of parallel planes, and if the areas of the two sections cut by them on any plane parallel to the including planes are always in a given ratio, then the volumes of the two solids are also in this ratio. Hart (2013) has mentioned that if halves of a regular polygons with the same even number of sides are circumscribed about, and inscribed in, a semicircle, then by repeatedly doubling the number of sides of these polygons, and making the polygons always regular, the surfaces generated by the semi-perimeters of the polygons as they revolve about the diameter of the semicircle as an axis approach a common limit (Hart, 2013).

Wentworth & Smith (1913) has mentioned that the area of the surface of a sphere is equal to the product of the diameter by the circumference of a great circle (Wentworth & Smith, 1913). His method of representation is similar to Bartol (1893). Slaught & Lennes (2009) has mentioned the formula of computing surface area of a sphere as the product of the circumference of the circle obtained by plane section of a sphere with its altitude constructing inscribed polygon and circumscribed polygon with the circle that generates the sphere (Slaught & Lennes, 2009). If the altitude is taken as a diameter of the sphere then the surface area of the full sphere becomes$S = 2\pi R \times 2R = 4\pi R^2$, where R the radius of the sphere is. This process of finding the surface area of sphere is just like the finding out the lateral surface of a cylinder, in which the height of the cylinder is multiplied by the circumference of its base circle. Sykes & Comstock (2016) has mentioned the areas of the surfaces generated by a series of chains of equal chords inscribed in the same semicircle, and revolving about the diameter of that semicircle as an axis, have a definite limit if the number of chords is increased indefinitely (Sykes & Comstock, 2016). Beman & Smith (1903) has presented very clear proof for the surface area of a sphere using inscribed polygon and has represented the semicircle as the limiting position of the half polygon when the number of its sides are increased indefinitely (Beman & Smith, 1903). Then the surface of the sphere has been generated as the revolution of the semi-sphere.

Milne (1899) has described the surface of a sphere as equivalent to the rectangle formed by its diameter and the circumference of a great circle (Milne, 1899). Then the area of the surface of sphere has been shown equal to the product of its diameter by the circumference of a great circle by the similar method as mentioned above. Godfrey & Siddons (1911) has not given any formal proof of the formula for computing surface area of a sphere but has left to the readers certain clues one of which states that the limit the surface of a finite belt of a sphere is $2\pi \times$ intercept on axis of sphere by planes bounding the belt $\times$ radius of sphere (Godfrey & Siddons,1911). According to Ford & Ammerman (1920), the surface area of a sphere is equal to the product of its diameter by the circumference of a great circle. He has presented a sphere as the revolution of a semicircle about its diameter. The proof has been given by using an inscribed half regular polygon in the semi-circle and then limiting position of the polygon as the number of side are increased indefinitely generates after revolution the surface area of the sphere. Bowser (1890) has mentioned that the area generated by a straight line revolving about an axis in its plane, is equal to the product of the projection of the line on the axis by the circumference whose radius is the perpendicular erected at the middle point of the line and terminated by the axis (Bowser, 1890). Then he has expressed the same proof as mentioned above in different literatures that the area of the surface of a sphere is equal to the product of its diameter by the circumference of a great circle. Gore (1898) has presented the same argument as mentioned by Bowser (1890). Stewart (1891) has mentioned that the surface area of a sphere is equal to four great circles each having for its radius the radius of the sphere (Stewart, 1891). He also has presented the similar argument as mentioned above.

The arguments in the most of the literatures mentioned above has presented the surface area of a sphere as the revolution of semi-circle about its diameter that results into a full sphere. The methods has been used as constructing an inscribed regular half polygon with even number of sides and then computing the area of this half polygon with respect to the product of the segment of diameter and radius of the semicircle. Then the concept has been extended to the sphere as the limiting position of increasing the number of sides indefinitely in the polygon. This method looks suitable for the whole sphere. However, if there is need to find the surface area of a solid obtained by the revolution of a plane geometric figure of different then the work becomes difficult. So, there was a need of presenting a suitable method that may facilitates the learners to find the surface area of the solid obtained by the revolution of plane geometric figures. This article has been presented to meet that need of the learners.

