Voice : A Biannual & Bilingual Journal
2025 July, Vol : 17, No. 1, pp.115-125
DOI : https://doi.org/10.3126/voice.v17i1.84781

Error Patterns in Algebraic Equation in Secondary Mathematics Education

Krishna Chandra Paudel[1]
Shree Prasad Ghimire[2]

Article Information : Received : June 17, 2025 **Revised :** June 29, 2025 **Accepted :** July 15, 2025

Abstract

This study investigates pattern of error in algebraic equation solving that occurs among Grade IX students. This study used Newman's Error Analysis Framework to investigate the errors. This study consist of 34 incorrect response sheets from a secondary school in Kathmandu, Nepal, as sampling frame and four representative cases were examined to identify particular misconceptions in solving algebraic equations. This study mainly concentrate on a two-digit number related word problem necessary for students to formulate linear equations and solve for unknown variables. The findings explore the systematic errors across different cognitive levels. The comprehension errors in the interpretation of digit reversal conditions, transformation errors related to converting verbal statements to mathematical equations, process skill errors was based on algebraic manipulations, and encoding errors is related to the finding of final solutions. The most serious challenge appears during the transformation stage, where students failed to properly translate the problems into correct mathematical forms. These determined misconceptions suggest the demand of targeted instructional strategies that address conceptual understanding rather than procedural memorization, emphasizing the significance of conceptual change theory in mathematics education.

***Key Words**:* Algebraic equation, conceptual change theory, mathematical misconceptions, Newman's Error Analysis Framework, Transformation errors

[1,2]Lecturer, Department of Mathematics Education, Mahendra Ratna Campus, Tribhuvan University
Corresponding Author : kcpaudel@ncit.edu.np | shreeprasad.ghimire@mrc.tu.edu.np

ISSN : 2091-2161

Published by Autar Dei Chaudharain Research Centre (ADCRC), Mahendra Multiple Campus, Nepalgunj, Banke

Introduction

The transdformation of word problems into algebraic equations is a crucial ability, in mathematics learning, that connects tangible real-world circumstances with abstract mathematical representations. This procedure includes multiple cognitive stages: comprehending the text, pinpointing relevant variables and connections, and converting verbal explanations into mathematical symbols and operations. Mathematical word problems have been an important part of mathematics curricula centuries (Tiwari & Fatima, 2019). Algebraic equations offer this advantage due to their wide use in academic settings and everyday life. Their significance arises from their capacity to contextualize abstract mathematical ideas and illustrate empirical uses of algebra. Nevertheless, learners frequently encounter difficulties with word problems, as the process of translating them poses a major obstacle to achievement.

Researches in mathematics education have recognized the transition from natural language to algebraic notation as especially challenging for students at all educational levels (Pournara, 2020). The skill to convert word problems into algebraic equations requires multiple competencies, such as understanding language, recognizing problem schema, proficiency in mathematical modeling, identifying and representing variables, and understanding of mathematical operations along with their verbal equivalents.

The pedagogical criticism primarily focuses on the teaching methods used, which have predominantly prioritized memorization techniques for solving algebraic problems instead of nurturing a true understanding of algebraic equations and their multiple perspectives as a long-term solution for addressing issues (Mathaba et al., 2024; Pournara, 2020). The gap between procedural abilities and conceptual understanding leads to ongoing errors that hinder mathematical proficiency. This study explores the gap in knowledge about the specific traits of students' mistakes in algebraic equation problems, vital for developing targeted instructional strategies to improve mathematical reasoning and problem-solving abilities.

This study expands this area of study by exploration of error patterns in solving algebraic equation problems through the Newman's Error Analysis Framework (Newman, 1977). While the prior research has highlighted common challenges with equation (Mathaba et al., 2024), the particular kinds of conceptual errors leading to repeated transformation mistakes have been less examined. Observing that many students have algorithmic skills but lack conceptual comprehension, this research investigates the way errors in students' transformations expose fundamental misconceptions about the mathematical relationship between initial and augmented values. In this context, this study aims to identify mistakes by analyzing students' responses to a specific algebraic equation problem requiring reasoning, which can update the focused teaching strategies in mathematics education. This study concentrated on analyzing student mistakes in solving algebraic equation problems using the Newman's Error Analysis Framework to identify mathematical misconceptions and develop targeted instructional suggestions for improving comprehension of algebraic relationships. In order to reach the research objective, the following research question has been formulated:

What specific types of errors do students exhibit at different levels of Newman's Error Analysis framework when solving algebraic equation problems?