Methodology

This study follows the deductive research methodologies. This study uses most of the relations and formulae that has been previously established and contextual to application under similar conditions. This study follows the pattern of inductive method while establishing the formulae of derivation of surface area and volume of a sphere. It has been tried to make the formulae of computation of surface area and volume of a sphere easily deducible so that the teachers of mathematics can explain in front of students clearly and easily understandingly. The main aim of using this method is to improve the classroom teaching and learning by using as simple method as possible. The results has already been established but the aim of doing this study is to provide with new tools or techniques in teaching and learning of the surface area and volume of solid geometry, specially, those concerned with sphere.

Theory and Discussion

Before doing this let us introduce some plane revolutions that will be the base for this concept. First, let us begin from the revolution of a triangle and observe the space solid figures generated such a way.

The revolution of a triangle about a point, not lying on the surface of triangle forms a prism: If a space triangle ΔABC be rotated about a point O not lying on the ΔABC through an angle of $360^0 or\ 2\pi$ then the solid so formed is of the nature of a triangular prism. (see fig. 1.1). Let $d(O,B) = r$ then let us observe the following figures:

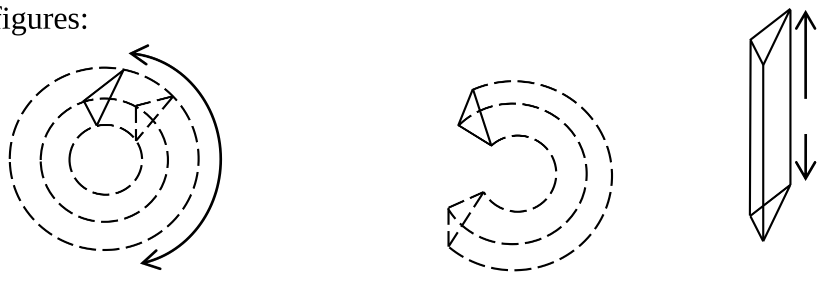


Figure 1: This heading contains total three figures in which fig. 1.1a presents the surface revolution of a triangle, fig. 1.1b shows the curved solid formed after surface revolution of a triangle and fig. 1.1c presents the triangular prism which is congruent with respect to its dimension to the solid of fig. 1.1b. This implies that the surface revolution of a triangle formed a solid that is a triangular prism.

The revolution of a triangle about a point lying on the triangle forms a pyramid: If a right triangle $\Delta ABC$with right angle at Cbe revolvedabout $z - axis$ fixing a point Pas invariant lying on the ΔABC through an angle of $360^0 or\ 2\pi$about $z - axis$then the solid so formed is of the nature of a circle of co-vertex pyramids, whose common vertex is the point O and their common height is the radius $OC = r$ say, of the circle$C(O, r)$. Let P be the foot of perpendicular drawn from the point A on the $YZ - plane$.

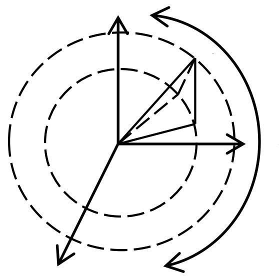


Figure 2: This figure presents the surface revolution of a right triangle taking one of its vertex fixed (or invariant) except the right angle formed a solid obtained by removing a right circular cone out of the cylinder whose height and radius are equal to the perpendicular and base of the right triangle.

The piecewise sectors from the point P perpendicular to the base of the triangle of revolution form pyramid of height BC (the radius of revolution) of the solid so formed.

Then the volume of each such pyramid $= \frac{1}{3} \times base\ area \times r$

But the revolution of the ΔABC about $B = O$ through an angle of 2π form a surface of area equal to the lateral surface area of a cylinder with radius r. Hence the volume of the solid obtained by the revolution of ΔABC about $Z - axis$ is equal to $V = \frac{1}{3} \times 2\pi rh \times r$,

Where, $h = height\ of\ \Delta ABC\ from\ A\ to\ the\ base\ BC.$

$$Or, V = \frac{2}{3}\pi r^2 h$$

The revolution of a rectangle about a side of the rectangle: If a space rectangle $\square ABCD$ be rotated about a side AB of the rectangle $\square ABCD$through an angle of $360^0 or\ 2\pi$ then the solid so formed is of the nature of circular prism and is called cylinder (see fig. 3). Let BD be a diagonal of the rectangle $\square ABCD$ then the solid obtained by the revolution of ΔABD form a right circular cone whose volume $V_1 = \frac{1}{3}\pi r^2 h$, where $h = AB = CD$the height of the cylinder.

Also, the volume of the solid obtained by the revolution of the ΔBCD has the volume $V_2 = \frac{2}{3}\pi r^2 h$. This solid is also called a cylinder with a *conical cavity*.

Thus the volume of the cylinder $V = V_1 + V_2$

$$i.e. V = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^2 h = \pi r^2 h$$

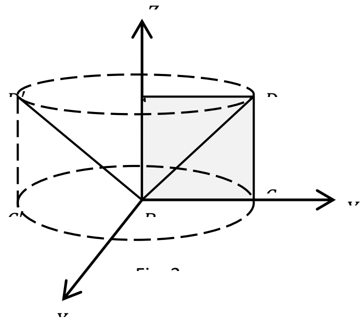


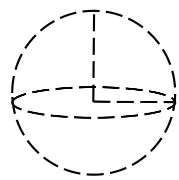
Figure 3: The rectangle $\square ABCD$ is rotated about one of its side, say AB. This revolution contributes a cylinder.

The revolution of quarter of a circle about its radius: If the quarter part of a circle with center O and radius R is revolved about its one of the radius (taking the radius as fixed line of the revolution) at O through an angle of 2π then the solid so formed is a sphere of radius R and center$O$. Let $C(O,R)$ be a circle and A and P be any two points on it so that AOP is a quarter sector of the circle (see fig. 4). Then let us rotate OP about OA through an angle of 2π. The solid so formed is a sphere.

Let us consider its hami-sphere first. Let A be a pole of the hami-sphere and B and C be any two points on its base circle.

If $\angle BOC = \theta$ then the area of the sector$BOC = \frac{1}{2}R^2\theta$.

Figure 4: The right sector AOP of a circle has been revolved about its radius AOthrough an angle of 2πthat contributes a solid hemisphere and twice of this revolution gives the full sphere has been shown in fig. 4.



The surface area of a sphere: If the arc AC is revolving about the base circle of the hami-sphere then the surface so generated is the surface area of a hemi-sphere.

The surface area of a hemisphere can be easily understood from the area a circle. At first, we shall find the area of a spherical triangle. For this purpose, we shall use the concept of the area of sector of a circle.

Let $C(O,r)$ be a circle in a plane. Let A and B be any two points on it and θ be the angle between OA and OB (Figure5). Then, the area of the sector $AOB = \frac{r^2}{2} \times \theta$. If the angle $\theta = 2\pi$ then region covers the circle and its area is equal to $A = \frac{r^2}{2} \times 2\pi = \pi r^2$. Thus, the area of a circle can be understood as the rotation of radius through an angle of 2π about its center.

Also, the length of the arc $AB = r\theta$.

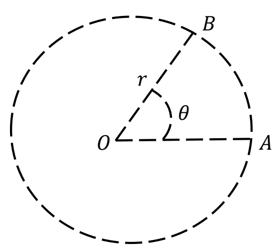


Fig. 5

Figure 5:The area of a circle as the plane rotation of a line segment about one of its fixed end point. The fixed point is center and the line segment is radius of the circle.