Conceptual Change Theory in Mathematics Education.

The conceptual change theory describes how previous knowledge affects learning and highlights the need to reorganize current knowledge structures to attain a scientifically accurate comprehension. It implies that learning happens through a significant restructuring of an individual's cognitive frameworks rather than merely incorporating updated information (Posner & Gertzog, 1982). This procedure includes acknowledging the boundaries of current mental frameworks, feeling cognitive disagreement, and subsequently reshaping understanding with a clearer conceptual perspective. To achieve effective conceptual change, learners need to view new concepts as distinct, reliable, and beneficial, offering superior explanatory power compared to their prior beliefs. Conceptual Change Theory provides a valuable framework for understanding the difficulties students face with mathematical problems, particularly the persistent misconceptions identified in this study.

In mathematics education, especially regarding equation word problems, a conceptual shift is essential because students typically tackle these issues with prior insufficient interpretations of the algebraic expressions. Vosniadou and Verschaffel (2004) suggested that learning mathematics often requires students to undergo considerable conceptual restructuring rather than simply obtaining knowledge. Numerous students use formulae instinctively without grasping the fundamental proportional connections. Lamon's (2007) extension of this theory to proportional reasoning indicated that students face "conceptual barriers" when their instinctive mathematical ideas conflict with formal mathematical frameworks. The mistakes identified during the transformation stage align with Chi and Slotta's (1993) idea of "ontological miscategorization," where students wrongly classify algebraic relationships as additive rather than multiplicative, leading to repetitive transformation errors even when they are proficient in calculations.

Newman's Error Analysis Framework

The Newman's Error Analysis (NEA) framework offers an organized approach to tackling these challenges by categorizing students' mathematical errors into various levels: reading, understanding, transformation, process skills, and encoding (Newman, 1977). Studies indicate that the stage where students convert verbal problems into appropriate mathematical equations is a significant challenge in solving equation problems, highlighting critical misconceptions about mathematical relationships (Karnasih, 2015). Parakitipong and

Nakamura (2006) categorized Newman's Error Analysis five stages into two groups. The problem-solving process in algebra come across two types of obstacles that hinder students in obtaining the correct answers: The first one is related to the smooth linguistics and conception barrier that align with a basic level of reading and taking the significance of problems, and other issues is related to mathematical computation including transformation of knowledge and skills, procedural abilities, and answer encoding. Karnasih (2015) concludes that employing the Newman Error Analysis (NEA) in education can serve as a strong diagnostic instrument for evaluating and analyzing students' challenges in resolving mathematical word problems.

Methodology

This study used a descriptive qualitative research approach design to explore students' errors in dealing with word problems related to algebraic equations via Newman's Procedure. Creswell and Creswell (2018) defined that a sample is a small group or subset of the population, describing its characteristics. A public secondary school in the Kathmandu district was chosen for the research. The researchers purposely selected response sheets from Grade IX students that had wrong answers to equation word problems. The analysis indicated that 40% of the students (34 out of 85) made error while solving the problem. Utilizing these incorrect answers, the researchers analyzed all the 34 answer sheets and randomly selected four sheets as representatives, which were subsequently given new codes: S1, S2, S3, and S4. Prior to commencing the study, the researchers obtained consent from the head teacher and the subject instructor, assuring them that their confidentiality would be maintained during and after the research. Researchers conducted qualitative content analysis to collect data for this study. As stated by (Nik Azis, 2014), the qualitative content analysis represents a subjective, qualitative, and scientific method for assessing the textual material content via classification processes. To ensure validity, experts in mathematics education confirmed the research instruments. The researchers similarly implemented measures to minimize potential biases throughout the analysis stage.

Data Analysis

The analysis of data adhered to the Newman's error analysis framework, providing an organized method for identifying and categorizing the different error types committed by students. These steps included reading errors, understanding errors, transformation errors, process errors, and encoding errors as suggested by Newman's Error Analysis Framework (Newman, 1977) . Utilizing this proven approach, the researchers managed to detect trends in the students' mathematical misconceptions and challenges in resolving algebraic equation word problems, which could guide instructional approaches. This research employed the Interactive Analysis models propounded by Miles, Huberman, and Saldana (2016) for data analysis, encompassing data collection, reduction, presentation, and findings. The errors made by the students were analyzed based on the Newman error analysis framework: reading error, understanding error, transformation error, procedural skills error, and encoding error.