Theorem 1: *The lateral area of a frustum of a right circular cone equals one-half the product of the slant height and the sum of the circumferences of its bases.*

If l is the lateral surface area of a frustum of a right circular cone, C_1 and C_2 are the circumferences of its upper and lower bases, respectively, and s is its slant height then $l = \frac{1}{2}s(C_1 + C_2)$. (see fig. 6)

If l', P_1, P_2, s be the lateral area, the perimeters of the upper and lower bases, and the slant height, respectively, of the circumscribed frustum F of a regular Pyramid, then $l' = \frac{1}{2}s(P_1 + P_2)$.

If the number of faces of F increases indefinitely then $l' = l$, $P_1 = C_1$, $P_2 = C_2$, while the slant height is the same. Hence, $l' = l = \frac{1}{2}s(C_1 + C_2)$.

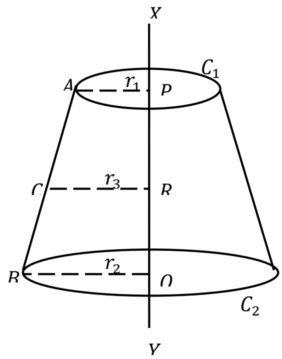


Fig. 6

If the radii of the upper and lower bases are r_1, r_2, respectively, then $l = \pi s(r_1 + r_2)$. If r_3 is the radius of he circle midway between the bases of the frustum, then, $l = 2\pi r_3 s$.

Figure 6:This figure presents frustum of a right circular cone with upper and lower circular bases having radii r_1 and r_2 with circumference C_1 and C_2 respectively.

Beman, W.W. & Smith, D.E. (1903) has described the surface area of a sphere by taking a semicircle with center O cut off by a diameter XX' and as it revolved about XX' as an axis then it generates a sphere (see fig. 7). Let AB be one of chord of the semicircle inscribed in arc XBX'. Let M be the midpoint of AB then OM perpendicularly bisects AB. Let AA', BB', MM' all be perpendiculars to XX' and AC be perpendicular to BB'. Then $AC \parallel XX'$.

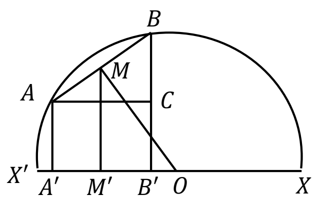


Fig. 7

If AB is revolved about the axis XX' then it generates the surface, say $l = 2\pi . AB . MM'$. Also, since $\Delta ACB \sim \Delta MM'O$ therefore, $\frac{OM}{M'M} = \frac{AB}{AC} = \frac{AB}{A'B'}$. This implies that $AB . M'M = A'B' . OM$, therefore, we get $l = 2\pi . A'B' . OM$.

Figure 7: This figure presents the computation of the surface area of circular frustum by taking a chord, say AB and perpendicular distance of its midpoint M from the center $O$of the semicircle that is to be revolutioned about the diameter $XX'$of the circle as an axis to describes a frustum of a solid with circular bases thatfacilitates the computation of lateral surface area of the frustum.

Now, summing for all the circular frustums obtained by revolving a side of an inscribed regular polygon having an even number of sides then the sum of their lateral surface areas $= 2\pi.OM.(X'A' + A'B' + \cdots \ldots) = 2\pi.OM.2r$.

But if the number of sides of the inscribed polygon increases indefinitely then the sum of the lateral surfaces becomes the surface S of the sphere and $OM = r$. Therefore, $S = 2\pi.r.2r = 4\pi r^2$ □

Milne, W.J. (1899), has described the surface area of a sphere by taking a half regular polygon with even number of sides inscribed in a semicircle with diameter AE and center O (see fig. 8). Then the sphere obtained by the revolution of the semicircle about its diameter AE as an axis, has center O and diameter AE. Then the perpendiculars drawn from the center O of the semicircle to its chords AB, BC, CD and DE are equal and bisects the sides of the regular polygons. Let one of them be of length $OM = a$.

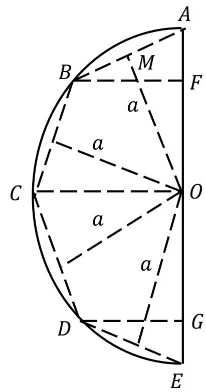


Fig. 8

Figure 8: It presents the regular polygon with even number of sides inscribed in a semicircle, when revolution about the diameter as an axis of the semicircle describes the sum of many circular frustums and as the number of sides increases indefinitely the sum of lateral surface area of these frustums becomes the surface area of the sphere.

Now, the surface generated by the chord AB is the lateral surface of the frustum of right circular cone. Let us denote this surface by surface AB and so on. Let F, O and G be the foot of perpendicular to the points B, C and D on AE. Then we get

$$surface\ AB = 2\pi.AF.OM$$
$$surface\ BC = 2\pi.FO.OM$$
$$surface\ CD = 2\pi.OG.OM$$
$$surface\ DE = 2\pi.GE.OM$$

But as the number of sides of the inscribed regular polygon increased indefinitely then theperpendicular length OM has length equal to the radius of the circle or sphere. $i.e. OM = R$.

Hence the surface of the sphere $S = surface\ AB + surface\ BC + \cdots \ldots$

$Or, \quad S = 2\pi.OM(AF + FO + \cdots \ldots)$

$= 2\pi.R.2R = 4\pi R^2$ □

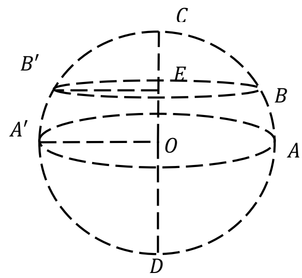


Fig. 9

If a sphere with diameter CD and center O is plane sectioned into two parts from the point E on CD such that $CE < OC = R$. Then clearly, $DE > OC = R$. Let $C(O,R)$ be the circle of section of the sphere with plane that contains two points B and B' (see fig. 9). Then the part of the solid BCB' obtained from the sphere has the lateral surface area $= 2\pi.CE.R$

Figure 9: It represents the plane section of a sphere divides it into two parts with different surface areas and their computations.

Also, let the remaining parts of the solid $B'DB$ obtained from the plane section of the sphere has surface area $= 2\pi.ED.R$.

Surface area of a sphere by the method of surface revolution

If the circle $C(O,R)$ is the circle on a sphere which separates the sphere into two hemi-spheres, then the radius of the sphere through the vertices of the sector BOCmeets the surface of the sphere at the points A, B and C (Figure 10 a). Then the regionABC on the surface of the sphere is a spherical triangle (Figure 10 b). Since the sectors AOB and AOCsubtends central angles $\frac{\pi}{2}$ at the center of the sphere, therefore the arcs $AB = AC = \frac{\pi}{2}R$. Let$\measuredangle BOC = \theta$for the sector on the base circle of the hemi-sphere. Then,$BC = \theta R$. Let us cut a sector $BO'C$ congruent to sector BOC on the surface of spherical triangle ABC(figure 10 c).Then area of this sector is $A_1 = \frac{R^2\theta}{2}$.

Again, if were not the surface from the corner A separating it into two parts alongAO' and denoting the new vertex, created in this way, byA', then there are two spherical triangles $BO'A$ and $BO'A'$. Also, if we join together $O'B$ and $O'C$ (Figure 10 d) then the total area of this region is A_2, say. Hence this area $A_2 = \frac{\pi R^2}{4}$.

Thus the total area of the spherical triangle ABC reconstructed as $ABA'$is $A = A_1 + A_2 = \frac{R^2\theta}{2} + \frac{\pi R^2}{4} = \frac{R^2}{2}\left(\theta + \frac{\pi}{2}\right)$. This implies that the sector of the sphere which subtends central angle $\frac{\pi}{2}$ revolves through an angle θ about the same radiusperpendicular to a circle on the surface of the sphere generates a surface of area $A = \frac{R^2}{2}\left(\theta + \frac{\pi}{2}\right)$.