Findings and Discussions

Looking at the student's answer sheets, the researchers analyzed the answers using the NEA framework, which classifies mathematical errors into five categories: *reading, comprehension, transformation, process skills, and encoding* (Newman, 1977). The answer sheets of four students were examined according to the various types of the Newman's error analysis. As the researchers examined the answer sheets, analyzed them based on the four stages, except the reading stage. The students were asked to find the solution to the problem:

A two-digit number has digits that add up to 16. When 18 is added to the number, the digits are reversed. The student had to: a) Represent these conditions as linear equations b) Determine the original two-digit number. c) Calculate the difference between the original number and its reversed form.

The student's effort is evident as they attempt to solve this by setting up the equations where x denotes the tens digit and y signifies the units digit. Their problem-solving approach included multiple errors in establishing the correct equations, handling algebraic expressions, and ultimately determining the result. The error analysis looks at where and why the student errored during the solution process by employing Newman's Error Analysis method to classify and comprehend these errors. The errors made by the four students on their answer sheets shown in the following image and the errors analysis were presented below.

Newman's Error Analysis of Student S1

Comprehension Error: The student seems to grasp what the problem is asking for, accurately formulating initial equations where x and y denote the digits

Transformation Error: The student begins with $10x + y = 16$ (initial line).This is incorrect. The total of the digits must be $x + y = 16$, rather than $10x + y = 16$. This is a transformation error- misunderstanding the mathematical meaning of the problem conditions.

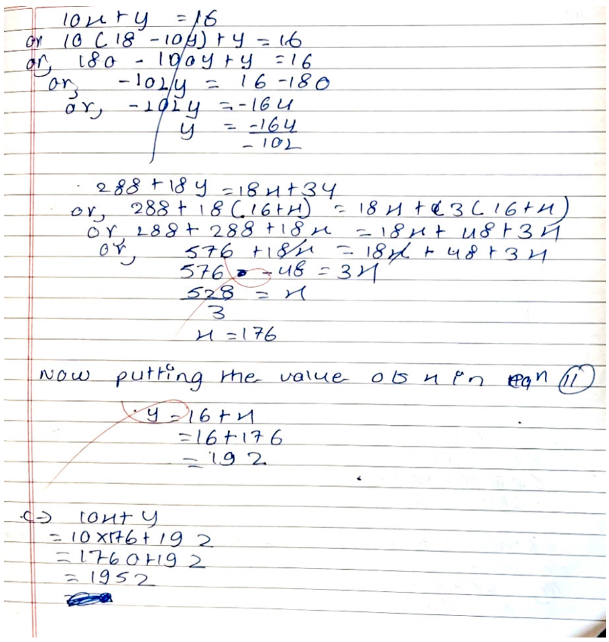

Figure 1. *Analysis of Error of Student S1*

Process Skills Errors: Beginning from the incorrect equation ($10x + y = 16$), the student executed a sequence of manipulations that include further calculation error:

In the middle part, he stated, when substituting the value of $y= 16-x$, the student incorrectly used $y = 16+x$ as the second condition. However, the algebraic manipulations performed later include computational mistakes. He also committed an error by writing "$y = 16 + x$," which is illogical.

Encoding Error: In the last part, the student concludes "$10x + y = 10\times176 + 192 = 1952$, which does not even align with his own computation. Their final response states that 1952 is incorrect (the accurate answer should be 79).

Summary of S_1 response

The primary errors in this work are the fundamental transformation error of expressing $10x + y = 16$ rather than $x + y = 16$. The subsequent type of error in algebraic operations involves inconsistent computations resulting in a incorrect result. Newman's framework suggests that the main challenge lies in the transformation stage, where the student did not accurately convert the problem statement "sum of digits is 16" into the proper mathematical equation. This initial error propagated through their later work, rendering a correct solution unachievable no matter how accurate their calculations were afterward.

Newman's Error Analysis of Student S2

Comprehension Error: The student realized that a two-digit figure could be expressed as $10x + y$ where x is the tens digit and y is the unit digit. The total of the digits equals 16, therefore $x + y = 16$, and the misunderstanding arose because the student misread the condition "when 18 is added, the digits are reversed." The student ought to have developed an equation indicating that $10x + y + 18 = 10y + x$.