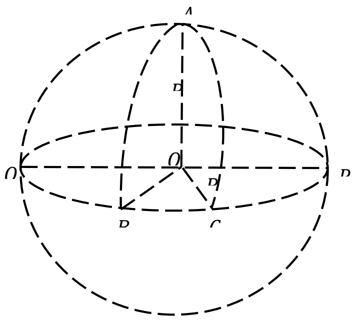


Fig. 10 a

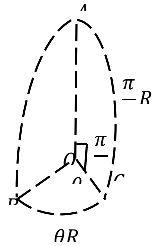


Fig. 10 b

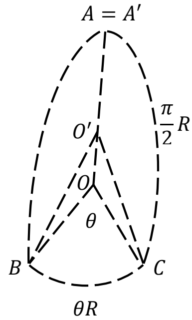


Fig. 10 c

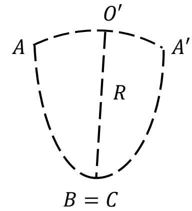


Fig. 10 d

Figure 10: The spherical triangle formed on the surface of a sphere determined by a sector BOC of the base circle $C(O,R)$ of the hemisphere has been shown in fig. 10 a, the spherical triangle with specification has been shown in fig. 10 b, surface partition of the spherical triangle to compute its area has been shown in fig. 10 c and the computation of the remaining area except the sector BO'C on the surface of the spherical triangle has been shown in fig. 10 d.

Let $C(O,R)$ be a sphere and AOB be a sector of the circle which separates the sphere into two hemi-spheres. Let us consider one of them, say H_1, whose radius is perpendicular to a point E on the surface of the sphere not lying on the circle $C(O,R)$. Let there be another circle on H_1 obtained by a plane section of H_1 parallel to the plane of $C(O,R)$ which intersect the circle through A and B on the surface of H_1 at points C and D respectively.

Then we have to find the area of the quadrilateral $ABCD$ on the surface of the hemi-sphere H_1. This quadrilateral is determined by two sectors AOB and AOC of the hemi-sphere H_1. Let θ_1 and θ_2 be the angles of the sectors AOB and AOC respectively (see fig. 11). Then it can be observed that the quadrilateral $ABCD$ has been obtained by the revolution of the sector AOC through an angle θ_1 at O. i.e. the angle $\measuredangle\theta_2$ is revolved at O through an angle of θ_1 in the anticlockwise direction. Therefore the surface area of the region $ABCD$contains the sum of two angles θ_1 and θ_2, $i.e. \theta_1 + \theta_2$ by the same way as we used in finding the area of a circle.

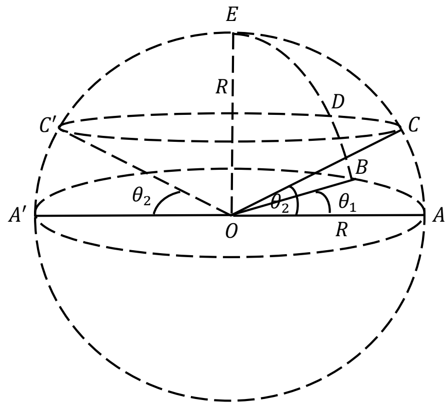


Fig. 11

Figure 11: The construction of a spherical rectangle $ABCD$ on the surface of a sphere and its parts to compute its area.

Thus the surface area of the rectangular region $ABCD$on the surface of the hemi-sphere $= \frac{R^2}{2}(\theta_1 + \theta_2)$.

If $\theta_1 = \frac{\pi}{2} = \theta_2$, then $A = \frac{R^2}{2} \times \pi = \frac{\pi R^2}{2}$. This is the surface area of the quarter part of the hemi-sphere.

If $\theta_1 = \pi = \theta_2$, then $A = \frac{R^2}{2} \times 2\pi = \pi R^2$. This is the surface area of the half part of the hemi-sphere.