Expected Solution :
Let 10x + y be the original number where x and y are the digits
x + y=16 (sum of digits)
10x + y + 18 = 10y + x (when 18 is added, digits are reversed)
Simplify, 10x +y+18 = 10y + x
⇒10x – x +18 = 10y-y
⇒9x +18 = 9y
⇒x+2 = y
Substitute into x + y=16:
⇒ x + (x+2)=16
⇒ 2x +2=16
⇒ 2x =14
x =7, which gives y=9
Therefore, the original number is 79, and the reverse is 97
The difference would be 97-79=18

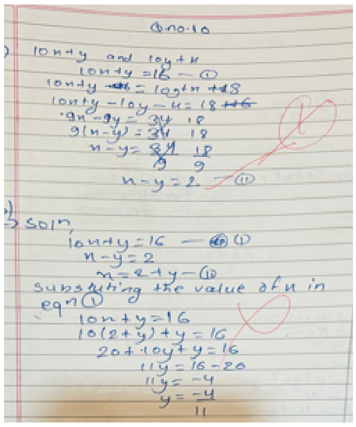

Figure 2: *Solution of Problem of Students S2*

Transformation Error: The student started with accurate expressions: $10x + y$ for the initial number and $x + y = 16$ for the digits' total. Instead of establishing the equation $10x + y + 18 = 10y + x$ for the digit reversal condition, the student mistakenly formulated "$10x + y - 10x - x = 18$," indicating a misunderstanding of how to correctly transform the reversal condition into an equation.

Process Skills Error: The student made several algebraic manipulation errors: Incorrectly simplified "$10x + y - 10x + y = 18 + 6$" to "$9x - 9y = 9y$". Made error in manipulating "$9 (x - y) = 9y$" to "$x - y = 3y$" (should be $x - y = y$ after dividing by 9). Incorrectly derived "$x - y = 2$" which does not follow from previous steps. As the result deriving "$10 (2 + y) + y = 16$" the steps contain arithmetic errors.

Encoding Error: The final answer *y = -4* fails to effectively respond to the question, which requested the original two-digit number and the disparity between the original number and its reverse. The student failed to complete encoding the solution in reference to the original problem.

Summary of S2 response

Newman's analysis of Student S2 uncovers interrelated errors throughout problem-solving phases. While precisely expressing a two-digit number as $10x + y$ with $x + y = 16$, the student misunderstood the digit reversal condition, resulting in an incorrect equation rather than $10x + y + 18 = 10y + x$. This error in transformation resulted in later skill issues in the process, including numerous algebraic manipulation errors while simplifying expressions and dividing by 9. Eventually, the student's yielded the solution *y = -4,* failing to identify the original two-digit number or the difference between it and its reverse as asked—underscoring how initial misunderstandings propagated throughout the solution process.

Newman's Error Analysis of Student S3

Comprehension Error: The student correctly grasped that a two-digit number can be expressed as *10x + y* and that the digits sum equal to 16 i.e. *10x + y = 16*. Nevertheless, misunderstood the condition regarding digit reversal.

Transformation Error: While formulating the equation for "when 18 is added, the digits are reversed," the student wrote "*10x + y = 10y + x + 18*," but then mistakenly altered it to "*10x + y - 10y - x = 18 + 6*," indicating a basic misunderstanding of the equation's setup.

Process Skills Errors: The student incorrectly reduced "*10x + y - 10y - x = 18 + 6*" to "*9x - 9y =18*." Another mistake is evident in the expression "9*(x - y)* = 18," where an attempt to factor appears to be made. Incorrect computation when dividing by 9: *"x - y =* 18/9" yield "*x - y =* 2." Erroneous inference of "*x - y = 2*" that lacks logical consistency. While solving "*10(2 + y) + y = 16*", there are calculation mistakes; *20 + 10y + y = 16* should turn into *20 + 11y = 16*, leading to the incorrect relation *11y = -4*, resulting in *y = -4/11.*

Encoding Error: The concluded answer *y = -4* is wrong, and the student failed to fulfill the task by determining the original two-digit number and the difference from its reverse, as asked in the initial problem.

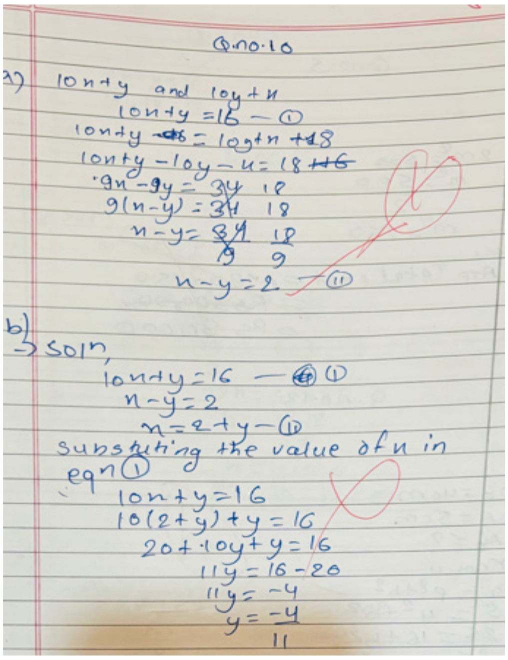

Figure 3. *Solution of Problem of Students S3*

Summary of S3 response

The Newman's framework of analysis of errors of student S3's work uncovers a series of errors affecting the solution method. The student accurately expressed the two-digit number as $10x + y$ with $x + y = 16$ but misinterpreted the digit reversal condition, wrongly changing it to "$10x + y - 10y - x = 18$ (or $+ 6$)" rather than the correct equation $10x + y + 18 = 10y + x$. This error in transformation resulted in several skill errors during the process, such as incorrect algebraic manipulations in expression simplification, wrong factoring, and calculation mistakes while dividing by 9 and resolving for y. The conclusive result of $y = -4/11$ indicates calculation mistakes and an inadequate solution that does not identify the original two-digit number or the sought difference showing how early misunderstandings were carried through the solution process. The student's work demonstrates a chain of mistakes stemming from an initial misunderstanding of how to depict the digit reversal condition, resulting in erroneous algebraic manipulations and, ultimately an incorrect answer that fails to respond to the original question accurately.

Newman's Error Analysis of Student S4

Given the minimal effort displayed in the image for Student S4, the subsequent Newman's Error Analysis is presented as:

Comprehension Error: Student S4 showed a limited grasp by trying to symbolize the problem in mathematical terms. Nonetheless, their equation "$10x + y = 16$" reflects a misunderstanding between the actual two-digit number and the total of its digits. The student seems to misunderstand what the question asked.

Transformation Error:

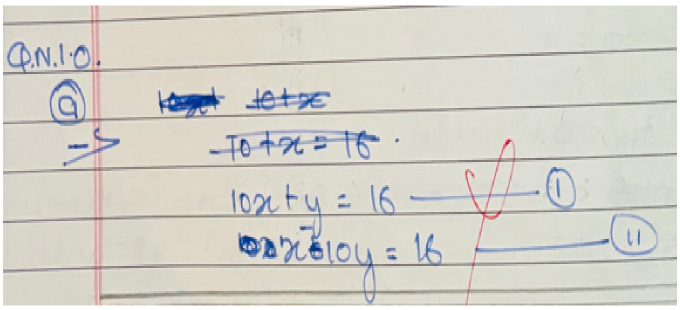

Figure 4. *Solution of Problem of Students S4*

Transformation Error: The student demonstrates notable transformation errors in converting the problem into mathematical equations. The second equation "$x + 10y = 16$" mistakenly swaps the coefficients of the variables, indicating a basic misunderstanding of how two-digit numbers are expressed algebraically.

Process Skills Error: There is no enough evidence to assess process skills comprehensively since the student stopped after creating the initial incorrect equations without executing any algebraic manipulations or solving for variables.

Encoding Error: The student does not finish the solution process or give any response to the given question. There is no effort to ascertain the two-digit number or compute the difference between the numbers as asked in the problem.

Summary of S4 response

Newman's evaluation of Student S4 uncovers noteworthy conceptual errors in problem solving. The student demonstrated comprehension error by mixing up the two-digit number with the total of its digits in the equation "$10x + y = 16$." The transformation skill exhibit considerable deficiencies when he mistakenly wrote "$x + 10y = 16$," uncovering basic misunderstandings of place value in two-digit numbers. Process skills cannot be assessed completely since the solution halts after these initial incorrect equations, with no algebraic manipulations made. The total lack of additional work signals an encoding error, since the student neither isolated variables nor identified the required values, indicating substantial misunderstandings about number representation that hindered any significant advancement in solving the problem. Some work with crossed-out sections indicates potential carelessness or doubt about how to approach the problem.