If $\theta_1 = 2\pi = \theta_2$, then $A = \frac{R^2}{2} \times 4\pi = 2\pi R^2$. This is the surface area of the hemi-sphere.

Thus the total surface area of a sphere is $A = 4\pi R^2$.

The volume of a sphere by the method of surface revolution

The volume of a sphere can easily be obtained by using the concept of obtaining the surface area of a hemi-sphere. As shown in fig. 11, if we join the corner points A, B, C and D with the center O of the hemi-sphere H_1, then it becomes a pyramid whose base is the region $ABCD$ and the height is the radius R. Hence the volume of this solid pyramid is $\frac{1}{3} \times \frac{R^2}{2}(\theta_1 + \theta_2) \times R = \frac{R^3}{6}(\theta_1 + \theta_2)$.

If $\theta_1 = \frac{\pi}{2} = \theta_2$, then $A = \frac{R^2}{2} \times \pi = \frac{\pi R^2}{2}$. This is the surface area of the quarter part of the hemi-sphere. Therefore the volume in this case is $V = \frac{1}{3} \times \frac{\pi R^2}{2} \times R = \frac{\pi R^3}{6}$.

If $\theta_1 = \pi = \theta_2$, then $A = \frac{R^2}{2} \times 2\pi = \pi R^2$. This is the surface area of the half part of the hemi-sphere. Therefore the volume in this case is $V = \frac{1}{3} \times \pi R^2 \times R = \frac{\pi R^3}{3}$.

If $\theta_1 = 2\pi = \theta_2$, then $A = \frac{R^2}{2} \times 4\pi = 2\pi R^2$. This is the surface area of the hemi-sphere. Therefore the volume in this case is $V = \frac{1}{3} \times 2\pi R^2 \times R = \frac{2\pi R^3}{3}$.

Since the total surface area of a sphere is $A = 4\pi R^2$ therefore the total volume of a sphere is $V = \frac{4\pi R^3}{3}$.

Results and Applications

The surface rotation method of finding the surface area and volume of sphere facilitates to compute the area of any rectangular region on the surface of the sphere and volume of the solid pyramid subtended by that region, if the angles subtended by the adjacent sides of the rectangular region are given.

Example: find the area of the region $ABCD$ on the surface of a sphere with radius $7cm$ such that arcs AB and arc AD subtends angles 30^0 and60^0, respectively, at the center O of the sphere. Also, find the volume of the pyramid with vertex O and base $ABCD$.

Solution:

Here, radius of the sphere $(R) = 7cm$,

Arc AB subtends central angle $(\theta_1) = 30^0$ and

Arc AD subtends central angle $(\theta_2) = 60^0$

Therefore, the surface area $ABCD$ on the surface of the sphere,

$$i.e.\, Ar(ABCD) = \frac{R^2}{2}(\theta_1 + \theta_2) = \frac{(7)^2}{2}(30^0 + 60^0)$$
$$= \frac{49}{2} \times 90^0 = \frac{49}{2} \times \frac{22}{14} = \frac{77}{2} = 38.5\ cm^2$$

Again, the volume of the pyramid with vertex O and base $ABCD = \frac{1}{3}.R.Ar(ABCD)$

$= \frac{1}{3} \times 7cm \times \frac{77}{2}\ cm^2 = 89.83\ cm^3$ □

Conclusion

Geometric solid figures have been presented as a surface revolution of plane geometric figures. It has been shown that solid figures can be obtained by the revolutions of a triangle, rectangle, right sector etc. Then it has been represented that the revolution of a triangle, rectangle and right sector are respectively a cylinder with a conical cavity, cylinder and a sphere. Then the formulae for computing their surface areas and volumes have also been verified on the basis of the area of the geometric plane figure from which it has been generated. Generating a solid by surface revolution of a plane figure better helps the understanding of solid figures and its properties as compared to the analytical method of representing it. This method facilitates understanding of concepts and easily applicable in classroom teaching and learning as compared to analytical method. Students unanswered questions about geometrically representation of surface area and volume of a sphere has been clarified.

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