In analyzing the students' works through the Newman's Error Analysis framework, the solution process reveals multiple error types. While no reading errors were evident, a significant comprehension error occurred when the student failed to interpret the digit reversal condition properly. The major transformation error appeared when they incorrectly translated the mathematical relationship of adding 18 to get the reversed number, failing to set up the equation $10x + y + 18 = 10y + x$. Subsequently, numerous process skill errors emerged, including incorrect algebraic manipulations when simplifying expressions and solving for variables. Although the students S_2 and S_3 obtained y= -4, this reflects encoding errors as they didn't convert their solution back to answer the original questions about finding the two-digit number and calculating the difference between the original number and its reverse. These systematic errors in transformation and process skills prevented the student from reaching the correct solution (Mathaba et al., 2024; Pournara, 2020). In this regards Posner & Gertzog (1982), and Vosniadou and Verschaffel (2004) also emphasized that for effective learning a substantial reorganization of a person's mental structures instead of just incorporating updated information. Learners must perceive new concepts as clear, trustworthy, and advantageous providing greater explanatory strength than their previous beliefs.

Conclusion and Implications

The solution process uncovers various types of errors while students solve mathematical problems. The Newman's Error Analysis showed that although the student read the problem well, significant errors happened in comprehension, transformation, and process skills. A major confusion regarding the digit reversal requirement and inability to formulate the right equation resulted in considerable conversion errors. These factors, along with mistakes in algebra and encoding problems, ultimately hindered the student from reaching the right answer. This examination emphasizes the significance of focusing on particular cognitive levels in mathematical problem solving to enhance students' conceptual comprehension and procedural correctness. Further, the results suggest that the focused teaching strategies are necessary to enhance students' understanding and ability to transform in mathematical problem solving. Educators can utilize the error analysis frameworks such as the Newman's one to identify learning deficiencies and create specific support strategies that enhance students' reasoning and precision for effective mathematics learning.

References

Chi M. T. H. and Slotta J. D. (1993). The ontological coherence of intuitive physics, Cognition Instruct., 10, 249–260.

Creswell, J.W. and Creswell, J.D. (2018) *Research Design: Qualitative, Quantitative, and Mixed* https://mgcub.ac.in/pdf/journals/mgcujss_1-1_april-september_2019-32-43.pdf

Karnasih, I. (2015). Analisis Kesalahan Newman pada Soal Cerita Matematis. *Journal PARADIKMA*, 8(1), 37-51.

Lamon, S. J. (2007). Rational numbers and proportional reasoning: Toward a theoretical framework. In F. Lester (Ed.), Second handbook of research on mathematics teaching and learning (pp. 629-668). Charlotte, NC: Information Age Publishing. Mahatma Gandhi Central University *Journal of Social Sciences,* 1(1), 22-33.

Mathaba, P. N., Bayaga, A., Tîrnovan, D., & Bossé, M. J. (2024). Error analysis in algebra learning: Exploring misconceptions and cognitive levels. *Journal on Mathematics Education, 15*(2), 575-592. http://doi.org/10.22342/jme.v15i2.pp575-592. *Methods Approaches.* Sage, Los Angeles

Miles, M., Huberman, M., & Saldaña, J. (2016). Activity 64 Evaluating Qualitative Analyses. *100 Activities for Teaching Research Methods*.

Newman, M.A. (1977). An analysis of sixth-grade pupils' errors on written mathematical tasks. In M. A. C. J. Foyster (Ed.), Research in Mathematics Education in Australia (Vol. 2, pp. 269-287). Swinburne College Press.

Nik Azis, N.P. (2014). *Penghasilan desertasi berkualiti dalam pendidikan matematik* (1 ed.). Kuala Lumpur: Penerbit Univesiti Malaya.

Parakitipong, N. and Nakamura, S. (2006). Analysis of Mathematics Performance of Grade Five Students in Thailand Using Newman Procedure. *Journal of International Cooperation in Education,* 9 (1), 111 – 122, CICE Hiroshima University

Posner, G. J., & Gertzog, W. A. (1982). The Clinical Interview and the Measurement of Conceptual Change. *Science education*, *66*(2), 195-209.

Pournara, C. (2020).Grade 9 learners' algebra performance: Comparisons across quintiles, insights from errors and curriculum implications. *South African Journal of Science*, 116, 1-7. http://www.scielo.org.za/scielo.php?script=sci_arttext&pid=S0038-23532020000600011&nrm=iso

Tiwari, C., & Fatima, R. (2019). Secondary school students' misconceptions in algebra concept. *Mahatma Gandhi Central University Journal of Social Sciences MGCUJSS* , Volume-I, Issue 1, April-September, 2019

Vosniadou, S., & Verschaffel, L. (2004). Extending the conceptual change approach to mathematics learning and teaching. *Learning and Instruction, 14*(5), 445-451. https://doi.org/https://doi.org/10.1016/j.learninstruc.2004.06.014